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A NUMERICAL MODEL FOR QUASI-BRITTLE MATERIALS

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Summary: *Complex analysis of massive historical structures that takes into account all geometrical details is still not computationally feasible. Instead, either coupled or uncoupled multi-scale homogenization analysis is often performed whereas the latter one in particular has proved its potential when searching for a reliable estimate of the response of large, generally three-dimensional, structures. In such a case the macroscopic analysis is carried out independently such that the driving material parameters of the macroscopic constitutive model are found from a detailed numerical analysis on the mesoscale. This step constitutes the most important part of the uncoupled multi-scale approach and its success is highly influenced by the proper representation of the material response on the level of individual phases, bricks and mortar. An extension of the constitutive models currently implemented in the ATENA finite element code is proposed. Particular attention is paid to the synthesis of an orthotropic damage model for the description of tensile failure with a new constitutive model capable of representing the material failure in shear and confining pressure. Several simple problems including tension and hydrostatic compression are solved to test the model capability.*

1 Introduction

Masonry structures are typical representatives of material systems where individual phases are classified as quasi-brittle materials characterized by complex failure mechanisms. Such mechanisms are usually promoted by evolution of nonhomogeneous local stress and strain fields. Such an unpleasant scenario arises especially when dealing with historical masonry structures with typical irregular distribution stone blocks, e.g. Charles Bridge in Prague (Šejnoha *et al.*, 2005). Although powerful homogenization tools capable of handling relatively complex geometrical irregularities are now available (Zeman and Šejnoha, 2007), it is clear that a successful prediction of the macroscopic response of such structures is highly influenced by the proper representation of the material response on the level of individual phases, bricks and mortar.

A reliable material model with the ability of representing most of the failure mechanism, at least on the micro level (the level of individual phases), is therefore of paramount importance for an adequate prediction of the homogenized structural response. This subject is addressed in this contribution on the bases of classical combination of damage and plasticity material models employing the concept of effective stresses.

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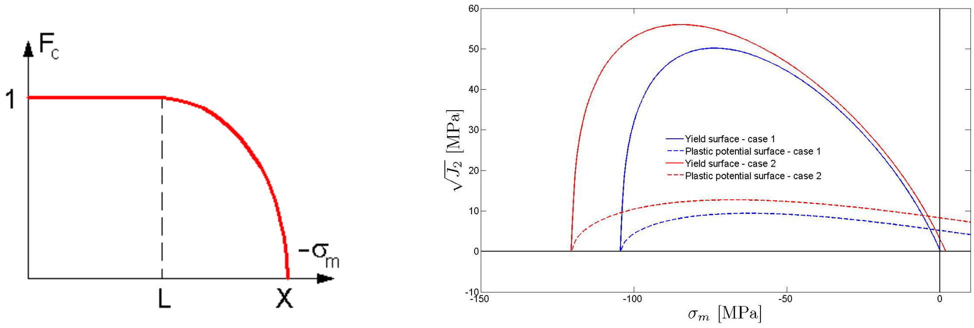


(a) (b)

Figure 1: (a) Dimensionless function representing the cap model, (b) Yield and plastic potential function in the deviatoric space

2 Model definition

Since the robust commercial software ATENA has been successfully used in the modeling of complex historical structures it appears natural to develop the theoretical formulation of the proposed model on the grounds of existing models already implemented in this code with the main goal of their improvements both from the modeling and implementation point of views. To that end, two major modifications - introduction of the cap model and representation of tensile failure through an orthotropic damage model - are introduced in the sequel.

2.1 Formulation of a yield smooth cap model for quasi-brittle materials

To begin with, recall the currently implemented representation of the shear failure in the ATENA finite element code (Červenka *et al.*, 2003) in the form of Menetrey-Willam yield surface

$$\begin{aligned} F_d(J_2, \sigma_m, \theta, c(\kappa_1)) &= \sqrt{J_2} + (\alpha J_2 + \sigma_m - \beta c(\kappa_1)) g(\theta, e) \\ &= \sqrt{J_2} + \alpha J_2 g(\theta, e) - F_d^c, g(\theta, e), \end{aligned} \tag{1}$$

where J_2, σ_m, θ are the 2^{nd} invariant of the deviatoric stress, mean effective stress and Lode angle, respectively. The model parameters α, β, c, e and the function $g(\theta, e)$ specifying the shape of the yield surface in the deviatoric plane are discussed in (Červenka *et al.*, 2003) and (Sýkora *et al.*, 2006).

To avoid an unrealistic prediction of the material response in hydrostatic compression the original formulation is enhanced by introducing a hardening cap, see (Schwer and Murray, 2002) and references therein,

$$F_c(J_2, \sigma_m, L, L(X(\kappa_2))) = \sqrt{J_2} - \frac{L - X}{R} \sqrt{F_c^c} g(\theta, e), \tag{2}$$

where the dimensionless function F_c^c, plotted in Figure 1, is provided by

$$F_c^c\left(\sigma_m, L(X(\kappa_2), R)\right) = 1 - \frac{[\sigma_m + L]\left[(\sigma_m + L) - |\sigma_m + L|\right]}{2\left(X - L\right)^2}. \tag{3}$$

The hardening cap parameters L, X defining the σ_m range of the cap are related by

$$X = L + R F_d^c(L) = L + R(L + \beta c) \Rightarrow L = \frac{X - R\beta c}{1 + R}, \tag{4}$$

where R is an additional material parameter that determines the ellipticity of the yield cap (geometrically given as the ratio of the horizontal and vertical axes of the elliptical cap). Evolution of the cap is again assumed in the strain-hardening format with κ_2 equal to the current volumetric plastic strain ε_v^{pl} and given by Schwer and Murray (2002)

$$X = X_0 - \frac{1}{D_1} \ln \left(1 + \frac{\varepsilon_v^{pl}}{W} \right), \tag{5}$$

where W is the maximum plastic volumetric strain (at hydrostatic compression 'lockup'), X_0 is the initial abscissa intercept of the cap surface and D_1 is a shape factor. Further note that L represents the absolute value of the mean effective stress at the point of interception of the two yield surfaces F_d and F_c. Only the hardening response of the material in compression is accepted so whenever $L < L_0(X_0)$ we set $L = L_0$ and likewise for X.

To describe the cap model requires identification of four parameters. These parameters can be obtain from a hydrostatic compressive test (X_0, D_1, W) and triaxial compressive tests (R) as offered in (Zaman *et al.*, 1982) taking into account the coupling effect between plasticity and damage.

Proceeding in the footsteps of Schwer and Murray (2002) the two functions, Eqs. (1) and (2) can be combined into a smooth cap model represented by a smoothly varying (continuous derivative) function in the form

$$F(J_2, \sigma_m, \theta, c(\kappa_1), L(X(\kappa_2)), R) = \sqrt{J_2} + \alpha J_2 g(\theta, e) - F_d^c \sqrt{F_c^c} g(\theta, e). \tag{6}$$

Projections of the proposed yield surface into deviatoric and meridian planes appear in Figure 1(b).

Providing the material point is loaded beyond the elastic regime a fully implicit integration scheme devised in (Sýkora *et al.*, 2006) can be used to bring the stresses found outside the yield surface back. The present formulation relies on a non-associated flow rule

$$\Delta \boldsymbol{\varepsilon}^{pl} = \Delta\lambda \frac{\partial G}{\partial \boldsymbol{\sigma}}, \qquad G = \sqrt{J_2} + (\sigma_m - C_{OR})\, M_{JP} \sqrt{F_c^c}, \tag{7}$$

where G represents the plastic potential surface independent of the Lode angle θ. The parameter C_{OR} is introduced to match both the yield and plastic potential surface for the current stress state, see Fig. 1(b). Although simple the proposed plastic potential function is capable of representing both plastic dilation and compression behavior together with a critical state the material will experience at a certain stage of loading accompanied by zero increment of the volumetric plastic strain.

2.2 Formulation of a damage law

Quasi-brittle materials are prone to progressive loss of material integrity due to propagation and coalescence of microcracks manifested by a degradation of material stiffness on the macroscale. This phenomenon can be well described by continuum damage mechanics.

In this section an orthotropic damage model developed by (Papa and Taliercio, 1996) is briefly outlined. We begin with the definition of free energy written in the principal coordinate system as

$$\rho\psi = \frac{1}{2}(1-d)\, K_0 \varepsilon_v^2 + G \boldsymbol{e}^{el} (\mathbf{I} - \mathbf{D})\, \boldsymbol{e}^{el}, \tag{8}$$

where ε_v is the volumetric strain and $\boldsymbol{e}^{el} = \left\{e_1^{el}, e_2^{el}, e_3^{el}\right\}^T$ lists the deviatoric components of the strain vector; $\mathbf{D}$ is the diagonal second order damage tensor and d stands for the scalar volumetric damage variable if loading the material in tension. Volumetric damage in compression is neglected. Finally, K_0, G_0 are the bulk and shear moduli of an undamaged material, respectively. The stress-strain relation written in the principal coordinate system then reads

$$\boldsymbol{\sigma} = \frac{\partial \rho\psi}{\partial \boldsymbol{\varepsilon}^{el}} = (1-d)\, K_0 \varepsilon_v + 2G\, (\mathbf{I} - \mathbf{D})\, \boldsymbol{e}^{el}, \tag{9}$$

where $\mathbf{P}$ is an auxiliary matrix that relates the deviatoric and Cartesian strains. Differentiating Eq. (8) with respect to the damage variables yields the damage driving forces conjugate to D and d, respectively:

$$\mathbf{Y} = -\frac{\partial \rho\psi}{\partial \mathbf{D}} = G_0 \boldsymbol{e}^{el^T} \mathbf{I} \boldsymbol{e}^{el}, \qquad y = -\frac{\partial \rho\psi}{\partial d} = \frac{1}{2} K_0 \varepsilon_v^{el^2}. \tag{10}$$

In the spirit of (Mazars, 1986) the overall strain and correspondingly also the driving forces and associated damage tensors are split into tensile and compressive parts as

$$\boldsymbol{e}^{el} = \boldsymbol{e}^+ + \boldsymbol{e}^-, \qquad \mathbf{Y} = \mathbf{Y}^+ + \mathbf{Y}^-, \qquad \mathbf{D} = \mathbf{D}^+ + \mathbf{D}^-. \tag{11}$$

The evolution law for the principal values of the damage tensor follow from the consistency condition in a manner much similar to classical plasticity. Here the "gauge functions" substituting the yield function assume the forms, see (Papa and Taliercio, 1996) for original formulation,

$$\begin{aligned} f_\alpha^h &= (1 - D_\alpha^h)(1 + A^h(\bar{Y}_\alpha^h - Y_0^h)^{B^h}) - 1 \leq 0, \quad \left\{ \begin{array}{l} \alpha = 1,\, 2,\, 3 \\ h = \text{"+"} \quad \text{or} \quad h = \text{"–"} \end{array} \right. \\ f_d &= (1 - d)(1 + a(\bar{y} - y_0)^b) - 1 \leq 0, \end{aligned} \tag{12}$$

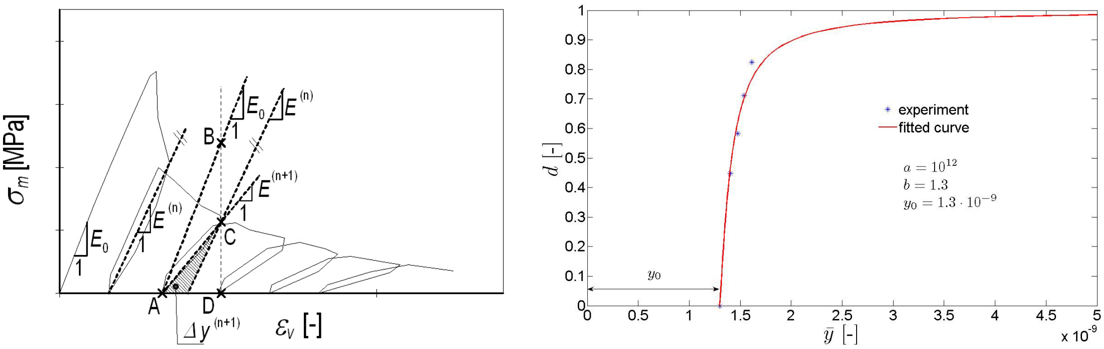
where $A^+, B^+, A^-, B^-, a \geq 0$ and $b \geq 1$ are non-dimensional material parameters governing the brittle damage evolution law. Y_0^+, Y_0^- and y_0 represent a thresholds for a non-dimensional damage forces $\bar{\mathbf{Y}} = \mathbf{Y}/E_0$ and $\bar{y} = y/E_0$.

2.3 Identification of damage model parameters

Since rather detailed description for the evaluation of the damage model parameters A^+, B^+, A^-, B^-, Y_0^+ and Y_0^- is provided in (Papa and Taliercio, 1996), we limit our attention to the volumetric parameters only. In particular, the model parameters a, b and y_0 can be derived from a uniaxial cyclic tensile test. The variation of nominal mean stress given by Eq. (9) depends only on the volumetric damage parameter d. Suppose that a typical stress-strain relationship, Figure 2(a), is available. Then, the damage force y and the damage parameter d are provided by

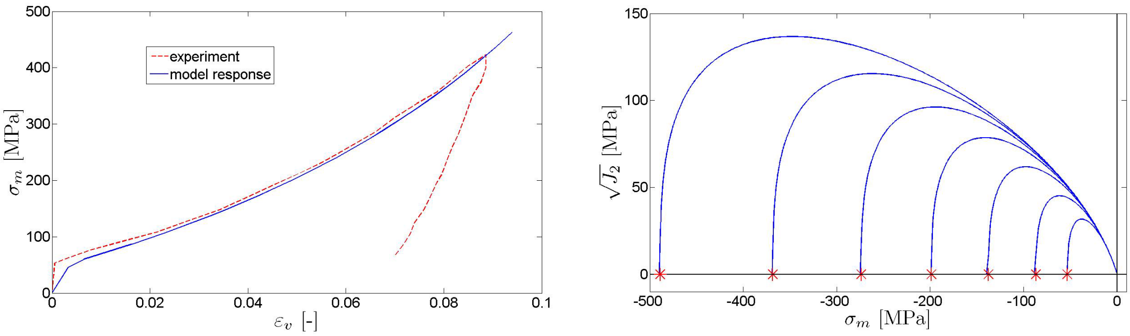
$$y^{(n+1)} = y^{(n)} + \Delta y^{(n+1)}, \qquad d^{(n+1)} = \frac{S_{ABD}}{S_{ABC}}, \tag{13}$$

where S denotes the relevant area and Δy a damage force increment. The unknown volumetric parameters are subsequently computed by means of the least square method from Eq. (12).



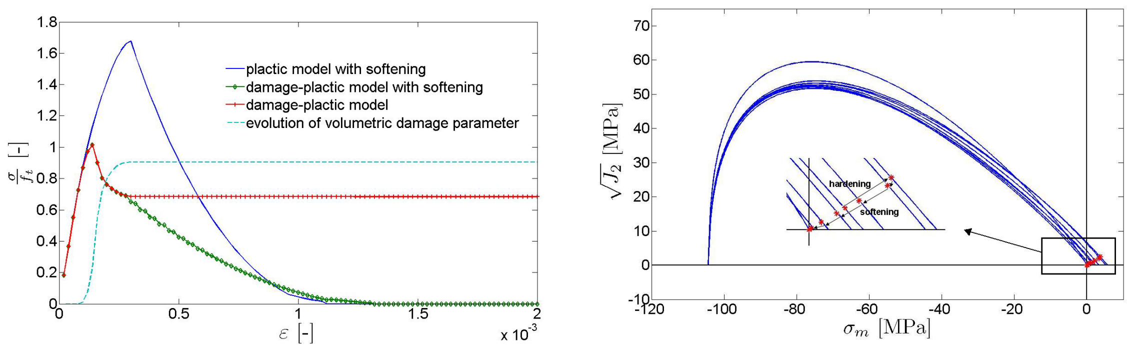
(a) (b)

Figure 2: Volumetric damage parameters: (a) stress-strain diagram, (b) evolution of damage parameter



(a) (b)

Figure 3: Hydrostatic compression test: (a) stress-strain diagram, (b) evolution of yield surface



(a) (b)

Figure 4: Tension test: (a) stress-strain diagram, (b) evolution of yield surface

3 Examples and conclusions

Two simple examples are presented for illustration. Figure 3 shows the ability of the model to represent rather well the classical hydrostatic compression test when employing the hardening law of the cap model. Figure 4 then displays the qualitative behavior of the plastic damage model when running the uniaxial tension test. Clearly, since the evolution of damage parameter (here the response is driven by the volumetric damage parameter d) is based on the elastic strains, it is inevitable to include softening in the evolution plastic yield surfaces to arrive at meaningful results. As evident, neither softening plasticity nor plastic damage model without softening provides acceptable results.

4 Acknowledgments

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