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PERFORMANCE-BASED DESIGN APPLIED FOR A SHAFT SUBJECTED TO COMBINED STRESS

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Summary: *"Performance-Based Design" (PBD) is based on the theory of probability which is connected with statistics. PBD is also based on the performance requirements which are usually defined as a synthesis of functionality, all-in cost, safety etc. Performance requirements can be expressed as an acceptable level of damage, which is defined by acceptable probability of possible failure. In the following example is used SBRA method (Simulation-Based Reliability Assessment Method, direct Monte Carlo Method, AntHill software). A shaft of unknown circular shape is exposed to bending moment, normal force and torque, which are given by truncated histograms. Yield limit of the material is also given by truncated histogram. The task is to calculate the nominal value of diameter which is given by normal truncated distribution* $\pm 1\%$. *The acceptable level of damage is related to the yield limit. The calculation of the diameter, with given acceptable probability of damage level (i.e. solution of the inverse problem of theory of probability), must be solved via iterative approaches (secant method, bisection method etc.). Hence, to get the solution of this type of inverse problem is much difficult than the solution of the classical problem of theory of probability.*

Introduction

In the last several decades, the science and engineering community has progressively ventured outside of traditional boundaries in terms of materials, loads, configurations etc. for structural systems in mechanics. Consequently, a new designer's approach called Performance-Based Design (PBD) can be defined as: "Design specifically intended to limit the consequences of one or more perils to defined acceptable

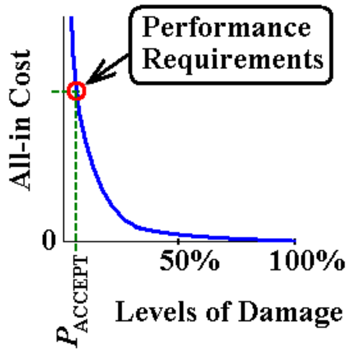

Fig.1

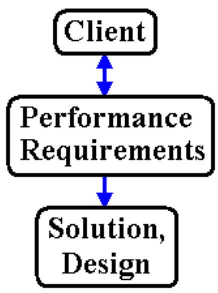

Fig.2 Structure of PBD.

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levels". PBD is based on the theory of probability and depends on many inter-connected issues including classification of constructed systems, definition of performance, tools for measuring performance, quantitative indices that may serve as assurance of performance, and especially, how to describe and measure performance especially under various levels of uncertainty which is connected with statistics. However, comprehensive approach of PBD is still in its infancy.

PBD is based on performance requirements which are usually defined as a synthesis of functionality, all-in cost, safety etc., see Fig.1 and 2. Performance requirements can be expressed as an acceptable level of damage, which is defined by acceptable probability of possible failure P_{ACCEPT}. For more details see reference (Hamburger, 1999).

Solved Example - PBD Applied For a Shaft Subjected to Combined Stress

In the following example is used SBRA method (Simulation-Based Reliability Assessment, direct Monte-Carlo method, AntHill software), see (Marek & Guštar & Anagnos, 1999; Marek & Brozzetti & Guštar, 2003).

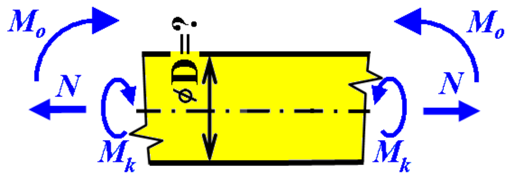


Fig.3 - Shaft Subjected to Combined Stress.

A shaft of unknown circular shape (see Fig.3), is exposed to bending moment $M_o = 581515.3^{+802256.6}_{-344596.4}$ Nmm, normal force $N = 157493.7^{+217277.8}_{-93328.2}$ N and torque $M_k = 367485.4^{+506981.6}_{-217765.8}$ Nmm, which are given by truncated histograms, see Fig.4 to 6. Yield limit of material is $R_e = 338.3^{+161.7}_{-90.3}$ MPa, see truncated histogram in Fig.7. Calculate the value of diameter D which is given by normal truncated distribution $\pm 1\%$ (i.e. $\mathrm{D}^{+1\%}_{-1\%}$) with accuracy 0.1 mm. The acceptable level of damage is $P_{\mathrm{ACCEPT}} = 0.0005 = 0.05\%$ (standard reliability level) is related to yield limit. In other words, 0.05% of all loading states can result in yielding.

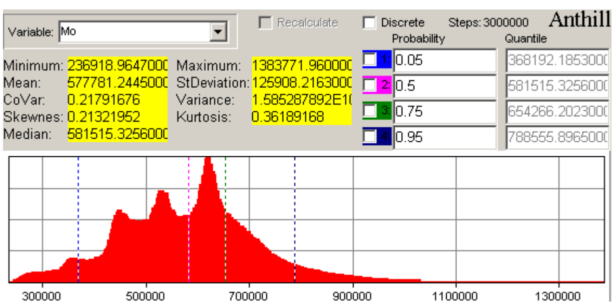


Fig.4 - Histogram of Bending Moment M_o.

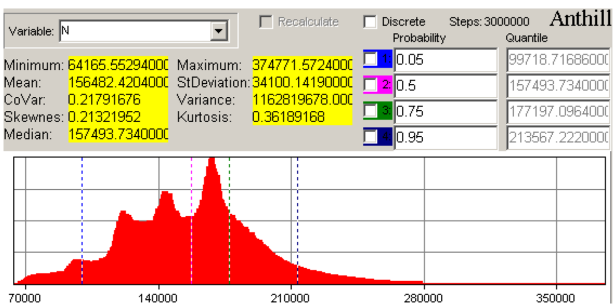


Fig.5 - Histogram of Normal Force N.

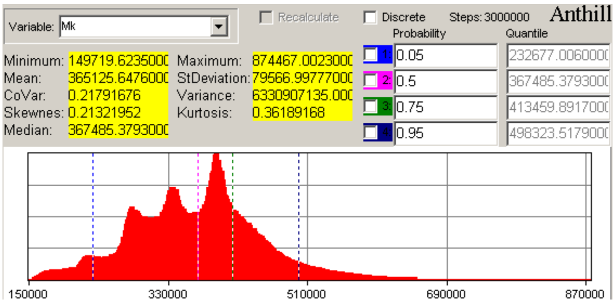


Fig.6 - Histogram of Torque M_k.

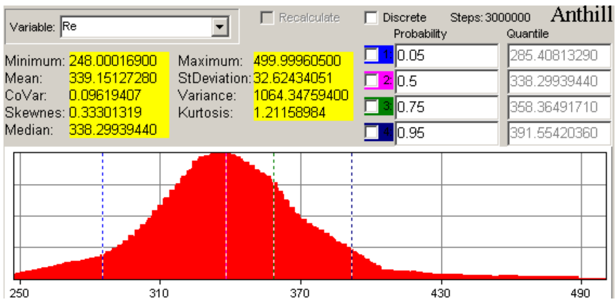


Fig.7 - Histogram of yield limit R_e.

According to the theory of small deformations (Frydrýšek & Adámková, 2007) can be written:

$$\sigma = \frac{N}{\frac{\pi \mathrm{D}^2}{4}} + \frac{M_o}{\frac{\pi \mathrm{D}^3}{32}} = \frac{4}{\pi \mathrm{D}^2}\left(N + \frac{8M_o}{\mathrm{D}}\right), \quad \tau = \frac{16M_k}{\pi \mathrm{D}^3}, \tag{1}$$

where σ /MPa/ is maximal normal stress and τ /MPa/ is maximal shear stress.

Hence, for equivalent von Mises stress σ_{HMH} /MPa/ can be written:

$$\sigma_{\mathrm{HMH}} = \sqrt{\sigma^2 + 3\tau^2} = \frac{4}{\pi \mathrm{D}^2}\sqrt{\left(N + \frac{8M_o}{\mathrm{D}}\right)^2 + \frac{48M_k^2}{\mathrm{D}^2}}. \tag{2}$$

Factor of safety (i.e. probability of situations when $R_e > \sigma_{\mathrm{HMH}}$) is defined as:

$$FS = P(\sigma_{\mathrm{HMH}} - R_e < 0), \tag{3}$$

where operator "P" means probability.

Hence, when $FS \geq 0$, it is evident that yield limit is not reached (i.e. in the shaft are not any plastic deformations).

The goal is to calculate diameter D which satisfy condition:

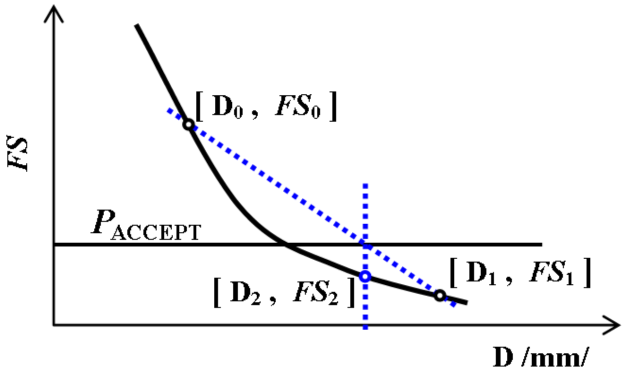
$$FS \leq P_{\mathrm{ACCEPT}}. \tag{4}$$

However, it is necessary to applied iteration methods (because from eq. (2) is not possible to express directly the unknown parameter D). Hence, iteration loop with application of secant method can be used, see Fig.8.

For chosen initial conditions (diameters): $\mathrm{D}_0 = 30 \pm 0.3$ mm a $\mathrm{D}_1 = 48 \pm 0.48$ mm (truncated normal distributions $\pm 1\%$) is possible to calculate (via SBRA method for 10^6 Monte Carlo simulations) the values of FS_0 and FS_1:

$$\mathrm{D}_0 = 30 \pm 0.3 \text{ mm}, \quad FS_0 = 0.869929 \geq P_{\mathrm{ACCEPT}},$$

$$\mathrm{D}_1 = 48 \pm 0.48 \text{ mm}, \quad FS_1 = 0.000032 \leq P_{\mathrm{ACCEPT}}.$$



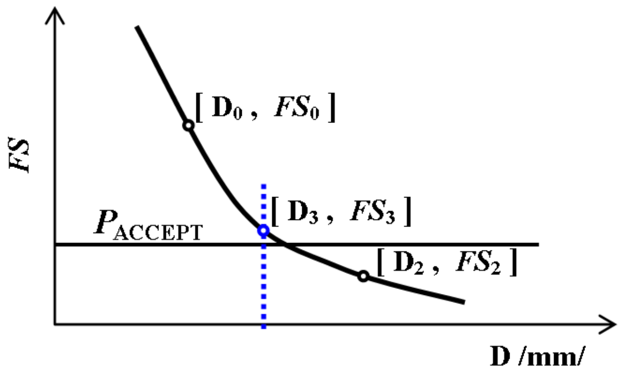


Fig.8 - Secant Method (Calculation of D_2). Fig.9 - Bisection Method (Calculation of D_3).

Hence, $FS_0 \geq P_{\text{ACCEPT}}$ and $FS_1 \leq P_{\text{ACCEPT}}$. It is evident that the required diameter D must be in interval: $D \in (D_0\,; D_1) = (30 \pm 0.3;\, 48 \pm 0.48)$ mm.

From Fig.8, can be derived new approximation of diameter (i.e. D_2) via secant method:

$$D_2 = f(P_{\text{ACCEPT}}) = \frac{D_1(P_{\text{ACCEPT}} - FS_0) + D_0(FS_1 - P_{\text{ACCEPT}})}{FS_1 - FS_0} = 47.9 \pm 0.48 \text{ mm} .$$

From the results of AntHill software follows:

$$D_2 = 47.9 \pm 0.48 \text{ mm}, \quad FS_2 = 0.000035 \leq P_{\text{ACCEPT}} ,$$

$$D \in (D_0\,; D_2) = (30 \pm 0.3;\, 47.9 \pm 0.48) \text{ mm} .$$

Next approximation of D (i.e. D_3) can be also calculated via bisection method, see Fig.9. Hence:

$$D_3 = \frac{D_0 + D_2}{2} = 38.95 \pm 0.39 \text{ mm} , \quad FS_3 = 0.048731 \geq P_{\text{ACCEPT}} ,$$

$$D \in (D_3\,; D_2) = (38.95 \pm 0.39;\, 47.90 \pm 0.48) \text{ mm} .$$

Next applications of bisection method give:

$$D_4 = \frac{D_3 + D_2}{2} = 43.42 \pm 0.43 \text{ mm} , \quad FS_4 = 0.002057 \geq P_{\text{ACCEPT}}$$

$$D \in (D_4\,; D_2) = (43.42 \pm 0.43;\, 47.90 \pm 0.48) \text{ mm} ,$$

$$D_5 = \frac{D_4 + D_2}{2} = 45.66 \pm 0.46 \text{ mm} , \quad FS_5 = 0.000292 \leq P_{\text{ACCEPT}} ,$$

$$D \in (D_4\,; D_5) = (43.42 \pm 0.43;\, 45.66 \pm 0.46) \text{ mm} ,$$

$$D_6 = \frac{D_4 + D_5}{2} = 44.54 \pm 0.45 \text{ mm} , \quad FS_6 = 0.000820 \geq P_{\text{ACCEPT}} ,$$

$$D \in (D_6\,; D_5) = (44.54 \pm 0.45;\, 45.66 \pm 0.46) \text{ mm} ,$$

Because the values of FS_5 and FS_6 are very close to given value of P_{ACCEPT}, it is wise to increase the number of Monte Carlo simulations to 3×10^6. Hence, the calculated values of FS will be more accurate.

Next application of bisection method gives:

$$\mathrm{D}_7=\frac{\mathrm{D}_6+\mathrm{D}_5}{2}=45.1\pm0.45\ \text{mm}\ ,\quad FS_7=0.000499\le P_{\text{ACCEPT}}\ ,$$

$$\mathrm{D}\in(\mathrm{D}_6\,;\mathrm{D}_7)=(44.54\pm0.45;\,45.1\pm0.45)\ \text{mm}\,.$$

Next application of secant method gives:

$$\mathrm{D}_8=f(P_{\text{ACCEPT}})=\frac{\mathrm{D}_7(P_{\text{ACCEPT}}-FS_6)+\mathrm{D}_6(FS_7-P_{\text{ACCEPT}})}{FS_1-FS_6}=45.098\pm0.45\ \text{mm}\ ,$$

$$FS_8=0.000544\ge P_{\text{DOV}}\ ,\quad \mathrm{D}_8=45\pm0.45\ \text{mm}\ ,$$

$$\mathrm{D}\in(\mathrm{D}_8\,;\mathrm{D}_7)=(45.0\pm0.45;\,45.1\pm0.45)\ \text{mm}\ .$$

Because $\mathrm{D}_8\cong\mathrm{D}_7$ and $FS_7\cong P_{\text{ACCEPT}}$. (with defined accuracy 0.1 mm), the diameter is:

$$\underline{\underline{\mathrm{D}=\mathrm{D}_7=45.1\pm0.45\ \text{mm}}}\ ,$$

see histograms shown in Fig.10 and 11.

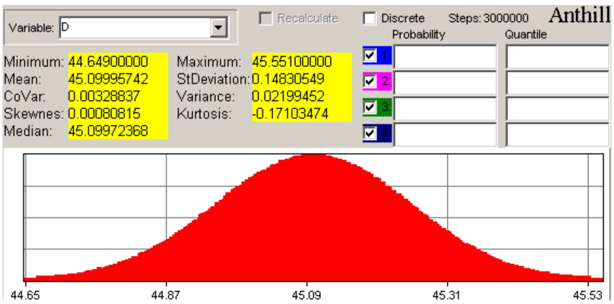


Fig.10 - Calculated Diameter.

$\mathrm{D}=45.1\pm0.45$ mm

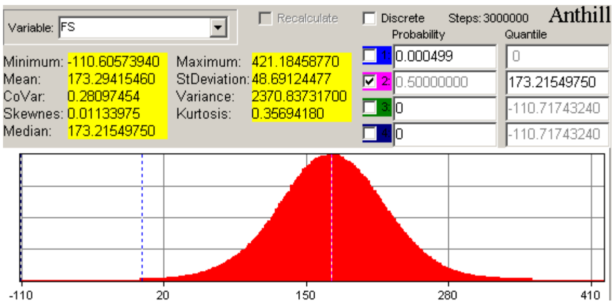


Fig.11 - Factor of Safety FS

Calculated for $\mathrm{D}=45.1\pm0.45$ mm.

Hence the diameter $\mathrm{D}=45.1\pm0.45$ mm is calculated with the acceptable level of damage $P_{\text{ACCEPT}}=0.0005=0.05\%$. In other words, 0.05% of all states will result in yielding.

The results of the presented iteration loop as a function $\mathrm{D}=f(FS)$ is shown in Fig.12.

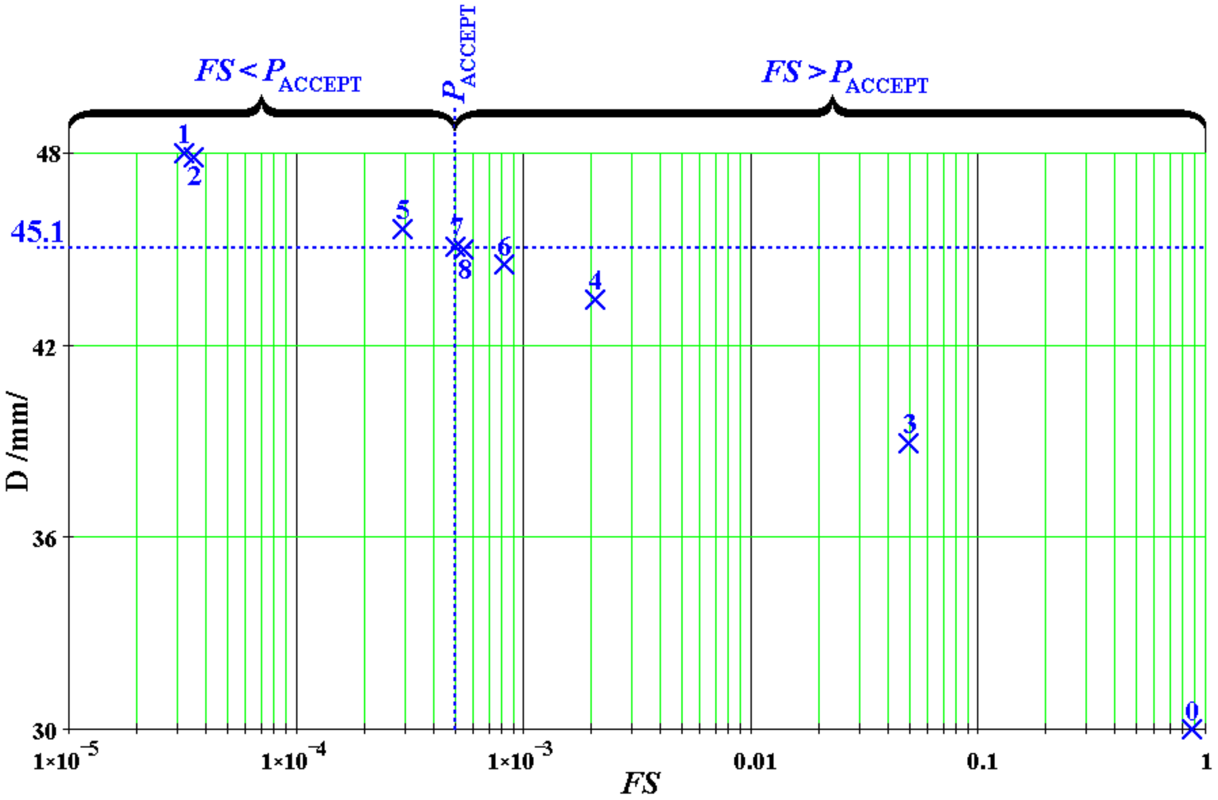


Fig.12 Diagram of Convergence for Calculated Diameter D (Results of Iterative Procedures).

Histogram of calculated stress $\sigma_{\mathrm{HMH}} = 166.89^{+259.90}_{-99.31}$ MPa and 2D histogram of $R_e = f\left(\sigma_{\mathrm{HMH}}\right)$ are presented in Fig.13 and 14.

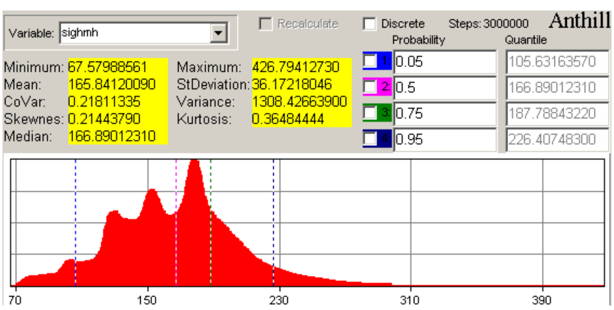


Fig.13 - Maximum Stress σ_{HMH} Histogram

Calculated for $\mathrm{D} = 45.1 \pm 0.45$ mm.

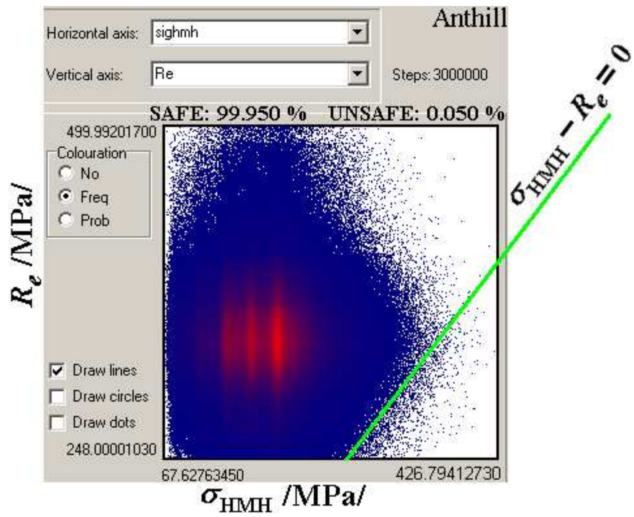


Fig.14 - 2D Histogram $R_e = f\left(\sigma_{\mathrm{HMH}}\right)$

Calculated for $\mathrm{D} = 45.1 \pm 0.45$ mm.

Conclusions

Performance-Based Design as a new and modern trend in mechanics is based on theory of probability and stochastic methods.

The calculation of the diameter D, with given acceptable probability of damage level P_{ACCEPT} (i.e. solution of the inverse problem of theory of probability), is solved via iterative approaches (secant method, bisection method). Hence, to get the solution of this type of inverse problem is much difficult than the classical problem of theory of probability.

Instead of secant method or bisection method can be used also another methods such as Regula-Falsi Method etc.

The whole iterative procedures and Monte Carlo simulations can be speed-up by application of parallel computers. However, on the present days, it is impossible to solve the large problems of mechanics via PBD. The reason of this is the low rate of present-day computers.

Acknowledgement

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References

[1] Hamburger, R., O.: *The Challenge of Performance-Based Design*, http://peer.berkeley.edu/news/1999jan/challenge.html.

[2] Marek, P., Guštar, M., Anagnos, T., *Simulation-Based Reliability Assessment for Structural Engineers*, CRC Press, Inc., Boca Raton, Florida, USA, 1995, pp.365.

[3] Marek, P., Brozzetti, J., Guštar M., *Probabilistic Assessment of Structures Using Monte Carlo Simulation Background, Exercises and Software*, ITAM CAS, Prague, 2003, Czech Republic, pp.471.

[4] Frydrýšek, K., Adámková, L.: *Mechanics of Materials 1 (Introduction, Simple Stress and Strain, Basic of Bending)*, Faculty of Mechanical Engineering, VŠB-Technical University of Ostrava, Ostrava, ISBN 978-80-248-1550-3, Ostrava, 2007, Czech Republic, pp. 179.