National Conference with International Participation

ENGINEERING MECHANICS 2008

Svratka, Czech Republic, May 12 – 15, 2008

NEW CONCEPTION OF OMNIDIRECTIONAL MOBILE ROBOT WITH MECANUM WHEELS: KINEMATICS AND DYNAMICS MODELLING

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Summary: *This contribution summarizes main results concerned with a design of a mobile robot with omnidirectional wheels and the key area covered in this work is kinematics and dynamics modelling of the robot platform. In the introductory part, there are presented main features of omnidirectional wheels and a new conception of a Mecanum-type wheel is introduced. Next sections are focused on a mathematical description – kinematical and dynamical model is then followed by a state-space model of the robot platform. Based on the mathematical description, a simulation model in Matlab/Simulink was created. The simulation results are shown and discussed in the final part of this work.*
Results in this work were acquired using the subsidization of the Ministry of Education Youth and Sports of the Czech Republic research plan MSM 0021630518 „Simulation modelling of mechatronic systems".

1. Introduction

This contribution deals with an omnidirectional mobile robot where the omnidirectional (also holonomic) movement is reached by means of omnidirectional wheels. The robot platform is capable of translating in both directions and rotating about its center of gravity. A practical benefit of this type of robots can be seen in providing very good mobility, especially in areas covered with static or dynamic obstacles, such as offices, workshops, warehouses and hospitals or for industrial applications such as surveillance, inspection and transportation tasks. One of the aims of the project will also be to verify how the omnidirectional capability may greatly reduce the amount of area and time required for maneuvers.

The content of the contribution is based on (Knoflíček, et al. 2006), though there has been completely redesigned the whole robot platform. Based on some simulation experiments, practical results and a feedback from the RAAD 2007 conference (Kubela & Pochylý, 2007), we decided to change the conception of the robot and instead of using standard and

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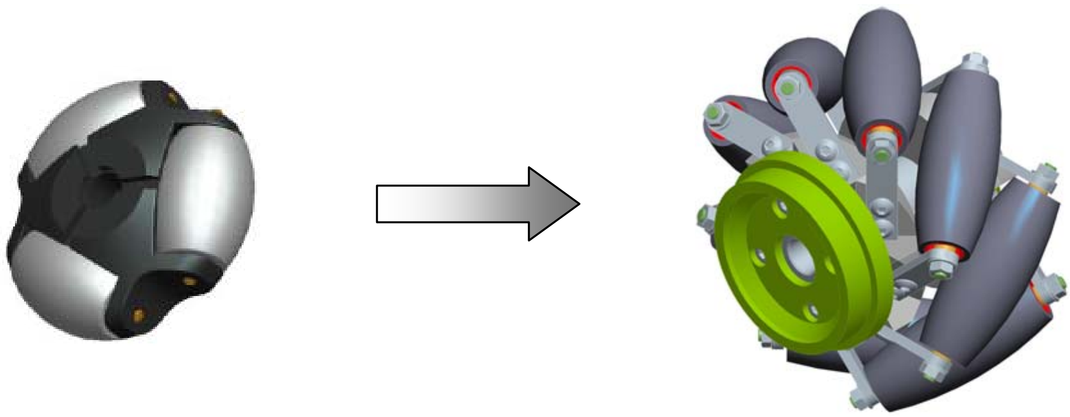
commercially available Interroll wheels (Fig. 1a), we newly rely on a new conception of Mecanum wheels of our own design structure (Fig. 1b).



a) b)

Fig. 1: Interrol wheel – former conception (a), Mecanum wheel – new conception (b)

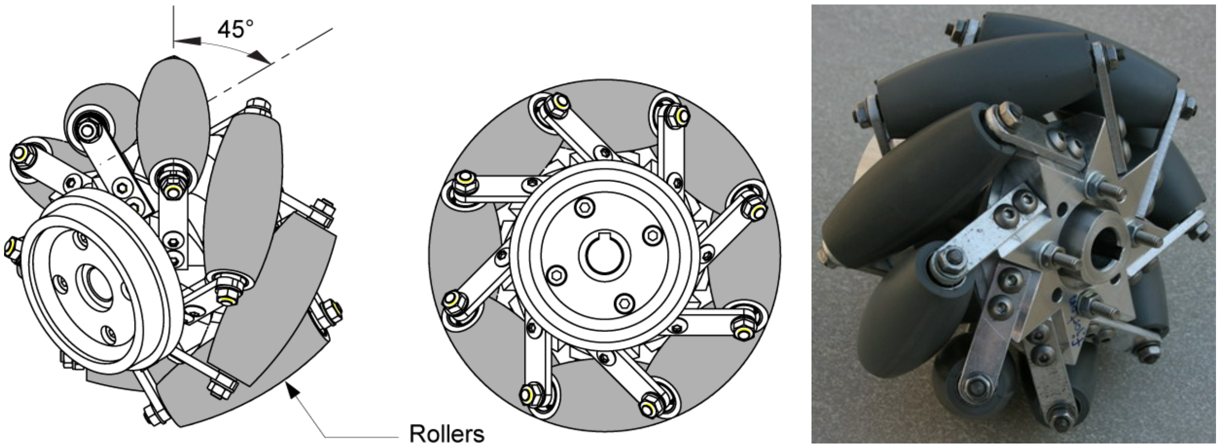
However, the Mecanum wheel itself is not a new conception but it was firstly introduced by Bengt Ilon in 1973. Another motivation to change the conception was the fact that the Interroll wheels were manufactured with poor quality, big clearance and other geometric inaccuracies; nevertheless, it was not clear from the technical documentation only. All the reasons stated above led to a search for a "new" wheel conception.

The advantages of Mecanum wheels are fully covered e.g. in the following works (Diegel, et al., 2002; Haendel, 2005; Kuppuswamy, et al., 2006; Woods, 2006). The following table (Tab. 1) represents a brief comparison of omnidirectional wheels we considered to use in this project. Based on our cooperation with "Mostly Harmless" RoboCup Team at the Graz University of Technology, we used the Mecanum wheels (Fig. 2) for the new conception of the robot (the wheel design is based on their intellectual property).

Tab. 1: Omnidirectional wheels comparison

Omniwheel (Interroll wheel)	+ low weight, compact design + simple mechanical design + commercially available	- discontinuous wheel contact - variable drive-radius (2 rows); complication for control and odometry – inaccuracies - poor quality of the wheel - sensitivity to floor irregularities - low payload
Mecanum wheel	+ compact + more robust, precise and reliable + heavy payload + smoother transfer of contact surfaces.)	- discontinuous wheel contact - sensitivity to floor irregularities - complex wheel design - wheel weight - less efficient use of the kinetic energy supplied to the wheels by the motors (higher power demands)

For the new conception of the robot platform we made a new kinematical model of the whole platform based on Mecanum paradigm and a more advanced dynamical model of the platform (all the constants used in the model are described in Tab. 2 at the end of the following chapter). In order to make a compact robot design, each Mecanum wheel is oriented such that the axes of wheel rollers (each at the top) are perpendicular to the respective axes towards the centre of rotation of the whole platform (Fig. 3).



a) b)

Fig. 2: Mecanum wheel; a) schematic wheel description, b) assembly of the wheel

2. Vehicle kinematics and dynamics

2.1. Kinematical model

Defining the velocity vector of the robot in the local coordinate system and the velocity vector of each motor:

$$u^* = \begin{pmatrix} v_x & v_y & \omega_r \end{pmatrix}^T \tag{1}$$

$$\phi = \begin{pmatrix} \omega_1 & \omega_2 & \omega_3 \end{pmatrix}^T \tag{2}$$

then the relation among the velocities is as follows; it can be considered as an inverse kinematics problem:

$$\phi = \frac{1}{R} \cdot \mathbf{G} \cdot u^* \tag{3}$$

where

$$\mathbf{G} = \begin{bmatrix} \sin\xi_1 - \cos\xi_1 & -\sin\xi_1 - \cos\xi_1 & -\sqrt{2} \cdot L \\ \sin\xi_2 - \cos\xi_2 & -\sin\xi_2 - \cos\xi_2 & -\sqrt{2} \cdot L \\ \sin\xi_3 - \cos\xi_3 & -\sin\xi_3 - \cos\xi_3 & -\sqrt{2} \cdot L \end{bmatrix} \tag{4}$$

The robot's wheels are positioned in a symmetrical way relative to the centre of gravity of the platform. The best omnidirectional performance will be reached if the wheels make an angle of 120°. The connecting lines between centre of gravity of the platform and each wheel make the following angles (in the local coordinate system of the robot) (Fig. 3):

$$\vartheta = \begin{pmatrix} 30° & 150° & 270° \end{pmatrix}^T \tag{5}$$

Mecanum wheel is based on the principle of a central wheel with a number of rollers (in our case there are 8 rollers) placed at an angle around the periphery of the wheel; this angle is 45° (Fig. 2). The ξ angles in matrix **G,** equation (4), are then as follows:

$$\xi = \vartheta + \gamma \tag{6}$$

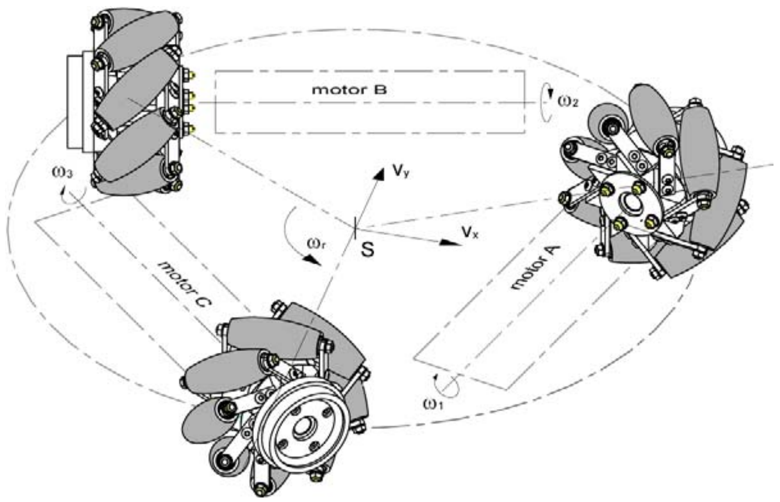


Fig. 3: Mecanum wheels arrangement on the robot platform

2.2. Dynamical model

The dynamical model of the robot is based on using the Lagrange equations of the second order:

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_j}\right) - \left(\frac{\partial E_k}{\partial q_j}\right) = Q_j \tag{7}$$

where the total kinetic energy of the system is described as follows:

$$E_k = 3 \cdot E_{ku} + E_{kx} + E_{ky} + E_{kR} \tag{8}$$

where

$$E_{ku} = \frac{1}{2}\omega_i^2\left(J_M + \frac{J_w + J_g}{n^2}\right) \tag{9}$$

$$E_{kx} = \frac{1}{2}m_R v_x^2 \tag{10}$$

$$E_{ky} = \frac{1}{2}m_R v_y^2 \tag{11}$$

$$E_{kR} = \frac{1}{2}J_R \omega_r^2 \tag{12}$$

The relation among velocities of the platform and velocities of the wheels is based on the equation (3) and it is described by the following equation; in this case the forward kinematics problem:

$$u^* = \frac{R}{n^2} \cdot \mathbf{G}^{-1} \cdot \phi \tag{13}$$

The motion equation for the vector ϕ is as follows:

$$\mathbf{J}\dot{\phi} = \mathbf{M} \tag{14}$$

where

$$J_{1,1} = J_{2,2} = J_{3,3} = J_M + \frac{J_w + J_g}{n^2} + \frac{4m_R R^2}{18n^2} + \frac{J_R R^2}{18L^2 n^2} \tag{15}$$

$$J_{1,2} = J_{2,1} = J_{3,2} = \frac{J_R R^2}{18L^2 n^2} - \frac{2m_R R^2}{18n^2} \tag{16}$$

$$J_{1,3} = J_{2,3} = J_{3,1} = \frac{2m_R R^2}{18n^2} - \frac{J_R R^2}{18L^2 n^2} \tag{17}$$

2.3. State-space model

Based on the standard description of a DC motor (description of the constants is in Tab. 2):

$$L_A \dot{i} + R_A i + \frac{\omega}{k_n} = u_n , \tag{18}$$

$$J_M \dot{\omega} + b\omega + \frac{M}{n} = k_m i \, . \tag{19}$$

The state-space model of the whole platform can be further described as follows:

$$\begin{matrix} \mathbf{H}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \end{matrix} \quad \text{or} \quad \begin{matrix} \dot{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{A}\mathbf{x} + \mathbf{H}^{-1}\mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \end{matrix} , \tag{20}$$

where (**u** – vector of inputs, **x** – vector of internal states)

$$\mathbf{u} = \begin{pmatrix} M_{w1} & M_{w2} & M_{w3} & u_{n1} & u_{n2} & u_{n3} \end{pmatrix}^T \tag{21}$$

and

$$\mathbf{x} = \begin{pmatrix} \omega_1 & \omega_2 & \omega_3 & i_1 & i_2 & i_3 \end{pmatrix}^T . \tag{22}$$

Further

$$\mathbf{H} = \begin{bmatrix} \mathbf{J} & 0 \\ 0 & \mathbf{L}_A \end{bmatrix}, \ \mathbf{L}_A = \begin{bmatrix} L_A & 0 & 0 \\ 0 & L_A & 0 \\ 0 & 0 & L_A \end{bmatrix}, \tag{23}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{b} & \mathbf{k}_m \\ \mathbf{k}_n & \mathbf{R}_A \end{bmatrix} \tag{24}$$

$$\mathbf{b} = \begin{bmatrix} -b & 0 & 0 \\ 0 & -b & 0 \\ 0 & 0 & -b \end{bmatrix},\ \mathbf{k}_m = \begin{bmatrix} k_m & 0 & 0 \\ 0 & k_m & 0 \\ 0 & 0 & k_m \end{bmatrix}, \quad (25), (26)$$

$$\mathbf{k}_n = \begin{bmatrix} -1/k_n & 0 & 0 \\ 0 & -1/k_n & 0 \\ 0 & 0 & -1/k_n \end{bmatrix},\ \mathbf{R}_A = \begin{bmatrix} -R_A & 0 & 0 \\ 0 & -R_A & 0 \\ 0 & 0 & -R_A \end{bmatrix} \quad (27), (28)$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{n} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix} \quad (29)$$

$$\mathbf{n} = \begin{bmatrix} -1/n & 0 & 0 \\ 0 & -1/n & 0 \\ 0 & 0 & -1/n \end{bmatrix} \quad (30)$$

$$\mathbf{C} = \mathbf{I}_{6\times6},\ \mathbf{D} = [0] \quad (31), (32)$$

2.4. Friction model

To the whole state-space model we added a "simplified" model of friction between the wheels and the surface and it is based on the Coulomb friction. The normal force is applied on each wheel:

$$N_i = \frac{m_R g}{3}. \quad (33)$$

The friction part of the normal force is described as follows:

$$|F_i| \le F_{i,max} = \mu N_i. \quad (34)$$

At this point the friction efforts u_m are introduced, which are a measure of how much torque the motors currently provide to the maximum torque:

$$F_i = u_m F_{i,max} \qquad u_m \in [-1,1] \quad (35)$$

The "friction" moment affecting each wheel is described as follows:

$$M_i = F_i R. \quad (36)$$

Tab. 2: Constant description

constant	value	unit	description
m_R	20	kg	mass of robot platform
L	0,225	m	radius of robot platform
R	61	mm	wheel radius
J_M	32,9	g.cm^2	motor inertia
J_w	7,44	kg.cm^2	wheel inertia
J_g	0,8	g.cm^2	gear inertia

n	14	-	gear ratio
L_A	0,28	mH	inductance of motor
R_A	1,53	Ω	resistance of motor
k_n	240	rpm / V	speed constant of motor
k_m	39,8	mNm / A	torque constant of motor
b	0,0461	mNm.s	damping constant of motor
μ	0,8	-	friction constant
g	9,81	m / s^2	gravitational constant

2.5. Robot model

In the following figure (Fig. 4) there is shown a scheme of the robot model in Matlab/Simulink as a synthesis of previous equations.

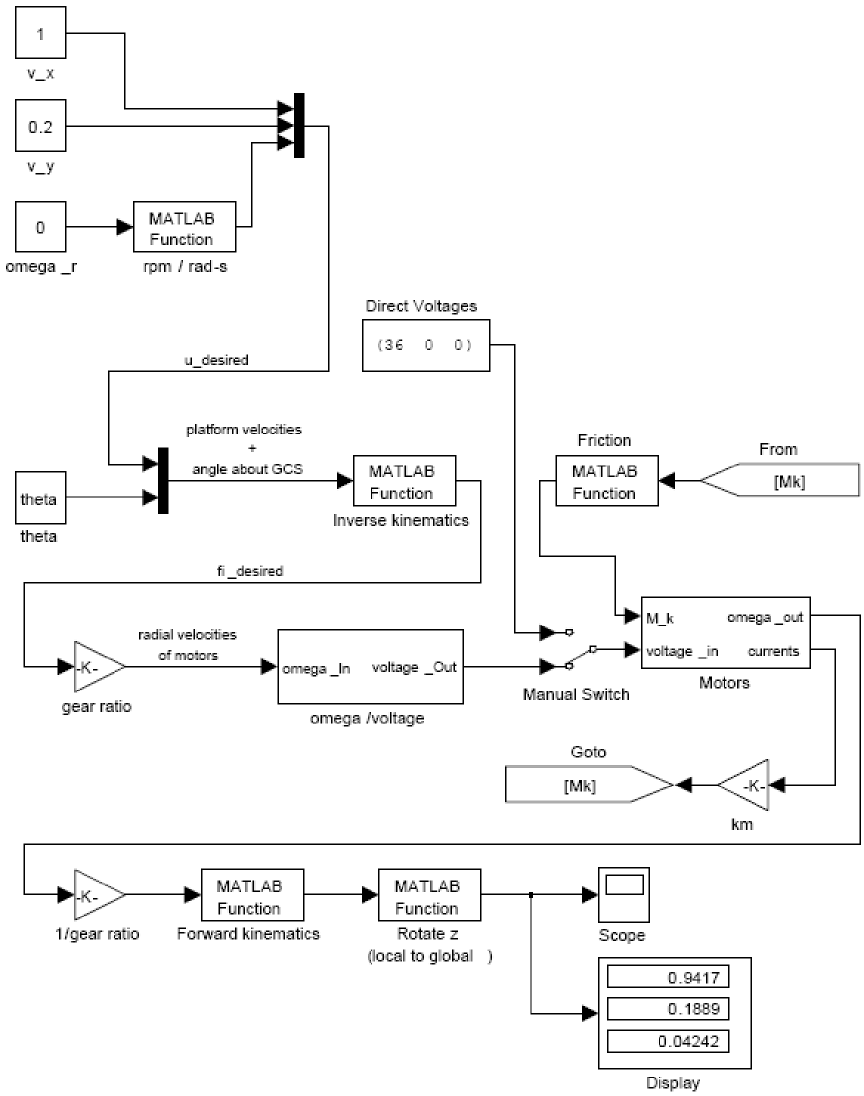
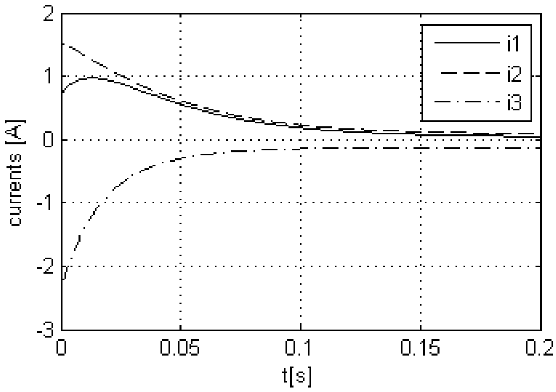


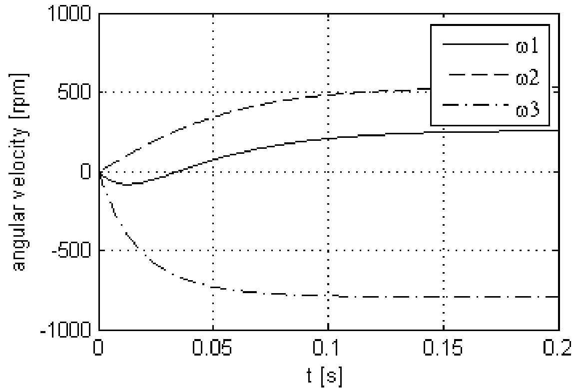
Fig. 4: Robot model in Matlab/Simulink

3. Results

In this section there are shown some simulation results in form of step responses of desired velocities of the robot (v_x = 1m/s , v_y = 0,2 m/s, ω_R = 0 rad/s) (Fig. 5 a,b,c).



a)



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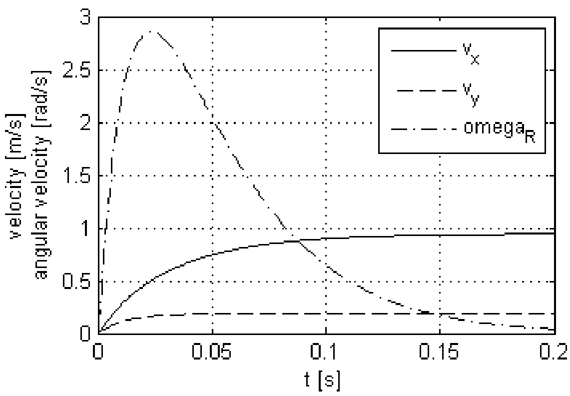

c)

Fig. 5: Simulation results – step responses

4. Future work

Future work will be focused mainly on:

- Assembly and completion of the whole robot with Mecanum wheels.
- Verification of the Robot model.
- Implementation of a control system for the motion control.
- Choosing the optimal strategy for the higher levels of robot control: localization, mapping, path planning. We will consider strategies based on Markov localization (Fox, et al., 1999), Monte Carlo localization (Dellaert, et al., 1999; Gutmann & Fox, 2002; Thrun, et al., 2001), hybrid mapping (integrating metric and topological maps), Rapidly Exploring Random Trees (LaValle, 1998), Case Based reasoning or GA-based approach to path planning. These works will be further based on our

cooperation with the whole robotic team in this project; utilizing the following contributions (Věchet, et al., 2006; Věchet & Krejsa, 2006; Krejsa & Věchet, 2005; Ondroušek, et al., 2006; Ondroušek, et al., 2007; Dvořák & Krček, 2006).

Acknowledgement

Published results were acquired using the subsidization of the Ministry of Education Youth and Sports of the Czech Republic research plan MSM 0021630518 „Simulation modelling of mechatronic systems".

The authors would also like to thank the "Mostly Harmless" RoboCup Team at the Graz University of Technology for their kind collaboration and for providing us with necessary information leading to the new conception of the robot with Mecanum wheels.

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