

## **STUDY OF DAMPING ELEMENT SHAPE INFLUENCE ON NATURAL FREQUENCY**

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**Summary:** *This paper deals with numerical simulations of non-linear structure vibrating. It was created a simple model of one-side fixed beam with the dovetail groove and the damping strap in it. There was contact included between side-walls of the strap and the groove. Main aim was to determine how the strap shape influencing the natural frequency and the damping coefficient. It was observed a time-dependency of the displacement. This signal was converted to the frequency-domain by FFT algorithm. In this Fourier's spectrum was easy to determine the response frequencies. The damping coefficient was determined pursuant to the resonance curve bandwidth in certain distance. It was performed several simulations with various parameters, and results were plotted to graphs.*

### **Introduction**

Bladed disk is a continuum. That means the number of degrees of freedom is infinite, therefore the number of natural frequencies is also infinite. The expressive oscillations occurs while the excitation frequency is equal to so called speed which is defined by ratio of natural frequency to the number of nodal diameters in appropriate mode shape [1]. In bladed disk this frequencies even depends on the number of blades, eventually on the number of cyclic sectors. Many resonance states occur when turbine is starting up. It can be happened, that the operating state will be close to one of these resonance state. Expressive vibrations are shortening disk's lifetime.

It exists several ways to damp the vibrations. In some cases a wire or several wires passing through the holes in the blades can be used. It is not suitable in high pressure stages of steam turbines, because the wire is impediment to the steam flow. It is better to locate the damping element in rotor end ring. Mostly used damping element shape is the strap which is placed in the dovetail groove. Strap shape is defined by these geometrical parameters: middle width  $b$ , high  $h$  and side-wall slope angle  $\varphi$  (Fig. 1), they are influencing dynamic behavior of whole bladed disk. Maximal vibration damping, in the particular operating state, can be probably obtained by suitable choice of these parameters. Damping quantity and relative motion, between the damping strap and the dovetail groove, hang together. Frictional force

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value in the contact surface is given by disk diameter, angular velocity of the disk and the strap mass. This mass is given by material density, middle width and high of the strap. The high is influencing the contact pressure value in additional, because if the high is changed, the contact area is changed too. Modification of the side wall slope angle  $\varphi$  doesn't change the strap mass, but change the area of contact surface, thus the normal force value, and frictional or tangential forces, are influenced too.

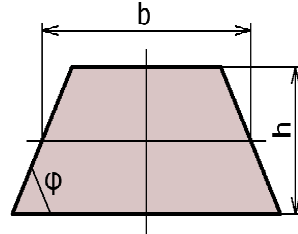


Fig. 1: Dimensions of damping element

This paper deals with the case that only one geometrical parameter (angle  $\varphi$ ) was changed. High and middle width was kept. Angular velocity of the disk  $\omega$  which affecting the centrifugal force was changed too. The problem was solved in the time-domain. The beam was excited by the shock load.

## 2. Computational model

### 2. 1. Model of geometry

The problem was solved in the ANSYS software. It would be difficult to compute response of whole bladed disk with bandage. On this account, it was created a very simplified model and the dynamic behavior was tested on this model. It was one-side fixed beam with the dovetail groove and the damping strap in it (Fig. 2). There was a contact modeled between the strap and the groove side-walls. In bladed disk, the strap is pressed to the groove side walls by pressure which is evoked by centrifugal forces. This effect was supplied by the pressure defined on the upper strap face. This pressure depends on strap mass, angular velocity and diameter of the fictive disk. Particular dimensions, which are staying constant, are in tab.1. Side wall slope angle  $\varphi$  was variable.

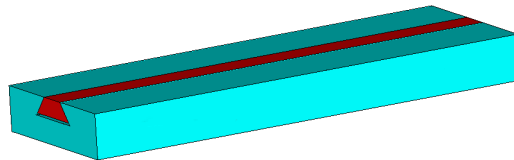


Fig. 2: Model of geometry

Tab. 1: Model dimensions

length of model	$l$	310 mm
width of model	$b_m$	110mm
high of model	$h_m$	30 mm
middle width of damping strap	$b$	35 mm
high of damping strap	$h$	12 mm

## 2. 2. Model of material

The bladed disk has been made of steel. Thus it was supposed that material of this model was steel too. It was presumed that area of linear-elastic behavior will be not exceeded, thus selected material was linear isotropic. Young's modulus  $E$  was selected  $E = 2.1e5MPa$  and Poisson's ratio  $\mu = 0.3$ . They are the most common steel constants. In problems where the mass matrix is in equations, the material density has to be defined, thus  $\rho = 7.85e-9t.mm^{-3}$ .

## 2. 3. Generating the mesh

Because the following simulations can be time-consuming, it was needed to create model with coarse mesh in order for decrease the number of degrees of freedom. Of course, the coarse mesh does not to produce accurate results, but in this paper was observed the global dynamic behavior. Particular quantity (for example quantity of displacement) was not examined. Mesh was generated by structural linear elements SOLID45.

## 2. 4 Model of contact

Between the side-walls of the damping strap and the groove was created the contact. The side-walls of the damping strap were meshed by TARGE170 and the side-walls of groove were meshed by CONTA173. The coefficient of friction depends on the relative velocity of the surfaces in contact. In the ANSYS software there is implemented the model of contact, which express this dependency as follows:

$$\mu = \mu_s \left( 1 + \left( \frac{\mu_s}{\mu_k} - 1 \right) \exp(-D \cdot v_{rel}) \right) \quad (1)$$

Where  $\mu_s$  is the static coefficient of friction,  $\mu_k$  is the kinetic coefficient of friction,  $D$  is decay coefficient and  $v_{rel}$  is relative velocity of the surfaces in contact.

This model presents a part of the disk bandage, with is in the high pressure stage of the steam turbine. There is a superheated stem in it. Thus between contact surfaces was supposed a solid friction. Parameters describing contact are in Tab. 2. A plot of friction coefficient dependency on relative velocity is in Fig.3.

Tab. 2: Contact parameters

static coefficient of friction	$\mu_s$	0.7
kinetic coefficient of friction	$\mu_k$	0.6
decay coefficient	$D$	0.8

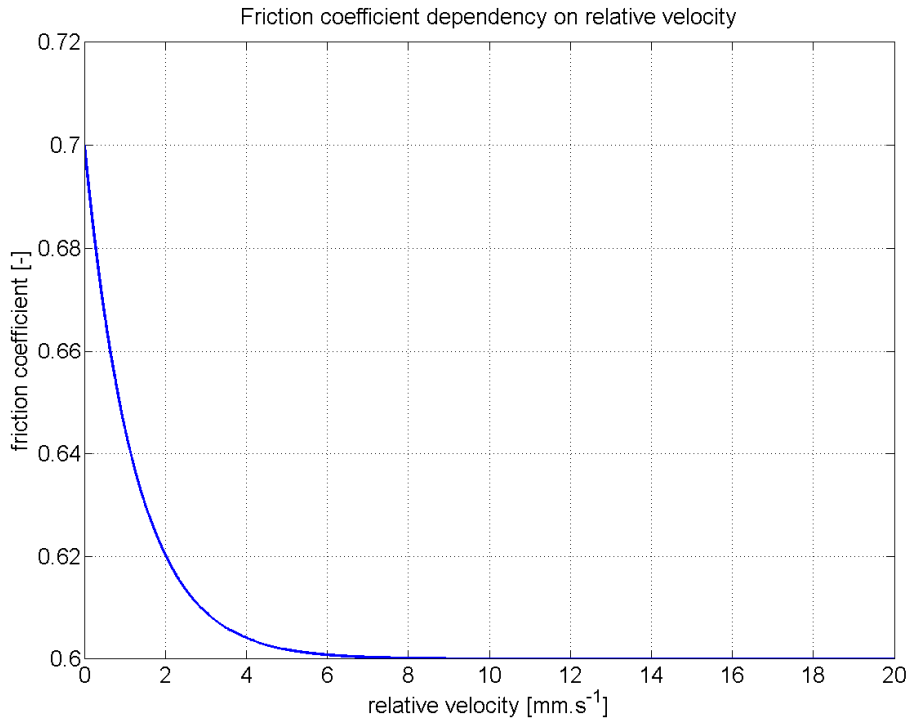


Fig. 3: Friction coefficient dependency on relative velocity

### 3. Boundary and initial conditions

The beam was fixed on the one side. It is shown in Fig. 4. Initial conditions present the state of the beam at time  $t = 0s$ . It was presumed, the beam is in quiescence state.

In the bladed disk, the damping strap is pressed to the side-walls of dovetail groove by centrifugal force  $F_o$ , which depends on the disk angular velocity  $\omega$  and the disk diameter  $d$  and the damping strap mass. In this simplified computational model this effect was obtained by applying pressure on the upper surface of the damping strap. The pressure was computed using equation (2).

$$p_o = \frac{\left(\omega^2 \frac{d}{2} \rho h b\right)}{\left(b - \frac{h}{\tan \varphi}\right)} \quad (2)$$

Where the  $p_o$  is pressure, witch supply the centrifugal force,  $\omega$  is the angular velocity of the disk,  $d$  is disk diameter,  $\rho$  is material density,  $h$  is the damping strap high,  $b$  is middle width and  $\varphi$  is the side-wall slope angle (see Fig. 1). The simulation was repeated with various angular velocities  $\omega$ .

### 4. Excitation and simulation

This problem has been nonlinear by reason of include the contact into the model. On this account it is impossible to talk about natural frequencies, because frequency in the nonlinear structures is depended on the deflection. It's only frequency whereat the structure oscillates by explicit amplitude. Modal analysis is applicable only on conservative or weakly non-

conservative systems. To be possible to assess the effect of the angle  $\phi$  change to the “natural” frequency, it was presumed small deflection about balanced state. The problem was linearized. It was observed the time dependence of the deflection on the end of the beam. Using the Fourier transform was obtained Fourier’s spectrum witch was analyzed. The model was excited by shock load. It’s time duration was  $9e-4s$  and magnitude of force was  $1000N$ . The loads were applied on model as two forces acting as is shown in Fig. 5.

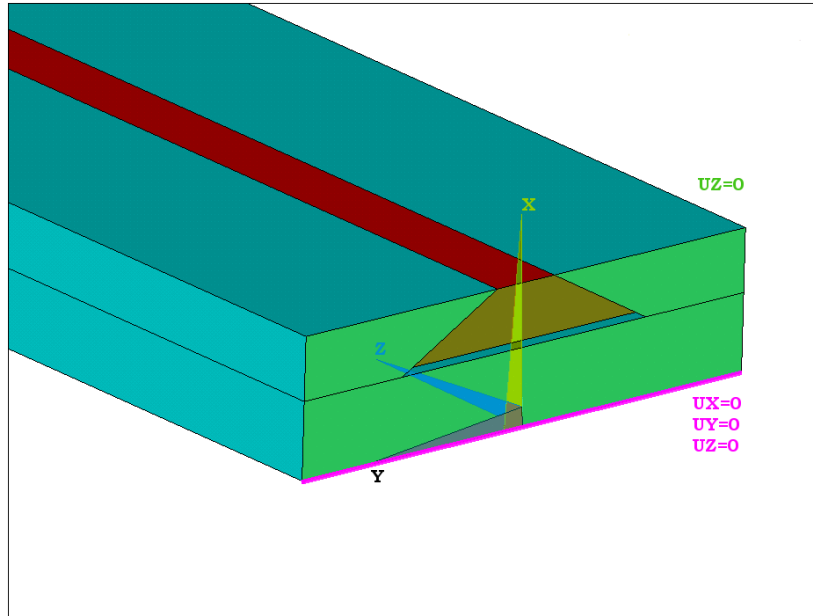


Fig. 4: Boundary condition

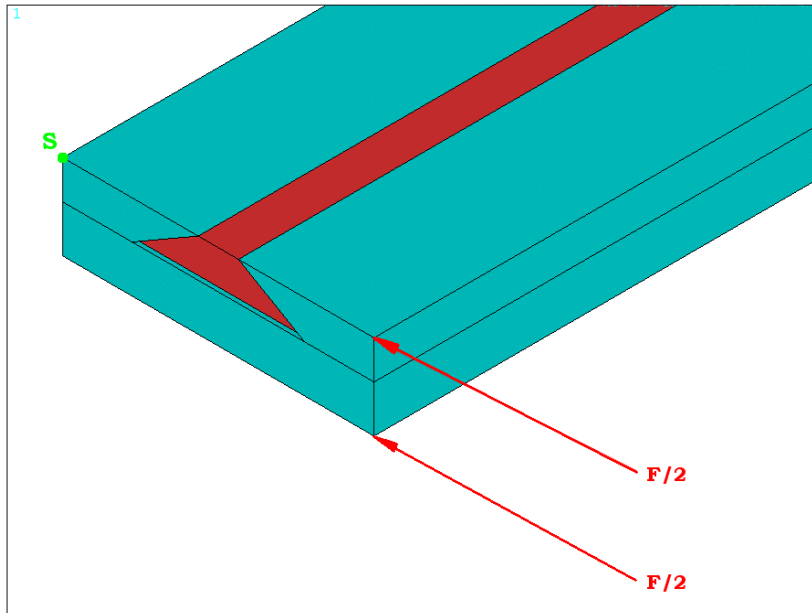


Fig. 5: Applying loads

Time of simulation was  $1s$ . The outside structural and the inner material damping coefficients was kept zero, Thus it was possible to observe the damping effect evoked only by friction between contact surfaces. The response was observed in point witch is in Fig.5 labeled as S. It

was observed the displacement in the transverse direction ( $x$ -direction). The solution was repeated with various values of the side-wall slope angle  $\varphi$  and angular velocity  $\omega$ . The values of these parameters are in Tab. 3 and Tab. 4.

Tab. 3 The side-walls slope angle dimensions

	$\varphi [^\circ]$
1	20
2	25
3	30
4	40
5	53
6	70

Tab. 4 The angular velocities values

	$\omega [\text{rad s}^{-1}]$
1	1
2	50
3	78
4	157
5	236

## 5. Modal analysis of conservative model

To identify the individual response frequencies in the Fourier's spectrum, the modal analysis was computed. Modal analysis was performed on a linear model. It was computed natural frequencies and corresponding mode shapes (see Fig. 6). Of course, the frequencies in Fourier's spectrum will be little different, because damping decreasing the natural frequency in general.

## 6. Results

### 6.1. Time-domain response

The response was observed in point witch is in Fig. 5 labeled as  $S$ . Example of response in the time-domain for  $\omega = 50 \text{ rad s}^{-1}$  and  $\varphi = 20^\circ$  is in Fig. 7. It shows the displacement in transversal direction ( $x$ -axis direction). The vibrations are damped in a relatively short time. The damping is caused only by friction in contact surfaces, because the coefficient of structural and material damping was set zero.

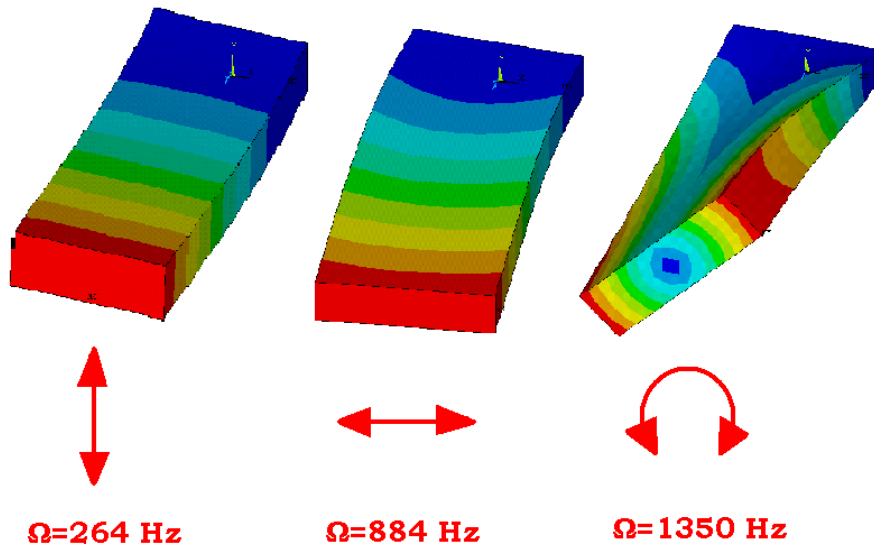


Fig. 6: Natural frequencies and mode shapes

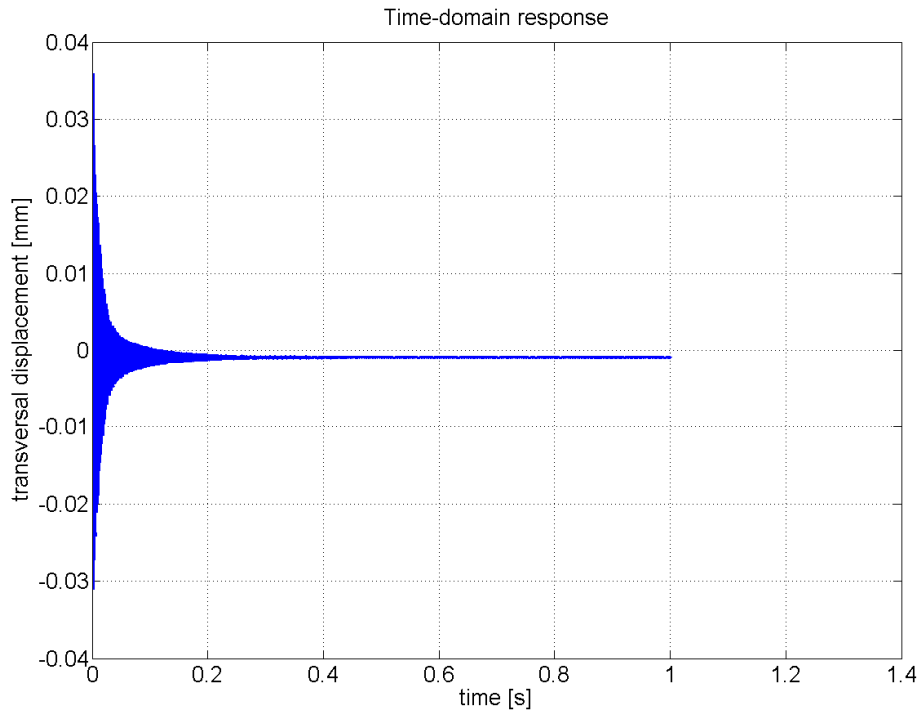


Fig. 7: The response in time domain

## 6.2. Frequency-domain response

Fourier's spectrum was created by applying FFT algorithm to the response signal in time-domain (see Fig. 8). Detail of interesting area is in Fig. 9. In the spectrum in Fig. 8 there are two significant frequencies. First  $\Omega_1 = 260.8 \text{ Hz}$  that is frequency of oscillation in vertical direction ( $y$ -axis direction). Second one,  $\Omega_2 = 802.3 \text{ Hz}$ , is frequency of oscillation in transversal direction ( $x$ -axis direction).

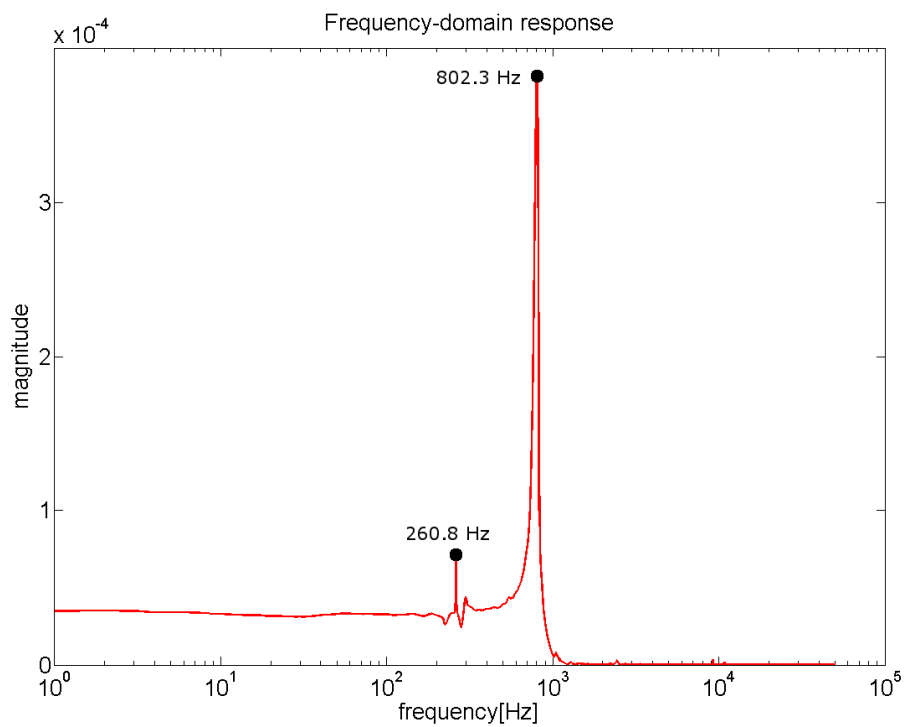


Fig. 8 The response in frequency domain

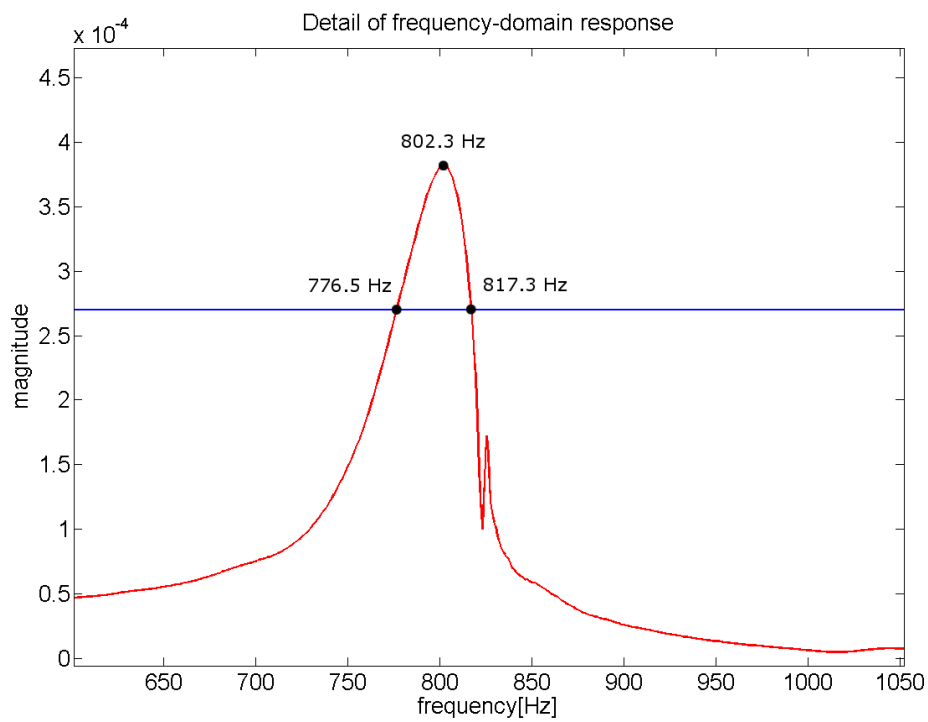


Fig. 9 Detail of interesting area in Fourier's spectrum

The damping coefficient was determined pursuant to the resonance curve bandwidth  $\Delta f$  in the distance  $h_b$  (see Fig. 9) which is defined as:

$$h_b = 0.707A \quad (3)$$

where  $A$  is the amplitude accordant with appropriate “natural” frequency  $\Omega_i$ . Damping coefficient is defined by relation:

$$b_p = \frac{1}{2} \left( \frac{\Delta f}{\Omega_i} \right) \quad (4)$$

In this case the damping coefficient is  $b_p = 0.025$ , that is 2.5%.

Damping coefficients for various values  $\varphi$  and  $\omega$  are in Tab. 5 and response frequencies are in Tab. 6. These values are plotted in graphs in Fig.10 and Fig. 11.

Tab. 5: Damping coefficient in dependence on side-wall slope angle and angular velocity

		angular velocity $\omega$ [rad s <sup>-1</sup> ]				
		1	50	78	157	236
side-wall slope angle $\varphi$ [°]	20	0,0019	0,0419	0,0528	0,0180	0,0120
	25	0,0037	0,0371	0,0273	0,0169	0,0170
	30	0,0027	0,0320	0,0261	0,0155	0,0124
	40	0,0027	0,0254	0,0198	0,0099	0,0094
	53	0,0045	0,0161	0,0100	0,0068	0,0060
	70	0,0055	0,0047	0,0036	0,0036	0,0030

Tab.6: Parameters influence to response frequencies

		angular velocity $\omega$ [rad s <sup>-1</sup> ]				
		1	50	78	157	236
side-wall slope angle $\varphi$ [°]	20	756	746	761	792	794
	25	799	771	788	799	796
	30	800	785	797	803	804
	40	821	802	815	815	816
	53	838	819	821	822	822
	70	850	826	826	827	826

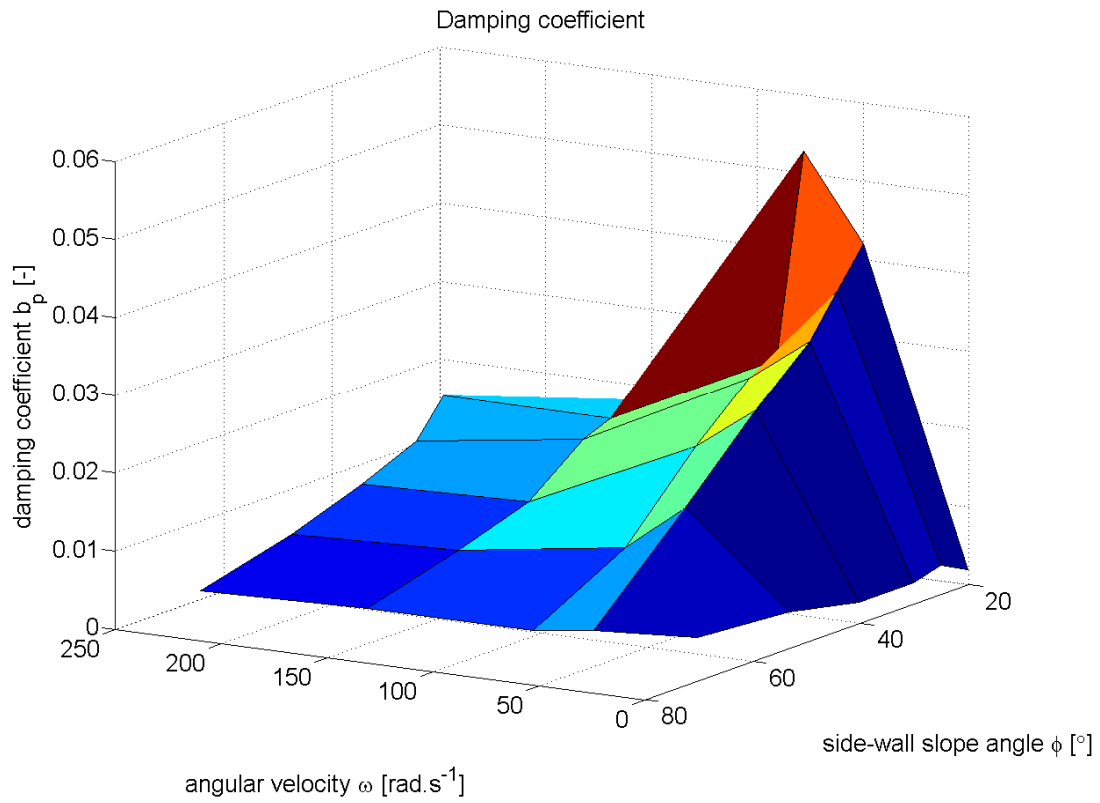


Fig. 10: Damping coefficient in dependence on side-wall slope angle and angular velocity

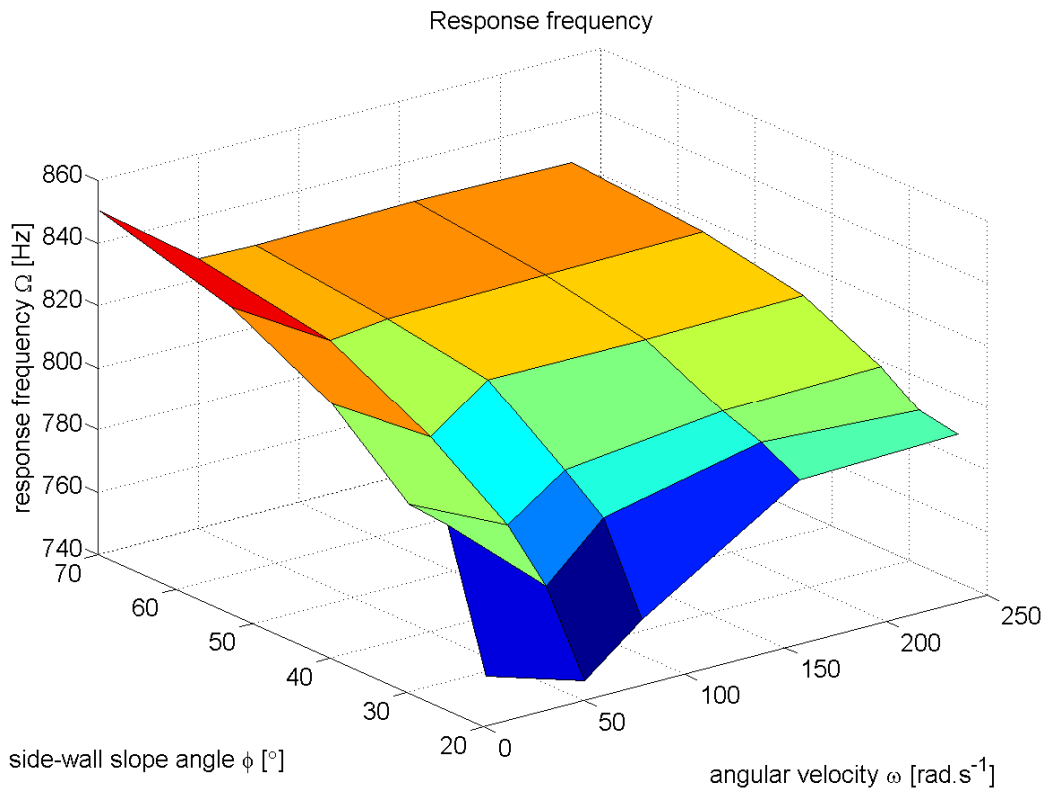


Fig. 10: Response frequency in dependence on side-wall slope angle and angular velocity

## **7. Conclusion**

In this paper was performed numerical simulations of model witch contains damping elements. It was studied the damping coefficient dependence on the change of selected geometrical parameters. Simulations time was 1s. Using the FFT algorithm on the response in the time domain was created the Fourier's spectrum. The damping coefficient was determined pursuant to the resonance curve bandwidth. It is show, the geometrical parameters modification have effect to the damping coefficient and response frequency values. It is possible to maximize the damping coefficient using parametrical optimization. This problem will be solved in future.

## **References**

[1] Hamid Mehdigholi, Forced vibration of rotating disc and interaction with nonrotaringstructures, Imperial College of Science, Technology and Medicine, London, 1991