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DYNAMICS OF MACHINE AGGREGATE DRIVE WITH SELF-LOCKING MECHANISM

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Summary: *Dynamic analysis of machine aggregate drive with self-locking mechanism is presented. The drive system with self-locking mechanism as a closed system with possibility of clearance adjustment in the gearing mechanism is considered. Two operating regimes of the drive are analyzed – traction regime and unbraking regime.*

1. Introduction

To apply the self-locking mechanisms (e.g. worm gearing) in drive systems requires to take in advance the appropriate measure to adjust the clearance. In this way the necessity of the presence of brakes in these systems is eliminated and the output members for steady-state motion of the system (regime of traction or unbraking regime) have to be driving members (Mudrik, 2007).

The same special phenomenon of transient processes in machine aggregates build up from self-locking mechanism of worm gearing with adjustment of lateral gearing clearance are analyzed in this paper. The self-locking of worm gearing and linearized dynamical moment characteristics of electric drive are considered.

2. Formulation of the problem

We consider the drive system with self-locking mechanism as a closed system with possibility of clearance adjustment in the gearing mechanism (Fig. 1).

The worms (members 1, 2) are kinematically coupled by bevel gearing 4 and together with the worm gear 3 they complete the worm gearing. This drive system is a closed kinematic chain. Force lock-up is created by spring 5 acting on axially movable (on shaft ②) worm 2 by which the lateral gearing clearance is adjusted. Then axial force of spring acts on worm gear 3 and creates the so called pre-stress moment M_p. Force effect is located only in self-locking mechanism. The worm 1 is mounted on shaft ① and is axially fixed against movement. For simplicity the transfer ratio of value $i = 1$ is considered.

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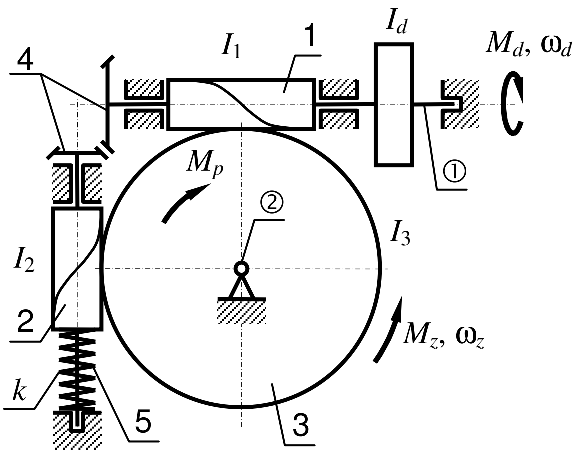

1 - driving worm,
2 - adjusting worm,
3 - worm gear,
4 - bevel gearing,
5 - pre-stressed spring,

I_d - inertia moment of rotor of drive system,
I_1, I_2, I_3 - inertia moments of drive system members

Fig. 1 Kinematic scheme of drive system with self-locking mechanism.

3. Operating regimes

Two operating regimes of the drive are analyzed. The following terms and descriptions will be used (Mudrik, 2007):

- Regime A - defined by inequality - $\mathbf{M}_p \cdot \boldsymbol{\omega}_z < 0$, (1)
  - worm gearing 1 → 3 operates in the ***traction regime***, i.e. $\mathbf{M}_p \cdot \boldsymbol{\omega}_z < 0$,
  - worm gearing 2 → 3 operates in the ***regime with brake released***, then
    A_1 - case defined by inequality - $\mathbf{M}_z \cdot \boldsymbol{\omega}_z < 0$,
    A_2 - case defined by inequality - $\mathbf{M}_z \cdot \boldsymbol{\omega}_z > 0$.
- Regime B - defined by inequality $\mathbf{M}_p \cdot \boldsymbol{\omega}_z > 0$, (2)
  - worm gear 1 → 3 operates in the ***regime with brake released***, i.e. $\mathbf{M}_p \cdot \boldsymbol{\omega}_z > 0$,
  - worm gear 2 → 3 operates in the ***traction regime***, then
    B_1 - case defined by inequality - $\mathbf{M}_z \cdot \boldsymbol{\omega}_z < 0$,
    B_2 - case defined by inequality - $\mathbf{M}_z \cdot \boldsymbol{\omega}_z > 0$.

The solution for both cases is realized for linear dynamic characteristics of electric drive in the form (Mudrik, 2006)

$$\tau_e \frac{dM_d}{dt} + M_d = \beta(\omega_0 - \omega_d), \tag{3}$$

where ω_d, ω_z - angular velocity of rotor and driven shaft (shafts I, II),
ω_0 - angular velocity of idling,
M_d - driving moment of electric motor,
M_z - loading moment acting on worm gear 3,
β - stiffness modulus of static characteristics.

4. Transient processes in the machine aggregate

As a drive motor, the separately excited DC motor is used. The dynamic model, static moment characteristics and run-up characteristics of this motor are shown on Fig. 2.

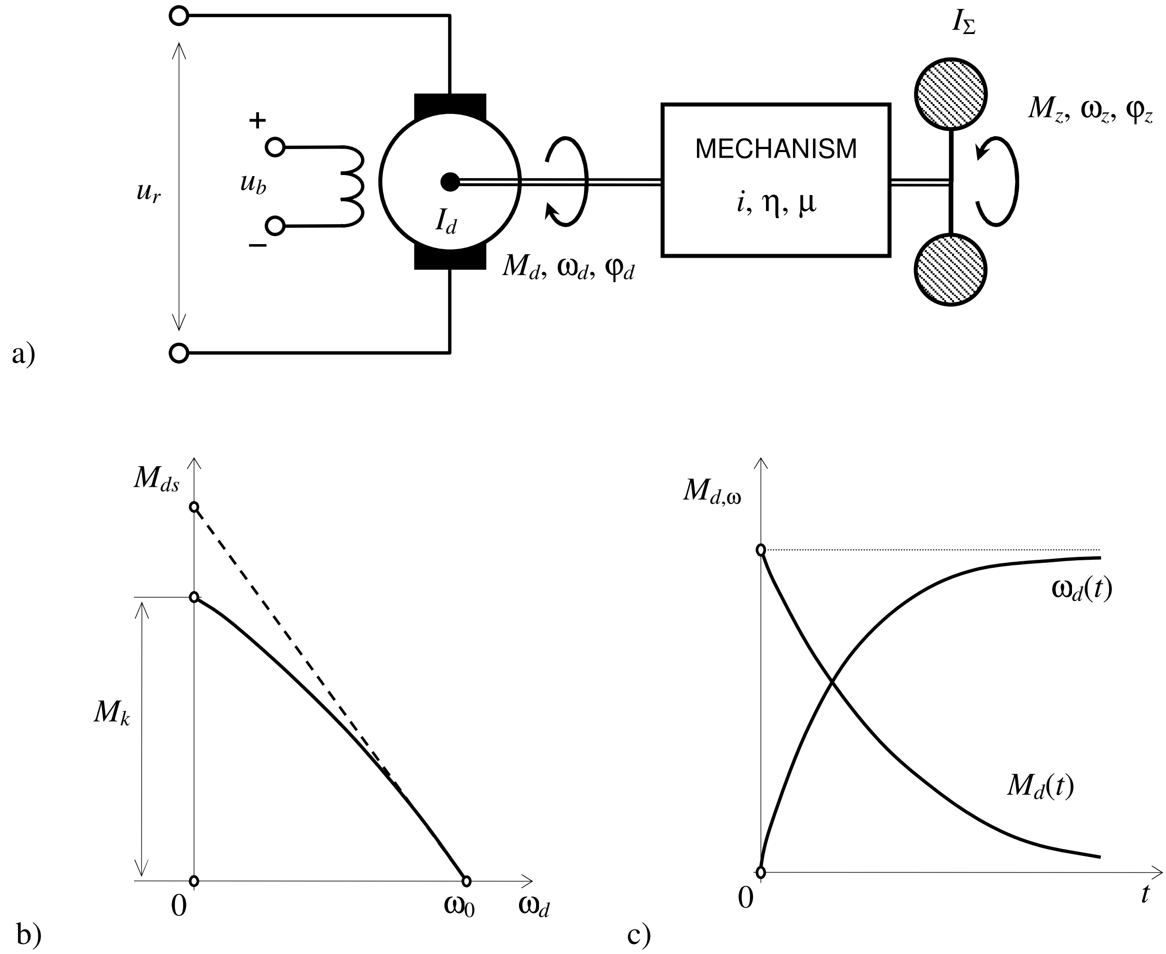

Fig. 2 Drive with separately excited DC motor.
a - dynamic model; b - static moment characteristics; c - run-up characteristics

In the first period, the electromagnetic transient process is performed in condition of standing rotor and for arbitrary operating regime is this period described by modified equation in the form (Mudrik & Naď & Labašová, 1998)

$$\tau_e \frac{dM_d}{dt} + M_d = M_k, \qquad (4)$$

where $\tau_e = \dfrac{L_\Sigma}{R_\Sigma}$ - electromagnetic time constant of transient phenomenon,

L_Σ - DC motor armature circuit inductance,

R_Σ - DC motor armature circuit resistance,

$M_k = \beta\omega_0$ - critical (maximal) moment corresponding to run-up characteristics,

$\omega_0 = \dfrac{u_r}{k\Phi}$ - angular velocity of ideal idling,

u_r - DC motor armature voltage,

Φ - magnetic flux of exciting winding,

$\beta = \dfrac{k^2\Phi^2}{R_\Sigma}$ - stiffness of static characteristics,

$k = \dfrac{p_p N}{2\pi n}$ - motor constant,

p_p - number of pole pairs,

N - number of active windings of motor armature,

n - number of parallel windings of the coil.

The solution of equation (4) under zero initial conditions is given by expression

$$M_d = M_k\left(1 - e^{-\frac{t}{\tau_e}}\right). \tag{5}$$

Time of the first period of transient process t^* can be determined form condition $M_d = M_{ds}$, i.e.

$$t^* = \tau_e \ln\left(\frac{M_k}{M_k - M_{ds}}\right), \tag{6}$$

where M_{ds} is a moment of motor in stationary equilibrium regime of the drive motion.

Next, the second period of machine aggregate as a whole can be described by following general equations of motion

$$\begin{aligned}
&\tau_e \frac{dM_d}{dt} + M_d = \beta(\omega_0 - \omega_d),\\
&(I_d + I_1)\frac{d\omega_d}{dt} = M_d + M_{31} + M_{k21},\\
&I_2 \frac{d\omega_d}{dt} = M_{k12} + M_{32},\\
&I_3 \frac{d\omega_z}{dt} = M_{13} + M_{23} + M_z,
\end{aligned} \tag{7}$$

where M_{31} - moment acting from worm gear 3 on worm 1,

M_{32} - moment acting from worm gear 3 on worm 2,

M_{13} - moment acting from worm 1 on worm gear 3,

M_{23} - moment acting from worm 2 on worm gear 3,

$M_{k21} = -M_{k12}$ - moments acting in bevel gear.

5. Amplitude-frequency characteristics

Let us introduce the following expressions

$I_\Sigma = I_d + I_1 + I_2$ - total reduced inertia moment of motor armature and worms,

$i = \dfrac{\omega_d}{\omega_z}$ - transmission ratio of worm gearing,

$\eta = -\dfrac{M_z \omega_z}{M_d \omega_d}$ - efficiency of worm gearing,

$\mu = \dfrac{M_d \omega_d}{M_z \omega_z}$ - brake release coefficient.

***Operating regime* A:**

For this operating regime the following expressions must be used (Mudrik, 2007)

$$\begin{aligned} M_{23} &= -M_p, \\ M_{13} &= -i\eta M_{31}, \\ M_{32} &= i^{-1}\mu M_{23}. \end{aligned} \tag{8}$$

Then using (8), the system of equations (7) has the form

$$\begin{aligned} &\tau_e \frac{dM_d}{dt} + M_d = \beta(\omega_0 - \omega_d), \\ &(I_\Sigma + I_3 i^{-2}\eta^{-1})\frac{d\omega_d}{dt} = M_d - M_p i^{-1}\eta^{-1}(1+\eta\mu) \mp M_z i^{-1}\eta^{-1}. \end{aligned} \tag{9}$$

The moment of stationary equilibrium drive motion ($d\omega_d / dt = 0$) is expressed from second equation of (9)

$$M_{d,A} = M_p i^{-1}\eta^{-1}(1+\eta\mu) \pm M_z i^{-1}\eta^{-1}, \tag{10}$$

where sign plus is used for regime A_1 and sign minus is used for regime A_2.

Characteristic equation of the system defined by (9) has the form

$$\tau_e T_{M,A}\lambda^2 + T_{M,A}\lambda + 1 = 0, \tag{11}$$

where electromechanical time constant in condition of regime A is expressed by equation

$$T_{M,A} = \beta^{-1}(I_\Sigma + I_3 i^{-2}\eta^{-1}).$$

When the transient process is oscillating, the characteristic equation has the form

$$T_A^2\lambda^2 + 2\delta_A T_A \lambda + 1 = 0, \tag{12}$$

where $T_A = \sqrt{\tau_e T_{M,A}} = \sqrt{\tau_e I_\Sigma \beta^{-1}}\sqrt{1 + \dfrac{I_3}{I_\Sigma i^2 \eta}} = T_0\sqrt{1 + \dfrac{\gamma}{\eta}}$ is drive time constant of regime A.

Next, we designate

$T_0 = \sqrt{\tau_e I_\Sigma \beta^{-1}}$ - basic time constant,

$\gamma = \dfrac{I_3}{I_\Sigma i^2}$ - ratio of the worm gearing inertia moments,

$\delta_A = \sqrt{\dfrac{T_{M,A}}{4\tau_e}} = \delta_0 \sqrt{1+\dfrac{\gamma}{\eta}}$ - decay coefficient of regime A, (13)

$\delta_0 = \sqrt{\dfrac{I_\Sigma}{4\tau_e \beta}}$ - basic decay coefficient.

The roots of characteristic equation (12) are

$$\lambda_{1,2} = \frac{-\delta_A \pm j\sqrt{1-\delta_A^2}}{T_A}, \tag{14}$$

where $j = \sqrt{-1}$ is imaginary unit.

***Operating regime* B:**

For regime B, the equations (8) are modified and they have the form

$$\begin{aligned} M_{23} &= M_p, \\ M_{13} &= i^{-1}\mu M_{31}, \\ M_{23} &= i\eta M_{32}. \end{aligned} \tag{15}$$

Using equations (15), the system of equations (9) has the form

$$\begin{aligned} &\tau_e \frac{dM_d}{dt} + M_d = \beta(\omega_0 - \omega_d), \\ &(I_\Sigma - I_3 i^{-2}\mu)\frac{d\omega_d}{dt} = M_d - M_{d,B}, \end{aligned} \tag{16}$$

where

$$M_{d,B} = M_p i^{-1}\eta^{-1}(1+\eta\mu) \mp M_z i^{-1}\mu \tag{17}$$

and sign minus is used for regime B_1 and sign plus is used for regime B_2.

The characteristic equation of regime B is analogical to characteristic equation (11)

$$T_B^2\lambda^2 + 2\delta_B T_B \lambda + 1 = 0$$

The electromechanical time constant is expressed by

$$T_{M,B} = \beta^{-1}(I_\Sigma - I_3 i^{-2}\mu). \tag{18}$$

Then $T_B = T_0\sqrt{1-\gamma\mu}$ is a time constant of regime B, (19)

$$\delta_B = \delta_0\sqrt{1-\gamma\mu} \quad \text{is a decay coefficient of regime B.} \tag{20}$$

The roots of characteristic equation of regime B

$$\lambda_{1,2} = \frac{-\delta_B \pm j\sqrt{1-\delta_B^2}}{T_B}. \tag{21}$$

Then, the natural angular frequencies for both regimes are obtained in the form

$$\begin{aligned} \omega_A &= \frac{1}{T_0}\sqrt{\frac{\eta}{\eta+\gamma} - \delta_0^2}, \\ \omega_B &= \frac{1}{T_0}\sqrt{\frac{1}{1-\gamma\mu} - \delta_0^2}. \end{aligned} \tag{22}$$

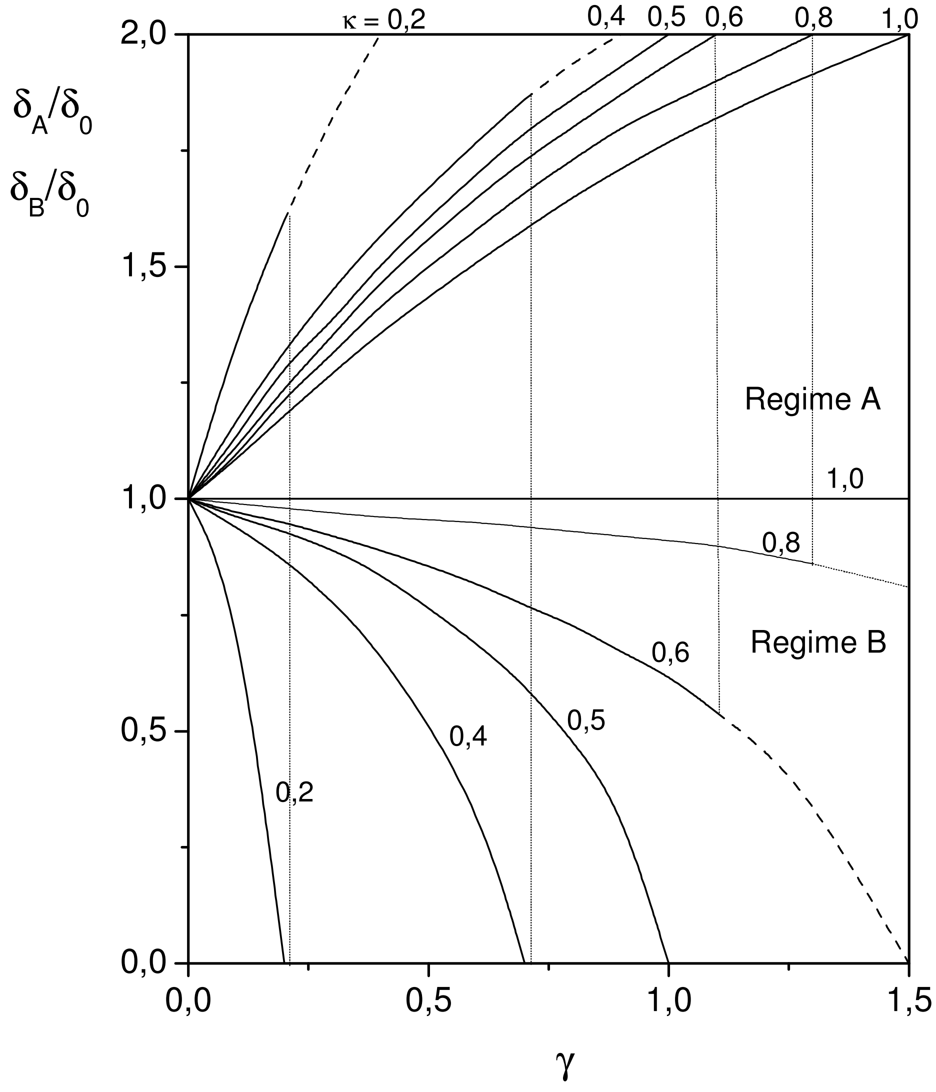

Fig. 3 Dependence of δ_A/δ_0 and δ_B/δ_0 *vs*. parameter γ.

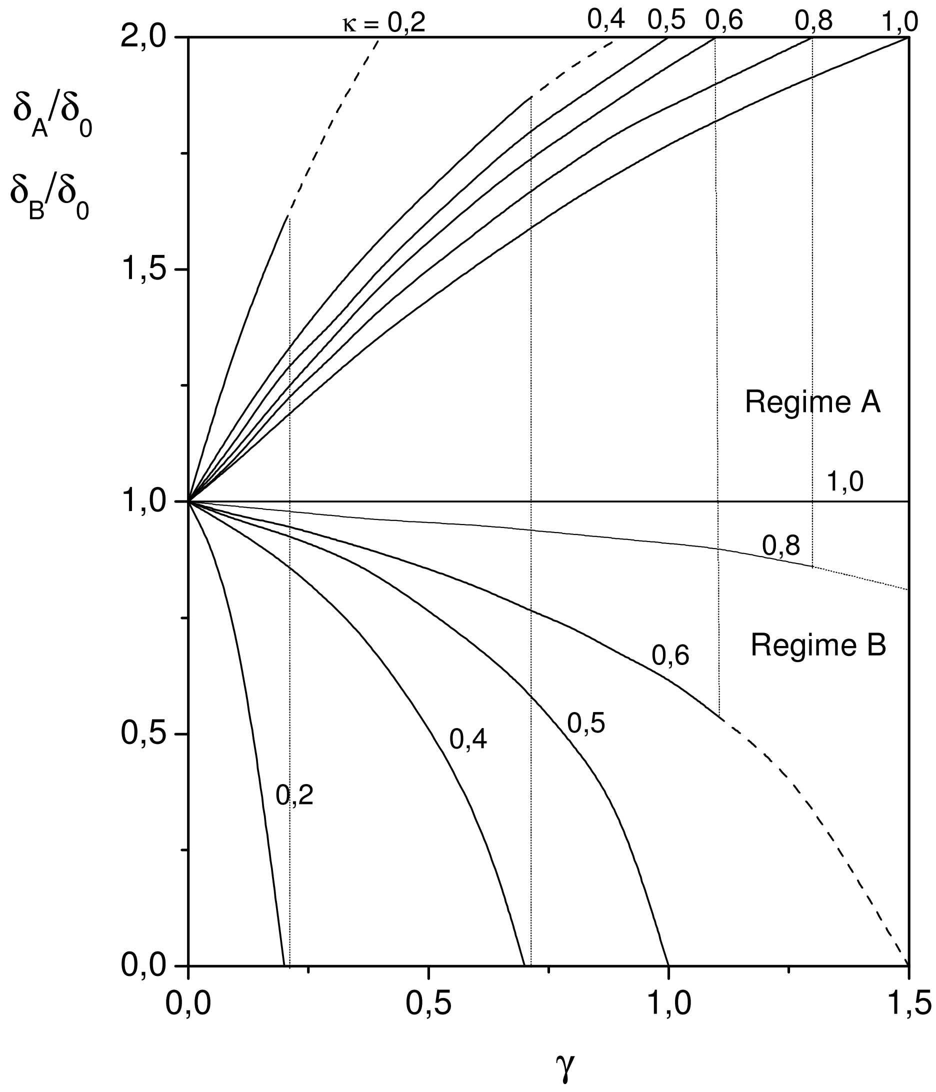

Fig. 4 Dependence of natural frequencies $\omega_A T_0$ and $\omega_B T_0$ *vs*. parameter γ.

6. Conclusions

The dependencies of δ_A/δ_0 and δ_B/δ_0 (ratio of decay coefficient to basic decay coefficient, $\delta_0 = 0{,}5$) on parameter γ for different values of safety factor κ_s are shown in Fig. 3. The safety factor is defined by

$$\kappa_s = \frac{\alpha}{\varphi_{red}}, \tag{23}$$

where α is a helix angle and φ_{red} is reduced angle of friction.

Decay coefficient depends on the direction of rotation. Generally, $\delta_A > \delta_B$ is held. From this assumption, the predisposition to generate of vibration in regime B is higher than in regime A.

From equations (13) and (20) representing the decay coefficients, it can be seen that the time electromagnetic coefficient τ_e of transient processes in machine aggregate affects vibration of the system. The time electromechanical coefficient T_m has the stabilizing effect, i.e. it is a damping factor of vibrating system. The growth of parameter γ causes, that the damping effect is constant for regime A. The damping effect for operating regime B is decreasing.

The analyzed mechanism of this drive is dynamic self-locking for condition $\gamma\mu > 1$ (Mudrik & Riečičiarová, 2007). The reversibility is ensured when the condition of $\gamma\mu < 1$, is satisfied. The ranges of drive reversibility are characterized by solid lines in Fig. 3.

The parameter values of γ (for which the existence of regimes A and B are possible) are determined by regime B when the safety factor $\kappa_s \le 0{,}5$ and for parameters

$$\kappa_s > 0{,}5\,; \quad \delta_A < 1{,}0\,,$$

are determined for conditions defined by regime A.

The dependencies of non-dimensional natural angular frequencies of both regimes ($\omega_A T_0$, $\omega_B T_0$) on parameter γ and for parameter $\delta_0 = 0{,}5$ are shown on Fig. 4. The natural frequency depends on the direction of rotation whereby the inequality

$$\omega_B \ge \omega_A\,,$$

is held.

The growing parameter γ causes the growth of the natural frequency ω_B and for

$$\gamma \to \frac{1}{\mu} \quad \text{we have} \quad \omega_B \to \infty \quad \text{and} \quad \omega_A = 0\,.$$

The values of parameter γ, for which $\omega_A = 0$, are the limiting values defining the change of vibrating processes on the aperiodic processes. The solid lines in Fig. 4 represent reversibility drive. The aperiodic processes in machine aggregate are identified by dashed lines in Fig. 3 and Fig. 4

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