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## ABOUT ANOTHER POSSIBILITY OF REDUCING THE PNEUMATIC SUSPENSION STIFFNESS

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**Summary:** In the paper is the application of the difference spring (two opposite running air springs) described. The requirement on stiffness reduction is realized by difference between characteristics of the effective areas of the air springs. Is deduced a mathematical model of the system and present the results of the numerical simulations.

## 1. Introduction

In the papers (Šklíba et al. 2006, Šklíba et al. 2005), there was analyzed a vibro-isolation system with three degrees of freedom. Its sprung suspension was realized with pneumatic springs. The requirement on stiffness reduction was solved with help of a cylinder spring with a formed piston, with help of change of transmission and with help of additional volume. Further significant possibility is a gyroscopic stabilization (Šklíba & Sivčák 2006). The last possibility is the application of difference pneumatic spring (Fig.1).

The function of a difference pneumatic spring is known (Krejčíř 1986). In the fig. 2, there is described so called double-acting spring, which is a limit case of difference spring. The double acting spring can be inflated with different pressures. In both cases, in the functions, there are two springs with opposite deflection and the resultant force is realized by the difference in forces of the both springs. We will perform a fundamental qualitative analysis.



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#### 2. General relations

The mass flow between the springs is described in the dynamic state by the first two equations and the third equation expresses the equilibrium of forces:

$$p_1 \dot{V}_1 + V_1 \dot{p}_1 = K (p_2 - p_1), \qquad p_2 \dot{V}_2 + V_2 \dot{p}_2 = -K (p_2 - p_1), \tag{1}$$

$$m\ddot{x} + b\dot{x} - S_1 p_1 + S_2 p_2 = -mg , \qquad (2)$$

where  $V_i$  is volume and  $S_i$  effective area of the *i*-spring,  $p_i$  is overpressure in *i*-spring and *K* is the resultant flow coefficient. In the equilibrium point  $l_{10}$  resp.  $l_{20}$ , we can develop the dependence of the volume and effective area on the length *l* in Taylor series.

$$l_{1} + l_{2} = l_{10} + l_{20} = l_{0} \quad l_{1} = l_{10} + x \quad l_{2} = l_{10} - x \quad V_{i} = \sum_{j=0}^{n} V_{ij} l_{i}^{j} \quad S_{i} = \sum_{j=0}^{n} S_{ij} l_{i}^{j} \quad i = 1,2$$
(3)

For given values  $p_1$  and  $p_2$  the equilibrium is defined as the solution of system

$$-p_1S_1 + p_2S_2 = 0, \ l_1 + l_2 = l_0.$$
<sup>(4)</sup>

The stability of the equilibrium state is given as the solution of the characteristic equation.

$$det \begin{vmatrix} m\lambda^{2} + b\lambda & -S_{10} & S_{20} \\ p_{1}S_{10}\lambda & V_{1}\lambda + K & -K \\ -p_{2}S_{20}\lambda & -K & V_{2}\lambda + K \end{vmatrix} =$$

$$= \lambda \begin{cases} \lambda^{3}(mV_{1}V_{2}) + \lambda^{2}(Km(V_{1} + V_{2}) + bV_{1}V_{2}) + \lambda(-p_{1}S_{1}^{2}V_{2} - p_{1}S_{2}^{2}V_{1} + bK(V_{1} + V_{2})) + \\ +K(S_{1} - S_{2})(p_{1}S_{1} - p_{2}S_{2}) \end{cases}$$

$$(5)$$

This system is a tatic and for the stability it must be valid  $S_1 > S_2$  and  $p_1 S_1 > p_2 S_2$ .

In the first limit case (the opened throttling), we can divide the characteristic equation by K and then make  $K \rightarrow \infty$ . In this case the natural frequency has the smallest value.

In the other limit case (the closed throttling, K=0), the characteristic equation has a double zero root and the natural frequency has the biggest value. This is the case of the double-acting spring. The springs are in the parallel position.

#### 3. Results of the numerical simulations

In simulations was described system from Fig. 1 with adding damper with next parameters:

 $S_{10}=0.005 \text{ m}^2$   $S_{20}=0.001 \text{ m}^2$   $S_{11}=0.0005 \text{ m}^2/\text{m}$   $S_{21}=0.0005 \text{ m}^2/\text{m}$   $V_0=0.003 \text{ m}^3$ m=80 kg, b=0.900 Ns/m, K=1e-9..1e-4



Fig. 3 Frequency spectrum (change K from 1e-9 /magenta/ to 1e-4 /red/).



Fig. 5 Deflection [m] (K=1e-9 /blue/ and 1e-4 /red/).



Fig. 7 Difference between pressures [Pa] (K=1e-9 /blue/ and 1e-4 /red/).

Fig. 4 Frequency spectrum (change damping coefficient b from 1/blue/ to 1000 /purple/ [Ns/m]).



Fig. 6 Velocity [m/s] (K=1e-9 /blue/ and 1e-4 /red/).



Fig. 8 Frequency spectrum (excitation frequency = 0.1/red/ - 10/blue/ Hz

## 4. Conclusion

The load on the difference spring or on the double acting spring presents an astatic system, which requires a position control.

The decreasing of the natural frequencies is nearby 50% and it as possible to expect, that we reach a bigger decreasing by bigger ratio  $V_2/V_1$ .

It is evident, that it exist an optimal throttling which ensure a maximal damping of system (the minimal transmission in resonance).

The application of proportional damper in this case is not necessary, but by controlled system will have significance.

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