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PARAMETRIZATION OF THE SIMULATION MODEL OF A ROTOR SYSTEM WITH JOURNAL BEARINGS

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Summary: *There are many ways how to model a rotor system with journal bearings, but this paper prefers an approach, which is based on the concept developed by Muszynska (1986). In contrast to the others, that prefer the lubricant flow prediction using a FE method, the Muszynska model can be employed to simulate a behavior of a journal vibration active control system by manipulating the sleeve position by piezoactuators, which are a part of the closed loop composed of proximity probes and a controller. The paper is focused on the problem how to estimate the true values of the simulation model parameters.*

1. Introduction

It is known that the journal bearing with an oil film becomes instable if the rotor rotation speed crosses a certain value, which is called the Bently-Muszynska threshold. To prevent the rotor instability, the active control can be employed. The arrangement of proximity probes and piezoactuators in a rotor system is shown in figure 1. It is assumed that the carrier ring is a movable part in two perpendicular directions while rotor is rotating. The carrier ring position is controlled by the piezoactuators according to the proximity probe signals, which are a part of the closed loop (Šimek, 2007) including a controller.

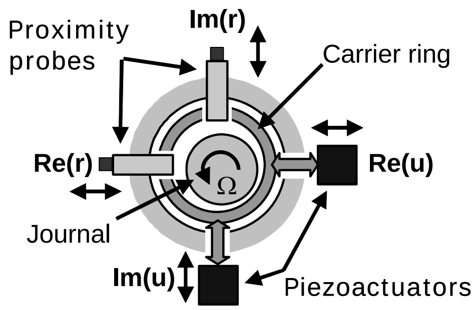

Figure 1. Journal coordinates

There are many ways how to model a rotor system, but this paper prefers an approach, which is based on the concept developed by Muszynska (1986, 2005) and Bently (1986), who were supported by Bently Rotor Dynamics Research Corporation or on the lubricant flow prediction using a FE method for Reynolds equation solution (Svoboda, 2007). An effective way to understand the rotor instability problem and to model a journal vibration active control system is an approach based on the Muszynska model.

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2. Lumped parameter model of the rotor system

Let the rotor angular velocity be designated by Ω. This paper proposes to use complex variables to describe motion of the rotor and the carrier ring in the complex plane. The position of the journal centre in the complex plane, origin of which is situated in the bearing centre, is designated by a positron vector $\mathbf{r}$. The position of the carrier ring is determined by a position vector $\mathbf{u}$.

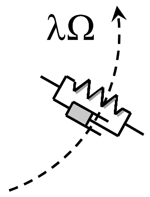

Figure 2. Model of oil film

The internal spring, damping and tangential forces are acting on the rotor. As Muszynska has stated these bearing forces can be modeled as a rotating spring and damper system at the angular velocity $\lambda\Omega$ (see figure 2), where λ is a parameter, which is slightly less than 0.5, see (Muszynska, 1986). The parameter λ is denominated by Muszynska as the fluid averaged circumferential velocity ratio. The external forces refer to forces that are applied to the rotor, such as unbalance, impacts and preloads in the form of constant radial forces. The fluid pressure wedge is the actual source of the fluid film stiffness in a journal bearing and maintains the rotor in equilibrium.

The validity of Muszynska's assumption can be verified by experiments. It is known that an oscillation starts when the rotor RPM crosses up some value and stops when RPM crosses down the other one. A sophisticated experiment shows that the resonance appears at the frequency, which is approximately equal to $\lambda\Omega$, when the rotor is excited by a non-synchronous perturbation force with respect to the rotor speed.

Fluid forces acting on the rotor in coordinates rotating at the same angular frequency as the spring and damper system are given by the formula

$$\mathbf{F}_{rot} = K\left(\mathbf{r}_{rot} - \mathbf{u}_{rot}\right) + D\left(\dot{\mathbf{r}}_{rot} - \dot{\mathbf{u}}_{rot}\right), \tag{1}$$

where the parameters, K and D, specify proportionality of stiffness and damping to the relative position of the journal centre displacement vector $\mathbf{r}_{rot} - \mathbf{u}_{rot}$ and velocity vector $\dot{\mathbf{r}}_{rot} - \dot{\mathbf{u}}_{rot}$, respectively. The equation of motion in stationary coordinates is obtained

$$M\,\ddot{\mathbf{r}} + D\left(\dot{\mathbf{r}} - \dot{\mathbf{u}}\right) + \left(K - jD\lambda\Omega\right)\left(\mathbf{r} - \mathbf{u}\right) = mr_u\omega^2 \exp\left(j\left(\omega t + \delta\right)\right), \tag{2}$$

where M is the total rotor mass. The unbalance force, which is produced by unbalance mass m mounted at a radius r_u, acts in the radial direction and has a phase δ at time $t = 0$. If $\mathbf{u} = 0$, the equation of motion turns to the form

$$M\,\ddot{\mathbf{r}} + D\dot{\mathbf{r}} + \left(K - jD\lambda\Omega\right)\mathbf{r} = mr_u\omega^2 \exp\left(j\left(\omega t + \delta\right)\right). \tag{3}$$

The equation (3) is solved by using Matlab-Simulink in the next part of this paper but firstly some experiments are discussed.

If the system were linear, then the unstable rotor vibration would spiral out to infinity when the rotor angular frequency crosses the Bently-Muszynska threshold

$$\Omega_{Crit} = 2\pi\, f_{Crit} = \frac{\sqrt{K/M}}{\lambda}. \tag{4}$$

The frequency transfer function relating a harmonic force $\mathbf{F}$ at the angular frequency ω to the centreline position $\mathbf{r}$ is given by the following formula

$$G_{Fr}(j\omega) = \frac{1}{K - M\omega^2 + j(\omega D - \lambda \Omega D)} \tag{5}$$

If the resonant frequency $\omega_0 = x\Omega$ exists then the dimensionless quantity x is the solution of a cubic equation and x is simultaneously fulfilling a condition as follows

$$\begin{aligned} &2(M\Omega)^2 x^3 + \left(D^2 - 2KM\right)x - \lambda D^2 = 0 \\ &6(M\Omega)^2 x^2 + \left(D^2 - 2KM\right) > 0 \end{aligned} \tag{6}$$

If the rotor angular frequency Ω is approaching the critical frequency Ω_{Crit} then the maximum of the $G_{Fr}(j\omega)$ magnitude is reached at the angular frequency $\omega_0 = \lambda\Omega$.

3. Experiments with Rotorkit system

To study motion of the shaft in a journal bearing the Rotorkit device, product of Bently Nevada, is employed. The Rotorkit shaft was equipped by two flywheels. To find out the model parameters, the measurements of the journal position time history during run-up and coast-down were carried out. The RPM profile is shown in figure 3 while the time history of the journal coordinates X ($\mathrm{Re}(\mathbf{r})$) and Y ($\mathrm{Im}(\mathbf{r})$) is shown in figure 4. The increase and decrease in RPM is at the constant ramp rate. As it is evident, the journal steady-state oscillation begins when the rotor RPM crosses up the threshold of 2400 RPM and ends when the rotor crosses down the threshold of 1700 RPM. This phenomenon is known as an oil whirl (Tůma &Biloš, 2007).

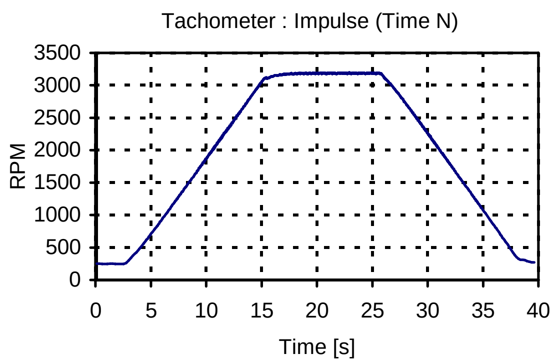

Figure 3. Time history of the rotor RPM

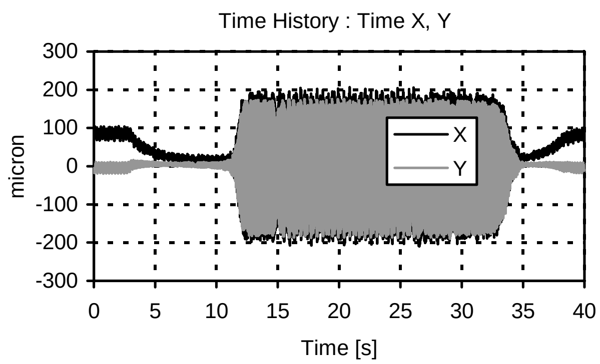

Figure4. Time history of the journal coordinates X and Y

The rotor has a residual unbalance which is exciting a component at the frequency Ω (1 ord) in the frequency spectrum of the journal centerline coordinates. The self excited vibration at the frequency $\lambda\Omega$ (0.48 ord) results from the oil fluid effect. Both these components (0.48 and 1 ord) dominate in the journal vibration frequency spectrum.

4. Simulink model of the rotor system

The equation of motion (3) contains a complex vector $\mathbf{r}(t)$, as an unknown function of time, and the equation parameters are complex quantities as well. The complex function can be replaced by the real and imaginary functions and solved as many similar models. In this paper, an approach based on Matlab-Simulink feature, which allows connecting blocks by complex signals, is preferred. Except of the integration function, all the blocks employed in the Simulink model for the motion equation (3) can work with the complex parameters and functions.. The complex signal is decomposed into the real and imaginary parts for individual integration operation and then they are combined to the complex signal again.

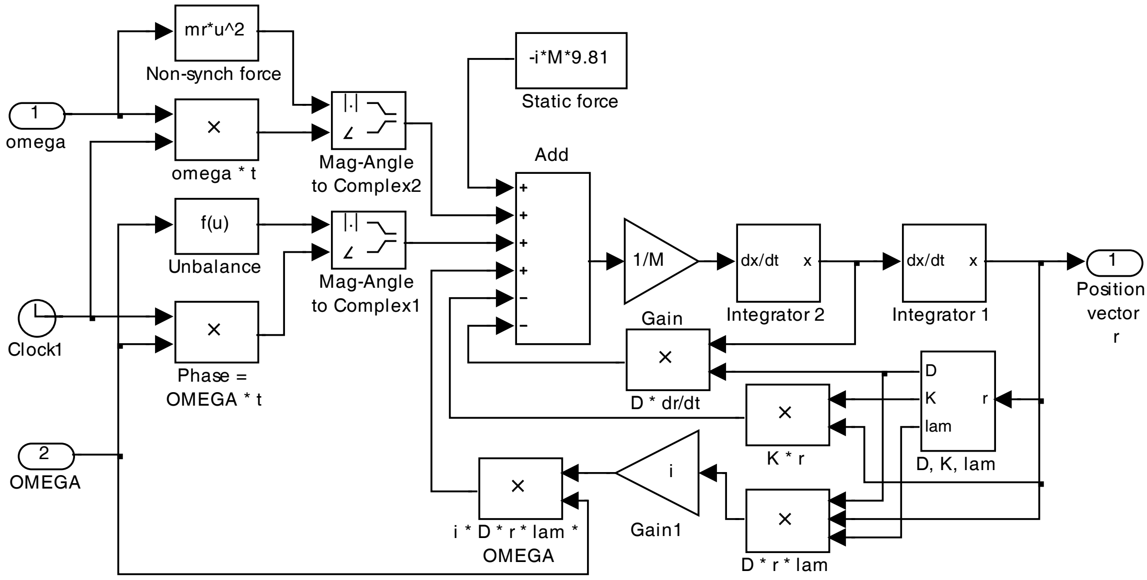

Figure 5: Model of a journal motion in a plane perpendicular to the rotor axis

The Simulink block diagram for the motion equation is shown in figure 5. The system is excited by an unbalance force rotating at the same angular velocity Ω (OMEGA) as the rotor and by the non-synchronous perturbation force rotating by the angular velocity ω (omega), amplitude of which is proportional to the square of the angular velocity.

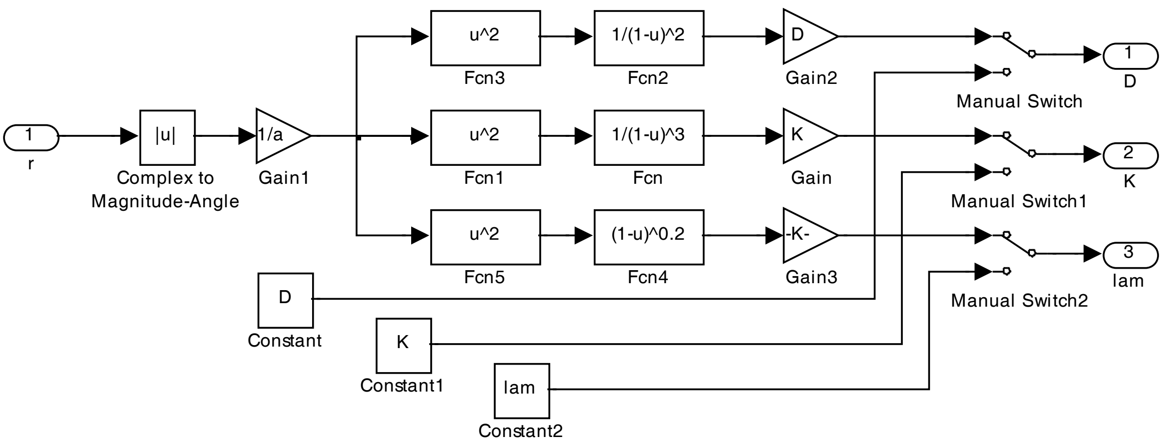

Figure 6. Non-linear block D, K, lam

The parameters K and D, specifying oil film stiffness and damping, are a function of the journal centerline position vector, namely the oil film thickness. It was proved that the closer position of the journal to the bearing wall and simultaneously the thinner oil film, the greater value of both these parameters. Some authors, such as Muszynska, assume that it is possible to approximate these functions by formulas

$$K = K_0 \Big/ \left(1-\left(|\mathbf{r}|/e\right)^2\right)^3, \quad D = D_0 \Big/ \left(1-\left(|\mathbf{r}|/e\right)^2\right)^2, \quad \lambda = \lambda_0 \left(1-\left(|\mathbf{r}|/e\right)^2\right)^{1/5} \tag{7}$$

where *e* is a journal bearing clearance. The non-linear block D, K, lam in figure 5 has its inner structure shown in figure 6. The factors, which are multiplying the parameter *K*, *D*, lam, as a function of the position vector relative magnitude, are shown in figure 7. The authors of this paper analysed the other formula structure as well (Tůma & Bilošová & et al, 2008).

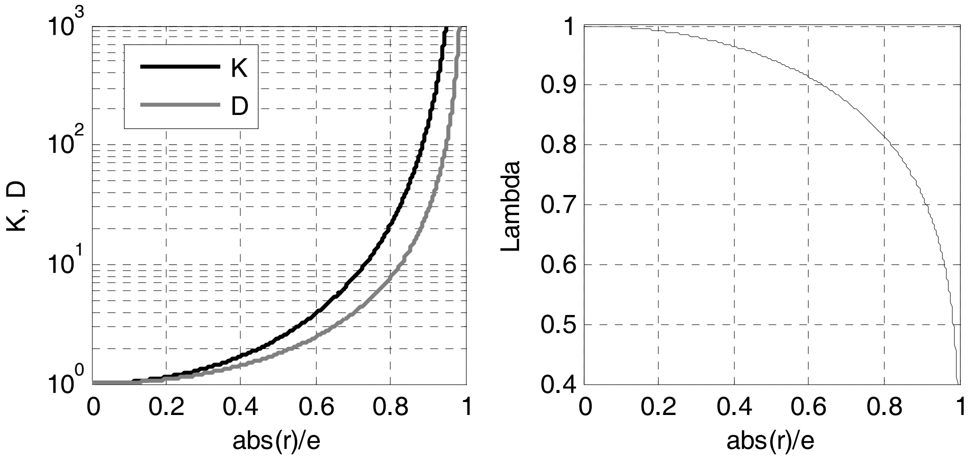

Figure 7. Effect of the position vector relative magnitude related to the bearing clearance on the relative value *K*, *D*, λ (Lambda) related to the initial value ($\mathrm{abs}(\mathbf{r}) = 0$)

5. Simulation study of the model behavior

The numeric solution of the equation of motion is obtained by using Matlab-Simulink. As the rotor system stability margin depends on the oil film stiffness and rotor mass, the first step is to estimate the parameter K. This task is not an easy problem due to the rotor static load by the gravity force and the dependence of the oil film stiffness on the rotor eccentricity. The second problem is an estimation of the parameter D, which predefines the rotor system vibration mode at the angular frequency, which is approximately equal to the half of the rotor angular frequency.

The agreement between the mentioned experiment and the simulation model is reached for the following values of the parameters

M =1.6;	% [kg] rotor mass
lam_0 = 0.475;	% [-] fluid averaged circumferential velocity ratio (lambda)
K_0 = 4000;	% [N/m] oil film stiffness
D_0 = 1000;	% [Ns/m] oil film damping coefficient
e = 0.0002;	% [m] clearance in the journal bearing
mr_u = 0.00001;	% [kgm] product of the unbalance mass m mounted at a radius r_u .

The value of the product mr_u corresponds to the ISO balance quality grade between G 1 and G 2.5 at 2500 RPM. The simulation starts at the zero value of the rotor speed. The simulation results are shown in figure 8. The initial journal position is situated in the point, where the real part of the position vector is as follows $\mathrm{Re}(\mathbf{r}) = 0$, while the imaginary part of the position vector is a value satisfying to the solution of the equation $K\,\mathrm{Im}(\mathbf{r}) = -Mg$. The experiments show that if the rotor is in an unstable state (vibration are limited only by the bearing wall), then the frequency of vibration is slightly less than half the rotor rotational frequency Ω. The ZOOMs of the position vector real and imaginary parts just before and after the vibration onset, which are shortened into the time interval of 0.2 s, are shown in figure 8 as well. Comparison of the number of waves in the time intervals of the same length shows that the frequency of vibration drops to half the frequency before the vibration onset. The effect of the damping parameter D_0 for K_0 = 4000 N/m on the shape of the journal centerline orbit plot during run-up is shown in figure 9. The orbit for D_0 = 2000 Ns/m is the most similar to the measurement results. It can be concluded that the behavior of the simulation model is almost the same as the true rotor system (Tůma & Bilošová et al, 2008).

All the simulations are done by using Matlab-Simuling with the variable integration step and the ODE45 integration method setting.

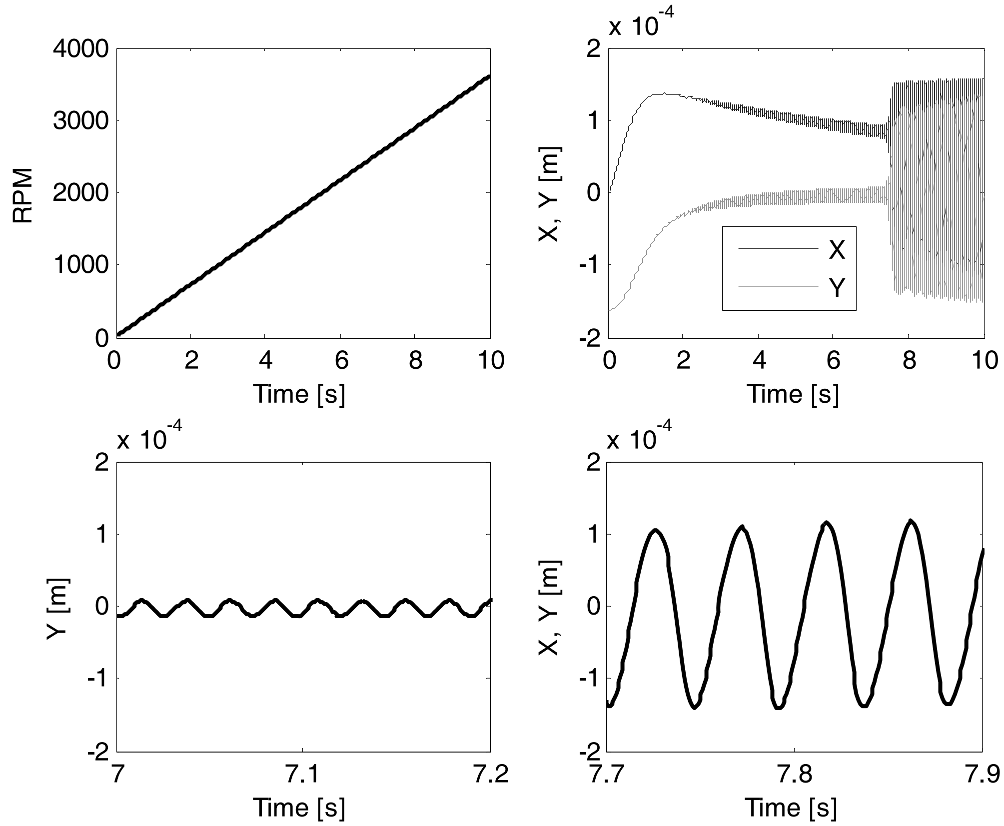

Figure 8. RPM profile and time history of the journal centreline coordinates till the onset of the fluid induced vibration starts up and ZOOMs just before and after the vibration onset

6. Analysis of the linear frequency response function

As the measurement of the rotor system response to the non-synchronous perturbation is not available yet, the simulation is replaced by the evaluation of the frequency response magnitude (5) as a function of the dimensionless frequency f/f_{rot}, which is shown in figure 10. The magnitudes of the frequency response on the figure left side are evaluated for the rotor steady-state speed 1800 RPM and for some multiples (1x, 2x, 5x and 10x) of the initial values of the parameters K_0 and D_0. The resonant frequency is approximately at the mentioned dimensionless frequency λ, i.e. slightly less then 0.5.

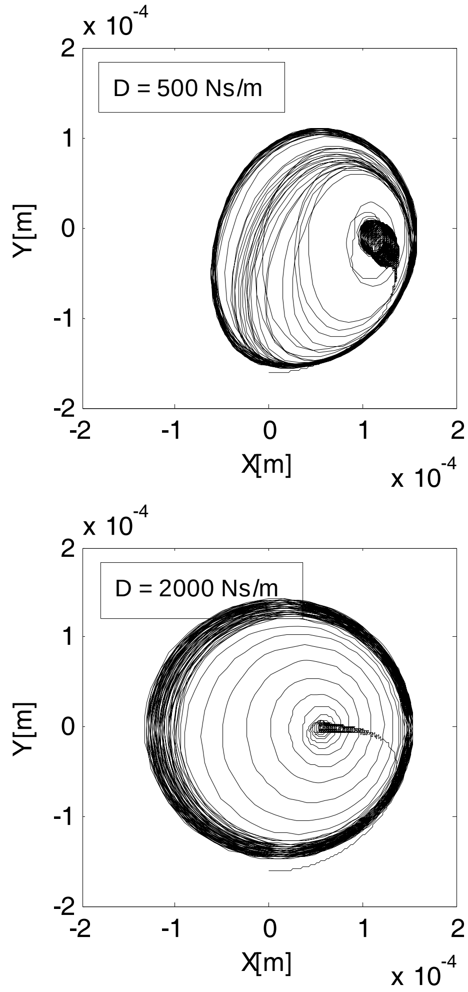

Figure 9. Orbit plot for the oil stiffness K_0 = 4000 N/m and various values of the damping

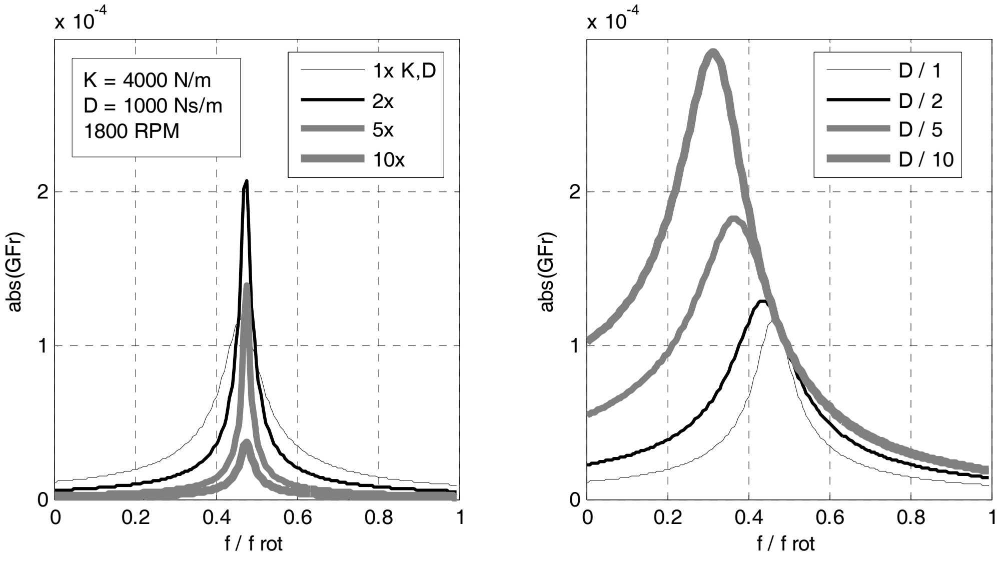

Figure 10. The frequency response magnitude as a function of the dimensionless perturbation force rotational frequency related to the rotor rotational frequency

The magnitude of the frequency response on the figure right side differs in the value of the parameter D_0. According to the experiments (Muszynska, 2005), the resonant frequency is greater than the dimensionless frequency 0.4. It can be concluded that the assumed relationship between the values of K_0 and D_0 seems to be satisfying.

7. Conclusion

The lumped parameter model of the journal centerline motion in the journal bearing is based on the Muszynska's theory. The equation of motion contains the complex vector and parameters. The main goal of the simulation study was to verify the model principle by comparing simulation results with results of experiments, which are described in many papers. The paper is focused on the coincidence between the model and experiment when the instability of motion and the vibration mode at the non-synchronous perturbation occur. The critical signification has assumption about the stiffness, damping and fluid averaged circumferential velocity ratio as a function of the journal centerline position vector magnitude. In comparison to the previously published paper (Tůma & Bilošová et al, 2008), the approximation submitted by Muszynska (2005) was tested.

The simulation of the rotor system using Matlab-Simulink confirms the agreement between the Muszynska's model and experiments.

Acknowledgement

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