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METHOD OF INTERPOLATED ELLIPSES BASED ON IMAGE CORRELATION TECHNIQUE

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Summary: *This paper presents the application of Digital Image Correlation (DIC) methodology for the measurement of strain fields using the Method of Interpolated Ellipses (MIE). It will be shown that MIE has the ability to measure strain fields with higher Signal to Noise Ratio in comparison with standard approaches of the strain field measurement. Consequently MIE is less sensitive on the measurement inaccuracy which is always present in real experiments.*

2. Introduction

The Digital Image Correlation (DIC) method (Russell, 1984) is nowadays a technique commonly applied for full field measurement of the displacement fields. DIC follows self-similar regions (subsets) in an image sequence, acquired during the measurement. A grid of control points is defined in the first (reference) image. Each control point serves as a centre of one subset surrounding this point. Subset centres (coordinates) are searched in the subsequent (target) images. Each subset has to cover the distinguishable image structure.

The search of self-similar places is mathematically based on the well known cross-correlation calculation. An number of algorithms exists for this search. One possible procedure is processed as follow. A normalized cross-correlation is calculated between reference and target subsets in the same reference point coordinates as well as between reference subset and target subsets surrounding the reference point. A matrix of cross-correlation coefficients is acquired by this way. Finally, the absolute sub-pixel peak of the cross-correlation matrix is found using a second order polynomial surface. The peak position is used as coordinates of a new reference control point. The described procedure is repeated for all images step by step (Vavřík et al. 2007).

The grid of control points is standardly defined as orthogonal; consequently the deformation fields are calculated using orthogonal X an Y components. Another approach can use the Method of Interpolated Ellipses (Vavřík & Zemanková, 2004), where a hexagonal grid is employed. The Method of Interpolated Ellipses (MIE) assumes that a small circle defined on the unloaded specimen surface will be deformed into an ellipse during loading. Each circle and consequent ellipse is interpolated by six neighboring dots of a hexagonal grid. Principal strains and their orientation with respect to the global coordinate system are determined

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directly from the actual geometry of the grid without necessity of determining the shear strain. The angle between ellipse's principal axis and global axis represents the angular rotation of the principal strains tensor. This is the main advantage of this approach which becomes important especially in the situation when non uniform strain fields surround the crack tip or if rigid body motion is presented. Although hexagonal grid was manufactured using photoresist technique in the past (Vavřík & Zemánková, 2004) the grid can be defined as well in meaning of control points grid defined by image correlation technique.

3. Methodology and Discussion

A comparison between two strain fields calculations using different definitions of the control points grid for DIC is based on numerically generated "experimental" image. Such image allows set the level of the strain intensity and orientation of the principal strain tensor against the global coordinate system. The image is generated as random spread of Gaussian speckles. This image simulates the unloaded plane strain state. A pure shear state was simulated by reshaping this image. The Fig. 1. shows an unloaded state where an hexagonal measuring grid is depicted. We tested the influence of grid rotation at three angle (0, π/8 and π/4) for an orthogonal grid rotated by π/4, see Fig. 2.

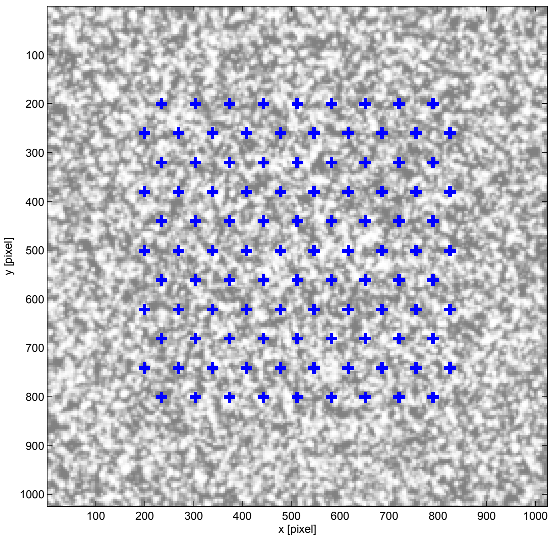

Fig.1: Hexagonal grid used for MIE

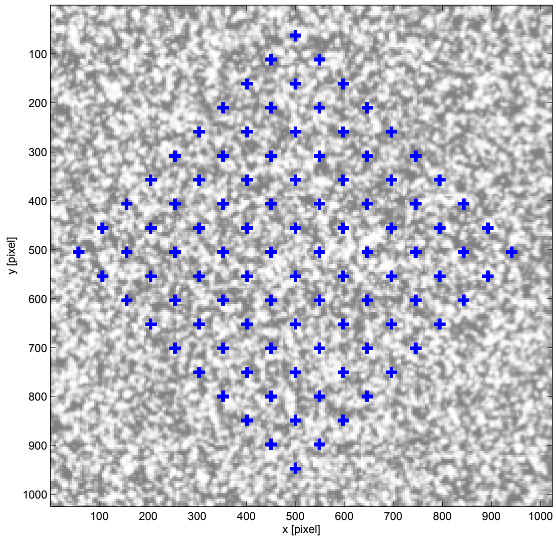

Fig. 2.: Orthogonal grid rotated by π/4.

All strain components are required for the full description of strain field under the general loading state. Only the first and second principal strain ε_1, ε_2 measured by MIE are necessary as the numerical experiment simulates the plane strain state. On the contrary four strain components ε_x, ε_y, γ_{xy}, γ_{yx} have to be measured if the orthogonal grid is used. Although shear components γ_{xy}, γ_{yx} should be equivalent for isotropic materials these can be different in experimental measurements due to the natural measuring error.

The so called Signal-to-noise the ratio (SNR) was used for characterizing the robustness of the methodology used against the measuring error. The SNR compares the square of the measured signal to the square of background noise: $SNR=S^2/N^2$. The higher the ratio, the less harmful the background noise is. This signal S is defined as root mean square (RMS) of the signal components:

$$S = \sqrt{1/n \sum s_i^2} \, , \qquad (1)$$

where s_i are the component signals therefore S is the average signal.

Let us focus on a pure shear strain state. In this case the strain components are governed by the well known equations:

$$\begin{aligned} \varepsilon_1 &= \sqrt{\varepsilon_x^2 + \gamma_{xy}^2} \\ \varepsilon_2 &= -\varepsilon_1 \end{aligned} \qquad (2)$$

$$\begin{aligned} \varepsilon_x &= \varepsilon_1 \cos 2\alpha, \\ \varepsilon_y &= -\varepsilon_1 \cos 2\alpha, \\ \gamma_{xy} &= \varepsilon_1 \sin 2\alpha, \\ \gamma_{yx} &= \gamma_{xy} \end{aligned} \qquad (3)$$

The noise N should be constant dependent on the measuring length only. However N varies in practice due to stochastic nature of the random speckle used by DIC. The SNR of first principal strain is taken as reference value below:

$$SNR_{\varepsilon_1} = \varepsilon_1^2 / N^2 \qquad (4)$$

The SNR for the second principal strain is equivalent by theory. Consequently, the SNR for other components can be expressed using equations (3) and (4) as follows:

$$\begin{aligned} SNR_{\varepsilon_x} &= SNR_{\varepsilon_1} \cos^2 2\alpha, \\ SNR_{\varepsilon_y} &= SNR_{\varepsilon_1} \cos^2 2\alpha, \\ SNR_{\gamma_{xy}} &= SNR_{\varepsilon_1} \sin^2 2\alpha, \\ SNR_{\gamma_{yx}} &= SNR_{\varepsilon_1} \sin^2 2\alpha, \end{aligned} \qquad (5)$$

It is clear from equations (5), that the SNR for strain components varying from zero up to the SNR of first principal strain depends on the measuring grid rotation. SNR equal to zero means that we are measuring noise only. Noise was taken as constant for the equations above just to show the relations between SNR for different components.

We can compare the composite SNR for MIE and standard orthogonal grid measurements of the strain field using a relation for signal (1) as follows:

$$\begin{aligned} SNR_{MIE} &= SNR_{\varepsilon_1}, \\ SNR_{ort} &= 1/2 \cdot SNR_{\varepsilon_1} \end{aligned} \qquad (6)$$

As arising from equation (6), the composite SNR is constant for both approaches. However, the SNR for MIE has to be two times higher than for the orthogonal grid measurement.

The noise N may be not constant for components measurement. Similarly, components which are equal by theory may be non equal in experiment. Therefore a different definition of the composite SNR has been accepted for the simulation purpose:

$$\begin{aligned} SNR_{\varepsilon_1} &= 1/2\left(\varepsilon_1^2 / N_{\varepsilon_1} + \varepsilon_2^2 / N_{\varepsilon_2}\right) \\ SNR_{ort} &= 1/4\left(\varepsilon_x^2 / N_{\varepsilon_x} + \varepsilon_y^2 / N_{\varepsilon_y} + \gamma_{xy}^2 / N_{\gamma_{xy}} + \gamma_{yx}^2 / N_{\gamma_{yx}}\right) \end{aligned} \qquad (7)$$

Simulations based on random speckle and DIC measurement showed dependence of the noise on measuring grid definition, summarized in Table 1:

Table 1: Composite SNR at different angles: image without noise.

Angle	0	π/8	π/4
SNR for MIE	1184	1276	1237
SNR for Orthogonal grid	365	422	336

The SNR for MIE appears always three times higher than the SNR for the orthogonal grid. It is better than it is arising from the equations (6). This is probably results from the fact, that six points are used for MIE calculations contrary to standard orthogonal grid calculations where just four points are used. Consequently better statistic can be expected from this reason.

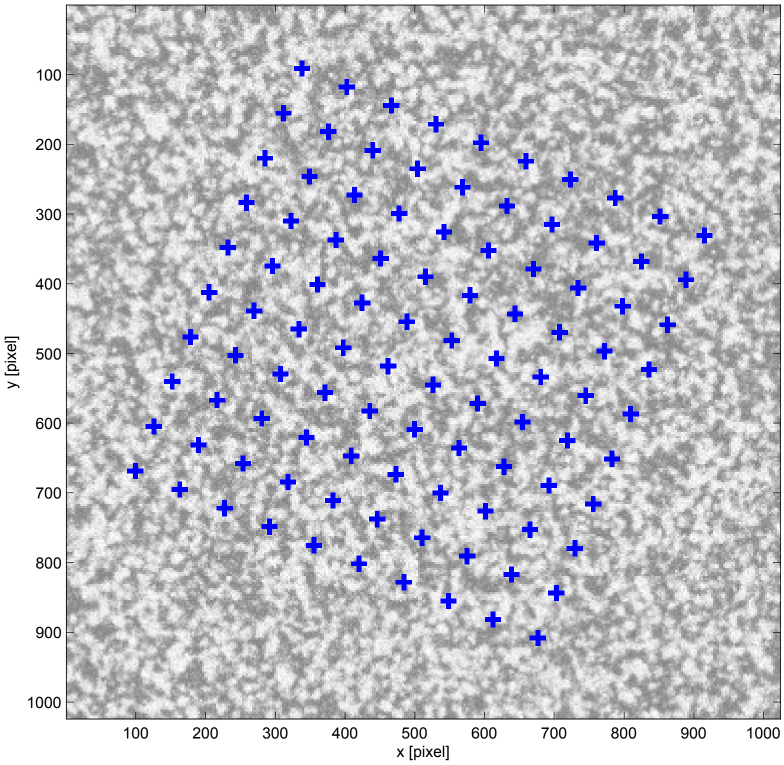

Fig. 2: Random speckle image with added white noise.

The influence of the image noise was simulated adding white noise into the image (see Fig. 2). Calculated SNR values are given in Table 2. Similarly as above, the SNR is approximately three times better for MIE than for orthogonal grid calculations.

Table 2: Composite SNR at different angles: noisy image.

Angle	0	π/8	π/4
SNR for MIE	6.8	4.3	4.24
SNR for Orthogonal grid	2	1.9	1.35

4. Conclusions

It was verified that the Method of Interpolated Ellipses can be successfully used for strain field measurements by the Digital Image correlation technique.

It was shown that Method of Interpolated Ellipses is significantly more robust against measurement errors in comparison with standard approaches employing an orthogonal measuring grid.

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References

Russell S. S. and Sutton M. A. (1984) Experimental Mechanics, Vol. 29, pp. 237-240

Vavřík D., Jakůbek J., Jakůbek M. and Jandejsek I. (2007) Radiographic Observation of 3D Displacement Field and Consequent Damage Zone Evolution, In Experimental Analysis of Nano and Engineering Materials and Structures, ICEM 13, Alexandropoulis, Greece, p.355

Vavřík, D. and Zemánková, J. (2004) Crack Instability in Ductile Materials Analyzed by the Method of Interpolated Ellipses., Experimental Mechanics, Vol. 44, pp. 327-335