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EFFECTIVE THERMAL CONDUCTIVITY OF CARBON-CARBON PLAIN WAVE TEXTILE COMPOSITES

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Summary: *An efficient approach to the evaluation of effective thermal conductivity of carbon-carbon (C/C) plain weave textile composites using the Mori-Tanaka (MT) method is presented. The method proves its potential even if applied to real orthotropic material systems with various types of imperfections including the non-uniform waviness of the fibre-tow paths. Uncertainties linked to other geometrical parameters are mostly reflected in the derivation of an optimal shape of the equivalent ellipsoidal inclusion using a family of ideal periodic unit cells (PUC). Evaluation of the effective properties in the absence of pores is presented first. Ellipsoidal pores are then introduced in the second step into a new homogenized matrix to account for the presence of large vacuoles in the composite. Several examples are presented to demonstrate this approach.*

1. Introduction

Carbon-Carbon plain weave fabric composites belong to an important class of high-temperature material systems. An exceptional thermal stability together with high resistance to thermal shocks or fracture due to rapid and strong changes in temperature have made these materials almost indispensable in a variety of engineering spheres including aeronautics, space and automobile industry. While their appealing thermal properties such as low coefficients of thermal expansion and high thermal conductivities are known, their prediction from the properties supplied by the manufacturer for individual constituents is far from being trivial since these systems are generally highly complicated (Fig. 1).

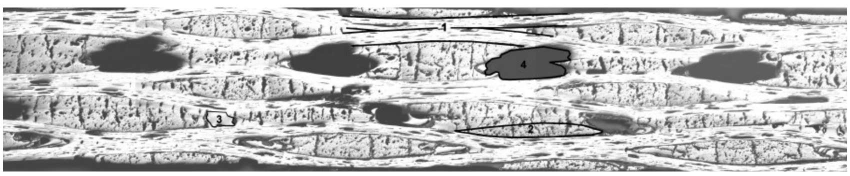

Fig. 1: Plain weave composite

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In the last decade, effective media theories, widely used in classical continuum micromechanics, have been recognized as an attractive alternative to time-consuming finite element based methods (FEM) (Skoček et al., 2008). All studies report good correspondence with experimental data with an error comparable to an experimental scatter. In this regard, the MT procedure introduced by Hatta & Taya (1986) for thermal conductivity is utilized to determinate the effective thermal coefficients for C/C composites on the micro- and meso-scale. On both scales a two step homogenization approach is adopted. It combines evaluation of effective properties in the absence of pores with a subsequent homogenization step in which the porous phase is introduced into a new homogenized matrix.

The paper is organized as follows. The second section presents an overview of the Mori-Tanaka method in the spirit presented for the evaluation of effective elastic properties of C/C plane weave composites in Skoček et al. (2008). Particular application to plane weave composites is discussed in the following section.

2. Mori-Tanaka method

The approach for the prediction of effective thermal conductivities of C/C composites presented hereafter follows the theoretical lines discussed in (Skoček et al., 2008), but is extended to account for the porous phase. Limiting our attention to a steady state heat conduction problem we write, in view of the Mori-Tanaka method assuming an ellipsoidal shape of the inhomogeneity, the local temperature gradient $\boldsymbol{H}$ as

$$\boldsymbol{H} = \boldsymbol{H}_0 + \boldsymbol{H}^f = \boldsymbol{H}_0 + \mathbf{S}\boldsymbol{H}^*, \tag{1}$$

where $\boldsymbol{H}_0$ is the average temperature gradient in the matrix and $\boldsymbol{H}^f$ represents the fluctuation part derived from and an equivalent inclusion problem (Hatta & Taya, 1986). Note that the second order tensor $\mathbf{S}$ and the vector $\boldsymbol{H}^*$ are analogous to the Eshelby tensor and transformation strain, respectively, for the elasticity problem. The Mori-Tanaka estimates are then provided by

$$\mathbf{K} = \mathbf{K}_0 + \left[\sum_{i=1}^{N} c_i (\mathbf{K}_i - \mathbf{K}_0) \langle \mathbf{P}_i \rangle \right] \left(c_0 \mathbf{I} + \sum_{i=1}^{N} c_i \langle \mathbf{P}_i \rangle \right)^{-1}, \tag{2}$$

where index 0, 1,…, i,…, N refer to individual constituents with 0 reserved for the matrix phase, c_i are the corresponding volume fractions and $\mathbf{K}_i$ are the thermal conductivity matrices. The $\langle \cdot \rangle$ brackets represent the orientation averaging to grasp the fibre-tow paths, see e.g. Skoček et al. (2008) and Zeman (2003), by means of the Euler angles (Fig. 2). Tensor $\mathbf{P}$ is then provided by

$$\mathbf{P} = \mathbf{I} - \mathbf{S}_i \mathbf{U}^{-1} \mathbf{f}_i, \tag{3}$$

where

$$\begin{aligned} \mathbf{f}_i &= \mathbf{K}_i - \mathbf{K}_0, \\ \mathbf{U} &= \mathbf{f}_i \mathbf{S}_i + \mathbf{K}_{0i}. \end{aligned} \tag{4}$$

The expressions above offer two possible approaches for the determination of the effective material parameters, one-step or multi-step method. The first treats all inhomogeneities simultaneously. The second one adopts the procedure where each inclusion is embedded into a new homogenized matrix in a certain hierarchical manner. The latter approach is employed henceforth.

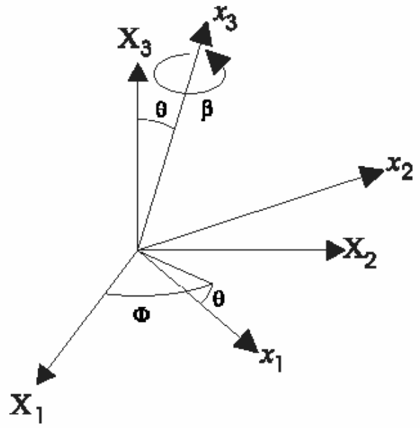


Fig. 2: Definitions of Euler angles

3. Application

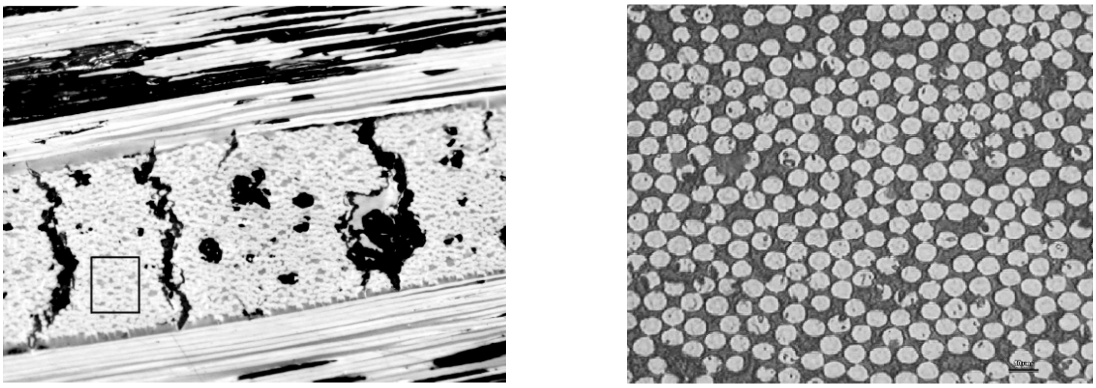
It has been demonstrated in our previous work, see e.g. Šejnoha et al. (2007), that image analysis of real, rather then artificial, material systems plays an essential role in the derivation of a reliable and accurate computational model. This issue is revisited here for the case of woven fabric C/C laminate with particular relation to the adopted uncoupled multi-scale solution strategy. In this paper two levels (mico-scale and meso-scale) are introduce to deal with the complicated material system.

A typical segment of the composite laminate appears in Fig. 2 showing characteristic porosity which may exceed at the structural level (macroscale) 30% and is often considered as an intrinsic property of this type of composite. Several such micrographs were processed by Tomková et al. (2007) to acquire information regarding the basic structural units like an average thickness of carbon tows, size of voids, shape and essential dimensions of fibre tow cross-section, distribution of transverse and delamination cracks etc. These parameters were exploited in Tomková et al. (2007) to construct equivalent periodic unit cells subsequently used in the FEM simulations (Fig. 4). The resulting effective conductivities derived on individual scales will be used herein as a benchmark for the comparison with the MT predictions.

3.1. Micro-scale

Starting with the fibre tow composite as the basic structural element we call Fig. 3 showing a typical shape of the fibre tow cross-section and significant amount of transverse cracks and voids resulting in a non-negligible porosity up to 15%. There two-step homogenization approach is used to determine the effective conductivity tensor as mentioned in previous section. The material parameters are taken from Tomková et al. (2007).

Tab. 1 summarizes effective thermal conductivities derived with the help of the Mori-Tanaka method. As expected, a reasonably good agreement with finite element simulations was achieved. Note, that on micro-scale the application of Mori-Tanaka method is particularly simple since in both homogenization steps the underlying matrix is isotropic. This, however, is not true when performing the same analysis on a meso-scale as discussed next.



(a) (b)

Fig. 3: Homogenization on micro-scale: (a) fibre tow composite, (b) fibre-matrix composite

Tab. 1: Effective thermal conductivity [$Wm^{-1}K^{-1}$] (micro-scale)

Method	Material		Thermal conductivity
	fibre	inclusion	
Multi-step MT	10x10x∞	10x10x∞	(1.54; 1.54; 19.13)
		10x10x∞ and 10x100x∞	(1.02; 1.63; 19.13)
One-step MT	10x10x∞	10x10x∞	(1.68; 1.68; 19.13)
		10x50x∞	(1.23; 1.82; 19.13)
FEM	-	-	(1.12; 1.77; 19.01)

3.2. Meso-scale

Having derived the effective material parameters for the fibre tow composite the assumed two level homogenization procedure continues along the same lines on meso-scale. An idealized geometrical model of a plain weave composite is utilized. For this particular case (as well as for almost all geometrical models available in the literature), the centreline of the warp tow is described using a simple trigonometrical function (Skoček et al., 2008).

The tree-dimensional model (PUC) is constructed to give a true picture of the material system (Fig. 4a,b). This periodic unit cell is employed in the first step of the homogenization procedure on the meso-scale. The acquired properties are used as a point of departure for the second step where the air voids are taken into account (Fig. 4c-f). Owing to the orthogonal arrangement of tows in the ideal unit cell the effective conductivity matrix is orthotropic. To evaluate $\mathbf{S}$ tensor thus calls for the solution of the problem of an ellipsoidal inclusion in an orthotropic matrix. This problem can be solved by introducing the following substitution

$$x_i = \sqrt{k_i}\,\tilde{x}_i\,, \tag{2}$$

where $i = 1,2,3$, $\tilde{x}_i$ are new coordinates and k_i.are the thermal coefficients of the matrix. Then, the $\mathbf{S}$ tensor is possible to derive through the relation written in Hatta & Taya (1986).

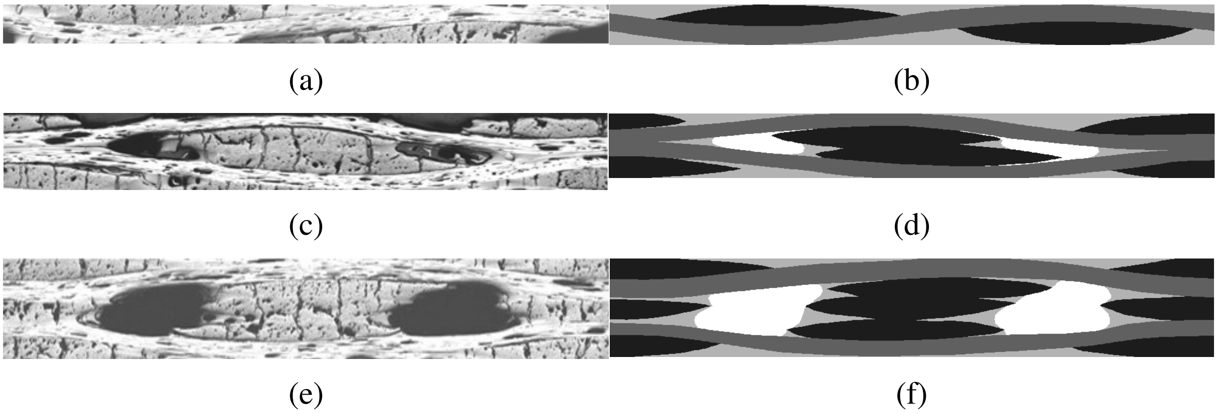


Fig. 4: Homogenization on meso-scale: (a)-(b) PUC1 representing carbon tow-carbon matrix composite, (c)-(d) PUC2 with vacuoles aligned with delamination cracks due to slip of textile plies, (e)-(f) PUC3 with extensive vacuoles representing the parts with textile reinforcement reduction due to bridging effect in the middle ply

The results are presented in Tab. 2 assuming the porous phase being represented by an ellipsoids with dimensions obtained from a simple trial and error method. To arrive at a better agreement with finite element simulations, a search for an optimal shape of the equivalent ellipsoidal inclusion, much similar to that described in Skoček et al. (2008), is needed. Also note rather different shapes of voids for various PUCs, which were not taken into account in the MT predictions.

Tab. 2: Effective thermal conductivity [$Wm^{-1}K^{-1}$] (meso-scale)

Analysis	Type PUC	Tow	Inhomogeneity		Thermal conductivity	
			mutual ratio	volume	longitudinal	transversal
MT	PUC 1	∞x400x33	-	-	9.14	1.87
MT	PUC 2		4x(150x150x30)	0.097	8.13	1.45
MT	PUC 3		8x(150x150x30)	0.141	7.69	1.31
FEM	PUC 1	-			9.46	2.27
FEM	PUC 2				9.03	1.47
FEM	PUC 3				7.29	1.53

4. Conclusions

In this contribution two levels of homogenization procedure are shown to determine the effective thermal conductivities for the plain weave composites. Different periodic unit cells are constructed to depict the material system for each scale.

The results presented in Tab. 2 demonstrate an good agreement with the data obtained by the finite element analysis. The difference between the results derived by means of FEM and MT analysis is less than the standard deviation for a material properties examined experimentally.

Acknowledgement

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