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DYNAMICAL ANALYSIS OF THE RAILWAY VEHICLE BOGIE

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Summary: *This contribution presents a method of dynamic analysis of a railway vehicle bogie which is based on the statistical approach. The excitation caused by vertical track irregularities is supposed. Using the linearized model of adhesion characteristics between wheels and rails and torque characteristics of engine, the complete mathematical model of vehicle bogie is transformed into the frequency domain. The dynamic response is expressed by upper limits of deformations and forces transmitted by couplings calculated using spatial power spectral densities of the track irregularities. The methodology is applied to a particular railway vehicle bogie with two individual wheelset drives excited by track irregularities measured along the track for left and right rails, respectively.*

1. Introduction

Modern high-speed railway vehicles show some dynamic phenomena characterized by frequencies in the mid-frequency range, as Claus & Shiehlen (2003) have showed. To describe these phenomena conventional models of the vehicles based on the basis of rigid multi-body systems are not sufficient. The vehicle bogie with two individual wheelset drives (Fig. 1) of the electric locomotive, developed for speeds about 200 *km/h* by the company ŠKODA TRANSPORTATION s.r.o. indicates some specific properties. Especially, spatial vibrations of all drive components, bogie frame and wheelsets supported on visco-elastic ballast as well as the elasticity of the hollow shafts and wheelsets are taken into account.

The mathematical model, modal properties and stability conditions of one individual wheelset drive were presented in the article of authors Zeman et al. (2007a). This contribution extends the methodology of modelling presented in the mentioned paper. Here, we deal with mathematical modelling of a complete railway vehicle bogie with two individual wheelset drives. The aim of this contribution is to present a suitable method for dynamic analysis of the whole bogie with two individual wheelset drives focused on excitation caused by vertical track irregularities from the statistic point of view.

2. Description of the excitation by track irregularities

Numerous measurements demonstrate that track irregularities can be understood as stationary stochastic processes described by power spectral density (PSD) function (Popp & Shielen

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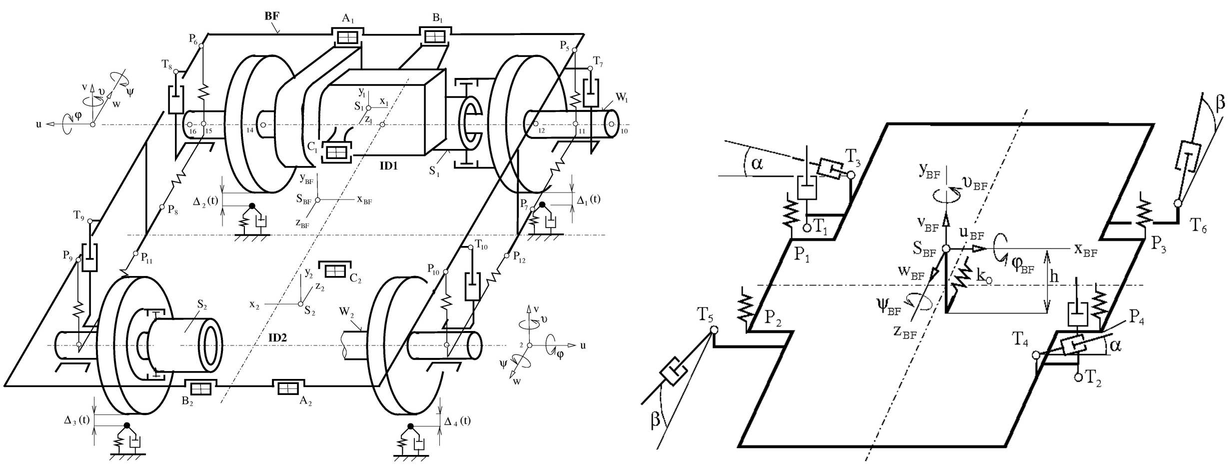

(a) (b)

Fig. 1: Model of the vehicle bogie (a) and bogie frame with secondary suspension (b)

1993). The corresponding single-sided spatial PSD $S(F)$ depends on spatial frequency $F = 1/\lambda$ given in cycle parameter (λ is a wavelength), because the track irregularity is a function of the distance measured along the track. Several track measurements have shown that $S(F)$ can be approximately expressed in the log-log coordinate system by piecewise straight line (Balda, 1993) in the analytical form

$$S(F) = S_i\left(\frac{F}{F_i}\right)^{\kappa_i}, \quad F \in \langle F_i, F_{i+1}\rangle, \quad \text{where} \quad \kappa_i = \frac{\log(S_{i+1}/S_i)}{\log(F_{i+1}/\mathrm{F}_i)} \tag{1}$$

and S_i (S_{i+1}) are PSD values for spatial frequencies F_i (F_{i+1}) as shown in Fig. 2. Keep in mind that the vehicle forward velocity v ms^{-1}, the frequency of the waves $f = vF$ Hz and the spatial PSD is transformed into the standard PSD, as Garg & Dukkipati (1984) have showed. According to (1) we obtain

$$S(f) = \frac{1}{v} S_i\left(\frac{f}{v}\frac{1}{F_i}\right)^{\kappa_i}, \qquad f \in \langle f_i, f_{i+1}\rangle. \tag{2}$$

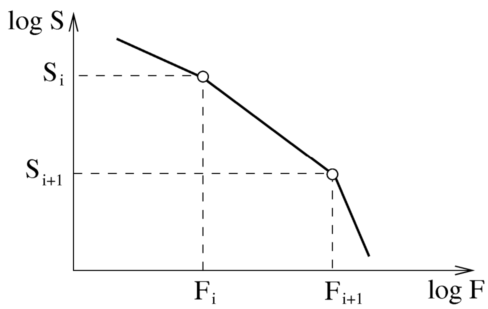

Fig. 2: Approximation of spatial power spectral density

3. Calculation of the dynamic displacements and load of the bogie components

Let us suppose an operational state of the railway vehicle running along the straight track in static equilibrium which is given by longitudinal creepage s_0 of all wheels, by forward velocity v of the vehicle and by vertical wheel force N_0. If the static equilibrium is disturbed by vertical track irregularities, the bogie vibrates and the vector of generalized coordinates can be expressed as a sum of static and dynamic displacements

$$\mathbf{q}(t) = \mathbf{q}_0 + \Delta\mathbf{q}(t), \tag{3}$$

where $\mathbf{q}_0$ satisfies the static equilibrium condition before the disturbance. After linearization of the creep forces and spin torque acting at the contact between rails and wheels and after linearization of the engine torque characteristics in the neighbourhood of the static equilibrium state we obtain full linearized model of the bogie.

The linearized model of the bogie with 165 DOF, written in perturbance coordinates $\Delta\mathbf{q}(t)$ in the neighbourhood of the static equilibrium state before the disturbance by track irregularities, has the form (Zeman et al., 2007b)

$$\mathbf{M}\Delta\ddot{\mathbf{q}}(t) + \left[\mathbf{B} + \mathbf{B}_M + \mathbf{B}_{ad}(s_0, v)\right]\Delta\dot{\mathbf{q}}(t) + \mathbf{K}\Delta\mathbf{q}(t) = \Delta\mathbf{f}(t). \tag{4}$$

Mass, damping and stiffness matrices have block-diagonal structure corresponding to subsystems - the first individual wheelset drive (ID1), the bogie frame (BF) linked by secondary suspension and dampers with a half of car body and the second individual wheelset drive (ID2) - completed by matrices of couplings among them (see Fig. 1).The matrices $\mathbf{B}_M$ and $\mathbf{B}_{ad}(s_0, v)$ express the influence of the linearized engine torque characteristics and creep forces between wheels and rails depending on longitudinal creepage s_0 defining the equilibrium state before the disturbance and on the vehicle velocity v.

Let us consider that the vertical track irregularities are expressed by deviations Δ_j of the rails (see Fig. 1). The whole track structure (rail, railpad, sleeper and ballast) is reduced to a single mass-spring-damper system defined by parameters m_R, b_R, k_R, as Feldmann et al. (2003) shows.

The excitation vector $\Delta\mathbf{f}(t)$ has non-zero components on positions 51, 63, 137, 149, respectively. They fulfill

$$f_j = m_R\ddot{\Delta}_j + b_R\dot{\Delta}_j + k_R\Delta_j, \qquad j = 1,2,3,4 \tag{5}$$

and correspond to vertical displacements of the wheels in the general coordinate vector $\Delta\mathbf{q}(t)$. The model (4) is then rewritten after Fourier transformation into the frequency domain

$$\left\{-\omega^2\mathbf{M} + \left[\mathbf{B} + \mathbf{B}_M + \mathbf{B}_{ad}(s_0, v)\right] + \mathbf{K}\right\}\Delta\mathbf{q}(\omega) = z_R(\omega)\boldsymbol{\Delta}(\omega), \tag{6}$$

where

$$z_R(\omega) = -\omega^2 m_R + i\omega b_R + k_R \tag{7}$$

is complex reduced track stiffness and

$$\boldsymbol{\Delta}(\omega) = \left[\ldots\Delta_1(\omega)\ldots\Delta_2(\omega)\ldots\Delta_3(\omega)\ldots\Delta_4(\omega)\ldots\right]^T \tag{8}$$

is the vector of Fourier transformations of the rail deviations with the above mentioned non-zero components $\Delta_j(\omega)$. Let us suppose a constant forward velocity v of the vehicle in direction of axis z_{BF} (see Fig. 1). Then deviations $\Delta_1(t)$ and $\Delta_2(t)$ can be then expressed as

$$\Delta_1(t) = \Delta_4(t - \Delta t), \qquad \Delta_2(t) = \Delta_3(t - \Delta t), \tag{9}$$

with the time shift $\Delta t = l / v$, where l is the wheelbase of a bogie. Fourier transformations of the dynamic response and excitation are related through the complex frequency response function as follows

$$\Delta \mathbf{q}(\omega) = \mathbf{G}(\omega) z_R(\omega) \mathbf{\Delta}(\omega), \tag{10}$$

where

$$\mathbf{G}(\omega) = \left\{- \omega^2 \mathbf{M} + \left[\mathbf{B} + \mathbf{B}_M + \mathbf{B}_{ad}(s_0, v)\right] + \mathbf{K}\right\}^{-1} = \left[g_{i,j}(\omega)\right] \tag{11}$$

is the transfer matrix function and in accordance with (8), (9)

$$\mathbf{\Delta}(\omega) = \left[\ldots \Delta_1(\omega) \ldots \Delta_2(\omega) \ldots \Delta_2(\omega) e^{i\omega\Delta t} \ldots \Delta_1(\omega) e^{i\omega\Delta t} \ldots\right]^T. \tag{12}$$

The Fourier transformation of an arbitrary dynamic displacement is

$$\Delta q_i(\omega) = G_{i,1}(\omega)\Delta_1(\omega) + G_{i,2}(\omega)\Delta_2(\omega), \tag{13}$$

where the corresponding frequency response functions are

$$G_{i,1}(\omega) = z_R(\omega)\left[g_{i,51}(\omega) + g_{i,149}(\omega) e^{i\omega\Delta t}\right],$$

$$G_{i,2}(\omega) = z_R(\omega)\left[g_{i,63}(\omega) + g_{i,137}(\omega) e^{i\omega\Delta t}\right]. \tag{14}$$

The vertical profile of the rails along the track can be understood as an ergodic Gaussian process with zero mean values with the cross correlation between the rail irregularities Δ_1 and Δ_2 equate to zero.

The power spectral densities for displacements Δq_i can be expressed as

$$S_{q_i}(\omega) = S_{\Delta_1}(\omega)\left|G_{i,1}(\omega)\right|^2 + S_{\Delta_2}(\omega)\left|G_{i,2}(\omega)\right|^2, i \in < 1, n >, \tag{15}$$

where n is the bogie number of DOF.

To design the bogie components, upper estimates of the dynamic forces and torques transmitted by couplings (gearing, clutches, supports of engine stators to the bogie frame etc.) can be calculated. As an illustration, the calculation of the forces transmitted by viscous-elastic supports of engine stator and of gear housing to the bogie frame (BF) is shown.

The force vectors transmitted by rubber silent-blocks A_1, B_1, C_1 in the first individual wheelset drive can be expressed in the form

$$\mathbf{f}_j = \mathbf{K}_t \mathbf{d}_j + \mathbf{B}_t \dot{\mathbf{d}}_j, \qquad j = A_1, B_1, C_1, \tag{16}$$

where $\mathbf{K}_t(\mathbf{B}_t)$ is diagonal stiffness (damping) matrix of one silent-block in a coordinate system which is parallel to coordinate system x_1, y_1, z_1. The vector of their translational deformations is

$$\mathbf{d}_j = \mathbf{u}_1 + \mathbf{R}_j^T \varphi_1 - (\mathbf{u}_{BF} + \mathbf{R}_{jBF}^T \varphi_{BF}), \qquad j = A_1, B_1, C_1. \tag{17}$$

Coordinates of vectors $\mathbf{u}_1$ and $\mathbf{u}_{BF}$ express displacements of mass centre S_1 of engine and mass centre S_{BF} of the bogie frame in directions which are parallel to the axes x_1, y_1, z_1 or x_{BF}, y_{BF}, z_{BF} and coordinates of the vectors φ_1 and φ_{BF} describe angle displacements around the mentioned axes. Skew-symmetric matrices $\mathbf{R}_j$ and $\mathbf{R}_{jBF}$ are determined by coordinates of elasticity centre of silent-blocks in the coordinate system x_1, y_1, z_1 (for $\mathbf{R}_j$) or in the coordinate system x_{BF}, y_{BF}, z_{BF} (for $\mathbf{R}_{jBF}$).

The force vectors $\mathbf{f}_j$ can be expressed as a sum of static and dynamic components

$$\mathbf{f}_j(t) = \mathbf{f}_{j,0} + \Delta\mathbf{f}_j(t), \tag{18}$$

where $\mathbf{f}_{j,0}$ corresponds to the bogie frame static equilibrium condition

$$\mathbf{K}\mathbf{q}_0 = \mathbf{f}_0$$

before the disturbance by track irregularities. According to (16) and (17) the force vector of dynamic components can be expressed in the form

$$\Delta\mathbf{f}_j(t) = \mathbf{K}_j \Delta\mathbf{q}_1(t) + \mathbf{B}_j \Delta\dot{\mathbf{q}}_1(t) - \mathbf{K}_{jBF} \Delta\mathbf{q}_{BF}(t) - \mathbf{B}_{jBF} \Delta\dot{\mathbf{q}}_{BF}(t), \tag{19}$$

where

$$\mathbf{K}_j = \left[\mathbf{K}_t \quad \mathbf{K}_t \mathbf{R}_j^T\right], \quad \mathbf{K}_{jBF} = \left[\mathbf{K}_t \quad \mathbf{K}_t \mathbf{R}_{jBF}^T\right] \in R^{3,6}, \tag{20}$$

and

$$\Delta\mathbf{q}_1(t) = \begin{bmatrix} \Delta\mathbf{u}_1(t) \\ \Delta\varphi_1(t) \end{bmatrix}, \qquad \Delta\mathbf{q}_{BF}(t) = \begin{bmatrix} \Delta\mathbf{u}_{BF}(t) \\ \Delta\varphi_{BF}(t) \end{bmatrix}. \tag{21}$$

Providing that the damping of silent-blocks is proportional to their stiffness with coefficient β, the Fourier transform of the $\Delta\mathbf{f}_j(t)$ is

$$\Delta\mathbf{f}_j(\omega) = (1 + \mathrm{i}\omega\beta)\left[\mathbf{K}_j \Delta\mathbf{q}_1(\omega) - \mathbf{K}_{jBF} \Delta\mathbf{q}_{BF}(\omega)\right]. \tag{22}$$

According to (10) and (12), the previous expression can be rewritten into

$$\Delta\mathbf{f}_j(\omega) = \mathbf{g}_{j,1}(\omega)\Delta_1(\omega) + \mathbf{g}_{j,2}(\omega)\Delta_2(\omega), \quad j = A_1, B_1, C_1, \tag{23}$$

where $\mathbf{g}_{j,1}(\omega), \mathbf{g}_{j,2}(\omega)$ are vectors of frequency response functions. These vectors are calculated from components of the transfer matrix function $\mathbf{G}(\omega)$, complex reduced track stiffness $z_R(\omega)$, matrices $\mathbf{K}_j$ and $\mathbf{K}_{jBF}$, time shift Δt and coefficient β.

The power spectral densities of dynamic forces transmitted by silent-blocks have the form

$$\mathbf{S}_{f_j}(\omega) = \mathbf{G}_j(\omega)\mathbf{S}_\Delta(\omega)\mathbf{G}_j^H(\omega), \quad j = A_1, B_1, C_1, \tag{24}$$

where

$$\mathbf{G}_j(\omega) = \begin{bmatrix} \mathbf{g}_{j,1}(\omega) & \mathbf{g}_{j,2}(\omega) \end{bmatrix} \in \mathrm{C}^{3,2}, \quad \mathbf{S}_\Delta(\omega) = \mathrm{diag}(S_{\Delta_1}(\omega), S_{\Delta_2}(\omega))$$

and superscript H denotes the transposition of conjugate matrix.

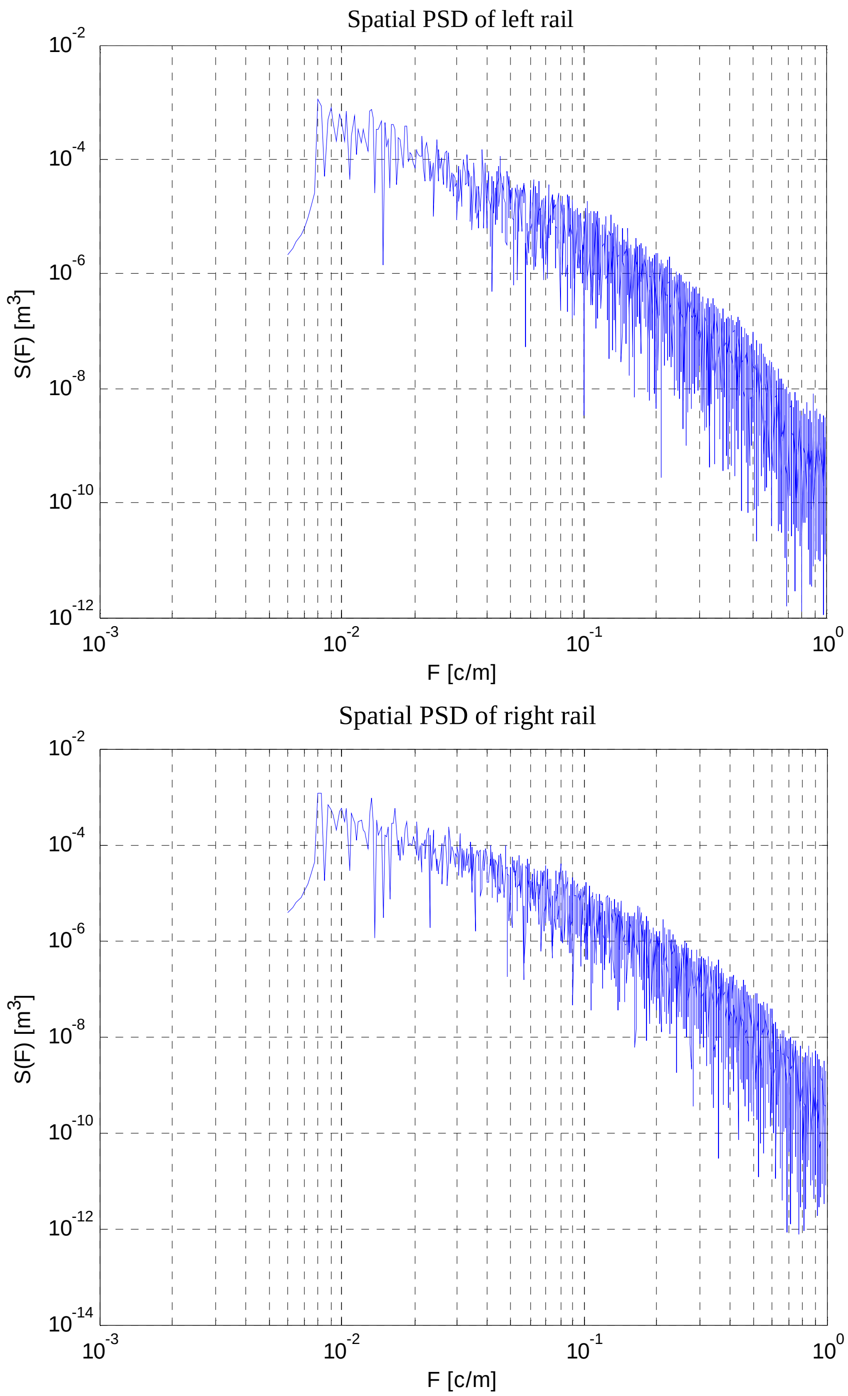

Fig. 3 Spatial PSD of left $S_{D_1}(F)$ (top) and right $S_{D_2}(F)$ rails (bottom).

Generally, the power spectral densities depending on frequency $f = \omega / 2\pi$ in Hertz $S_{q_i}(f)$ and $S_{Q_j}(f)$ of the dynamic displacement q_i and forces Q_j transmitted by couplings (gearing, clutches, viscous-elastic supports of engine stators and of gear housings to the bogie frame etc.) are calculated on the basis of PSD vertical rail irregularities in the form (2) and frequency response functions of the model (4). The upper limits of the displacements and forces transmitted by couplings are calculated by means of static displacements and load and corresponding standard deviations as follows

$$q_{i\max} = |q_{i\,\mathrm{st}}| + 2\sigma_{q_j}, Q_{j\max} = |Q_{j,0}| + 2\sigma_{Q_j},$$
$$\text{where } \sigma_{q_i}^2 = 2\int_0^\infty S_{q_i}(f)\mathrm{d}f,\ \ \sigma_{Q_j}^2 = 2\int_0^\infty S_{Q_j}(f)\mathrm{d}f \tag{25}$$

are corresponding dispersion variances.

4. Standard deviations of dynamic displacements and coupling forces of a particular vehicle bogie

It is efficient to investigate standard deviations of bogie components values in dependence on operational parameters - longitudinal creepage s_0 of the wheels and forward velocity v before the disturbance by track irregularities. For an illustration, further we will present the standard deviations of the displacements and coupling forces of the vehicle bogie shown in Fig. 1 caused by vertical track irregularities described by spatial PSD of left $S_{D_1}(F)$ and right $S_{D_2}(F)$ rails (Fig. 3). The coordinates of the breakpoints of the piecewise straight lines approximating the mentioned PSD are introduced in Table 1. The values of the equivalent parameters of the track structures were considered to be $m_R=38{,}3\ kg$, $b_R=8.10^4\ kgs^{-1}$, $k_R=8.10^7\ Nm^{-1}$ according to results gained from measurements presented in the article of authors Knote et al. (2003).

Tab. 1: Coordinates of breakpoints of spatial PSD of vertical rail irregularities

Rail	$F_i\,[c/m]$					$S_i\,[m^3]$				
	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
Left	$8\cdot10^{-3}$	$2\cdot10^{-2}$	0,1	0,2	1	$5\cdot10^{-4}$	10^{-4}	$3\cdot10^{-6}$	$5\cdot10^{-7}$	$3\cdot10^{-10}$
Right	$8\cdot10^{-3}$	$2\cdot10^{-2}$	0,1	0,2	1	$2\cdot10^{-4}$	10^{-4}	$3\cdot10^{-6}$	$5\cdot10^{-7}$	$3\cdot10^{-10}$

As an illustration, in Fig. 4 we show the frequency response functions $G_{i,1}(\omega)$ and $G_{i,2}(\omega)$ defined in (14) and the power spectral density $S_i(\omega)$ defined in (15) of the engine stator vertical displacement in the neighbourhood of the static equilibrium (before ground track irregularities) for longitudinal creepage of value $s_0=0{,}002$ of the wheels, forward velocity $v=200\ km/h$ and vertical wheel forces $N_0=10^5\ N$.

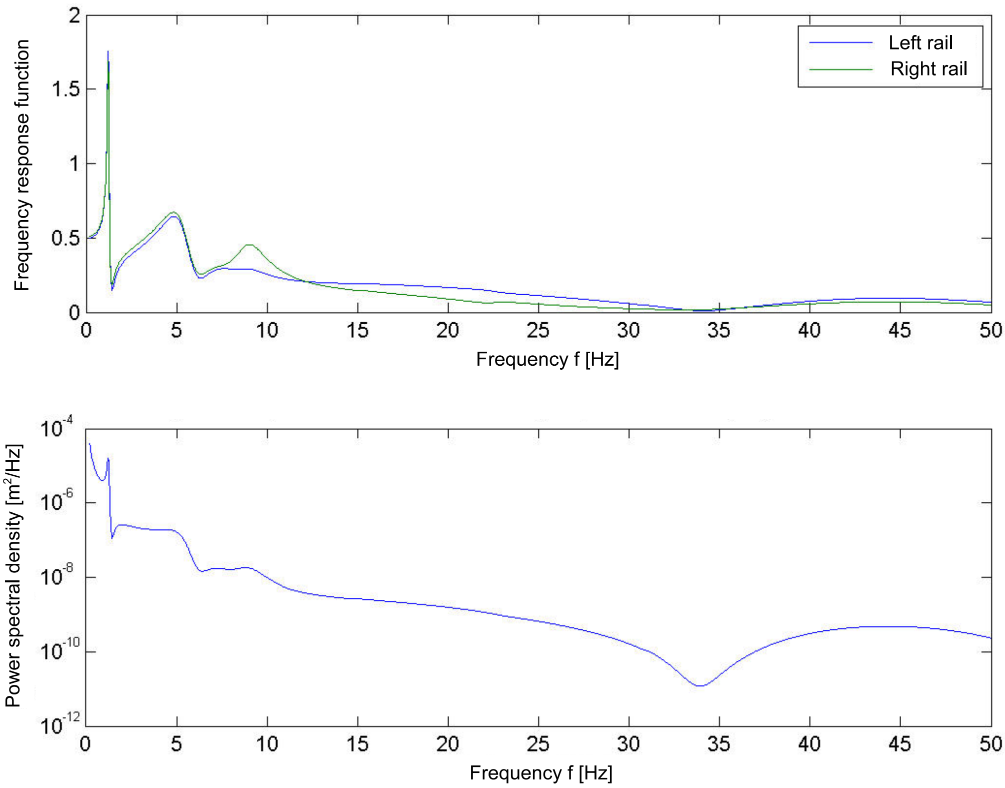

Fig. 4 Frequency response functions (above) and power spectral density (below) of the engine stator vertical displacement for $s_0=0{,}002$, $v=200\ km/h$ and $N_0=10^5\ N$

The standard deviations σ_{q_i} of the engine stator and bogie frame displacements for operational parameters $s_0=0{,}002$ and $v=200\ km/h$ are presented in Table 2. The standard deviations σ_{Q_i} of the forces transmitted by rubber silent-blocks are summarized in Table 3.

Tab. 2: Standard deviations of the engine stators and bogie frame displacements and static displacements

Displacement		Engine stator of ID1		Engine stator if ID2		Bogie frame	
mark	units	σ_{q_i}	$\lvert q_{i\,\mathrm{st}}\rvert$	σ_{q_i}	$\lvert q_{i\,\mathrm{st}}\rvert$	σ_{q_i}	$\lvert q_{i\,\mathrm{st}}\rvert$
u	mm	0,987	< 0,1	0,983	< 0,1	0,899	< 0,1
v		2,07	42,5	2,08	40,2	2,03	41,2
w		0,361	68,2	0,362	68,2	0,359	68,2
φ	$10^{-4}\ rad$	4,90	15,9	4,89	17,9	4,88	14,6
ϑ		3,11	~ 0	3,10	0,22	3,02	~ 0
ψ		17,2	0,27	17,2	2,51	17,2	~ 0

The second analyzed version of the railway vehicle bogie respects the radial static compliance of the wheels. The radial-elastic wheel consists of very stiff parts like the rim and the hub and the relatively soft connection between both parts. Therefore, the wheel rims and wheel hubs can be considered as rigid bodies. The flexible connection can be represented by linear massles springs and dampers with the coefficients k_W and b_W in radial direction. This influence can be approximately respected, in the vehicle bogie shown in Fig. 1, by means of the equivalent rail stiffness k_{Re} and damping b_{Re} calculated from formulas

$$\frac{1}{k_{\mathrm{Re}}} = \frac{1}{k_R} + \frac{1}{k_W}, \quad \frac{1}{b_{\mathrm{Re}}} = \frac{1}{b_R} + \frac{1}{b_W}$$

corresponding to springs and dampers connected in series.

The static forces (moments) (for $s_0=0{,}002$ and $v=200$ *km/h*) and standard deviations of the dynamic forces (moments) transmitted by chosen linkages depending on vehicle forward velocity v *km/h* for both alternatives radial-rigid and radial-elastic (for $k_W = 8.10^7$ N/m, $b_W = 10^{-5} k_W$) wheels are presented in Tab. 3.

Tab. 3: Static forces (moments) and standard deviations of the dynamic forces (moments) of chosen linkages of the first individual wheelset drive ID1 (Fig. 1) depending on vehicle forward velocity v *km/h*

Linkage	Forces 10^3 N Moments 10^3 Nm	static	Radial-rigid wheels			Radial-elastic wheels		
			v=100	v=150	v=200	v=100	v=150	v=200
Silent-block A_1	F_x	<0,1	0,169	0,250	0,431	0,180	0,317	0,459
	F_y	-14,7	0,803	1,56	2,44	0,839	0,231	2,97
	F_z	-1,12	0,575	0,678	1,95	0,462	0,669	1,67
B_1	F_x	<0,1	0,169	0,250	0,431	0,180	0,317	0,459
	F_y	-16,6	0,671	1,27	2,01	0,704	1,88	2,46
	F_z	-1,14	0,823	1,17	2,88	0,701	1,45	2,83
C_1	F_x	<0,1	0,185	0,272	0,425	0,203	0,332	0,462
	F_y	1,61	0,638	1,24	1,86	0,591	1,35	1,95
	F_z	1,88	0,241	0,399	0,795	0,233	0,630	0,823
disc-clutch	M_x	30,3	0,041	0,053	0,065	0,042	0,054	0,067
jaw-clutch	M_x		0,040	0,052	0,063	0,041	0,053	0,064
normal contact force	N_{12}	105,5	173	208	235	86,6	103,9	118
	N_{14}		147	160	168	73,8	80,1	83,9
force in gears	F_G	73,7	0,090	0,122	0,147	0,091	0,122	0,146

5. Conclusion

The paper presents an original method of mathematical modelling of dynamical load of the railway vehicle bogie components caused by vertical track irregularities. The method is based on linearized model of the system in perturbance coordinates with respect to operational state of static equilibrium before running of the bogie on a track with irregularities. The upper limits of the bogie component displacements and forces transmitted by couplings between bogie components are calculated on the basis of static load and the spatial power spectral density functions of the vertical rail irregularities measured along the track. The method is applied to dynamical analysis of the special bogie type with two individual wheelset drives with hollow shafts embracing the wheelset axles.

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References

Balda, M. (1993) Studie chování podvozku železničního vozidla s nelineárními vazbami při jízdě po náhodných nerovnostech. Inaugural dissertation, University of West Bohemia, Pilsen.

Claus, H. & Schiehlen, W. (2003) System Dynamics of Railcars with Radial and Laterelastic Wheels, in: System Dynamics and Long Term Behaviour of Railway Vehicles, Track and Subgrade (K. Popp & W. Schiehlen eds). Springer, pp. 65-84.

Garg, V. & Dukkipati, R. (1994) Dynamics of railway vehicle systems, London Academic Press, London.

Feldman, V. et al. (2003) Monitoring the dynamic of railway track by means of the Karhunen-Loéve-transformation, in: System Dynamics and Long Term Behaviour of Railway Vehicles, Track and Subgrade (K. Popp & W. Schiehlen eds). Springer, pp. 232-246.

Knote, K., Yu, M. & Ilias, H. (2003) System Dynamics of Railcars with Radial and Laterelastic Wheels, in: System Dynamics and Long Term Behaviour of Railway Vehicles, Track and Subgrade (K. Popp & W. Schiehlen eds). Springer, pp. 265-271.

Popp, K. & Schiehlen, W. (1993) Fahrzeugdynamik. B. G. Teubner, Stuttgart.

Zeman, V., Hlaváč, Z. & Byrtus, M. (2007a) Modelling and modal properties of a railway vehicle wheelset drive with a hollow shaft, in: Proc. of Conf. Engineering Mechanics 2007, Svratka, pp. 39-39, full paper on cd-rom.

Zeman, V., Hlaváč, Z. & Byrtus, M. (2007b) Modelling and modal properties of a railway vehicle bogie with two individual wheelset drives, Applied and Computational Mechanics 2007 1(1), pp. 371-380.