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SELECTED PROBLEMS OF NONLINEAR CONCRETE MODELLING

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Summary: *The paper discusses some of computational problems which can arise during non-linear computational investigation of numerical models of concrete with use of a smeared crack models. Despite the long–term researches in this area (good overviews are available in (Bažant and Planas, 1998) and in (Jirásek and Bažant, 2002)) there are still some areas which can make this type of modelling uneasy. The discussed constitutive model is based on smeared cracks and it uses equivalent constitutive one–dimensional material laws. Such models can be found both in professional (Vořechovský and Červenka, 2002) and academic finite element codes.*

1. Introduction

There are many commercial and academic finite element codes which are using smeared crack models and equivalent one-dimensional material laws. These models are pioneered by Červenka (Červenka, 1985) and later expanded by Bažant (Bažant and Planas, 1998) and other authors through inclusion of principles of non-linear fracture mechanics into the constitutive model. These models are generaly reliable and very usefull. But there still are cases when their use needs more attention and carefulness. Some of these cases are discussed in the paper.

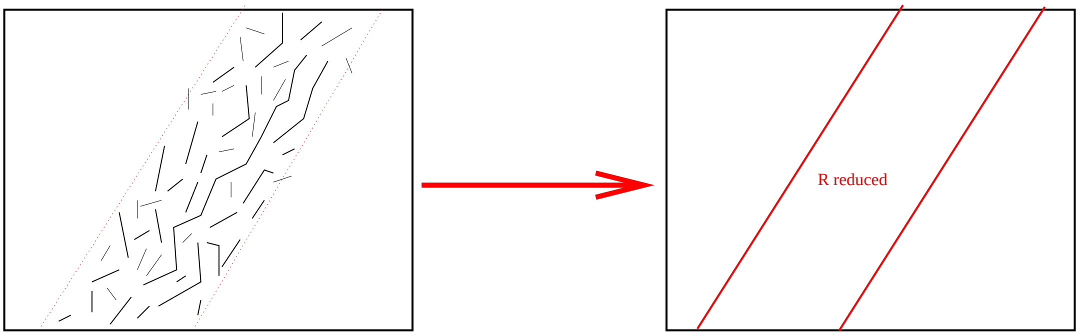

Figure 1: Smeared cracks model illustration

2. Constitutive model basics

There are many different variations of the mentioned constitutive model. It this paper we are working with modification which use a multilinear stress–strain and stress–crack width relations. In this case a damage of material is simulated through reduction of material properties.

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An orthotropic material behaviour is assumed for the material with tension cracks. It is assumed that the stiffness in the direction perpendicular to the direction of cracks is reduced (this direction should be obtained from analysis of principal stresses in the material).

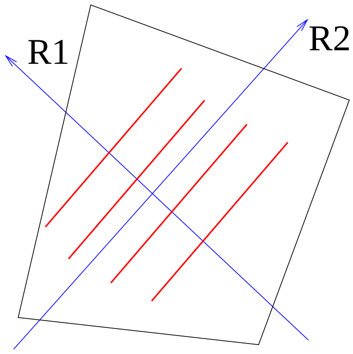

Figure 2: Orthotropic material axes

Stiffness in other direction can also be reduced due to material structure changes but this effect is less important in some cases. The stiffness matrix of orthotropic material can be in the form of (2). This matrix used in this paper is derived with use of methodology described in (Vořechovský and Červenka, 2002) but the result is slightly different than the matrix provided by the cited autors. The unmodified material stiffness matrix of orthotropic material (Bittnar and Šejnoha, 1992) is not very usefull here because of limited posibilities of measurement or computation of valid Poisson ratios.

There are several approaches for the creation of a material stiffness matrix. For example, Červenka (Vořechovský and Červenka, 2002) recommends a formulation which is shown in the equation (1).

$$\mathbf{D_{cr}} = \begin{bmatrix} \frac{R_1}{R_2} & \mu\frac{R_1}{R_2} & 0 \\ \mu\frac{R_1}{R_2} & \frac{R_1}{R_2} & 0 \\ 0 & 0 & \beta\, G \end{bmatrix} \tag{1}$$

Also other formulations of material stiffness materix are possible, for example we are using one which is described by equation (2). These alternative formulations give slightly different results but it is still investigated which one is better.

$$\mathbf{D_{cr}} = \frac{R_2}{R_2 - \mu^2\, R_1} \begin{bmatrix} R_1 & \mu R_1 & 0 \\ \mu R_1 & R_2 & 0 \\ 0 & 0 & \frac{\beta\, G}{R_2/(R_2 - \mu^2\, R_1)} \end{bmatrix} \tag{2}$$

It can be noted that there is also a more classic formulation of orthotropic matrix (3) for concrete which is recomended by many authors (Bittnar and Šejnoha, 1992). This formulation uses averaging for estimation of Poisson ratios. It is not recomended to use this matrix in conjunction with the discussed constitutive model because it results in incorrect results in many cases.

$$\mathbf{D_{cr}} = \frac{1}{1-\mu^2}\begin{bmatrix} R_1 & \mu\sqrt{R_1\,R_2} & 0 \\ \mu\sqrt{R_1\,R_2} & R_2 & 0 \\ 0 & 0 & \frac{1}{4}\beta(R_1 + R_2 - 2\,\mu\sqrt{R1\,R2})(1-\mu^2) \end{bmatrix} \quad (3)$$

The so-called fixed crack model is used here so crack direction is determined from principal stresses in the moment of solution when initiation of crack is detected.

The state of material is determined with use of equivalent one-dimensional law. Equivalent parameters are principal stresses and adequate strains and equivalent crack width (if applicable). Because studied problems assume two–dimensional stress state it is necessary to modify the one-dimensional law parameters to respect the actual 2D stress. It can be done with modification of equivalent law with respect to actual stress state. The most common way to accomplish this is a computation of limit parameters of equivalent law from parameters of failure condition for 2D. The Chen or Kupfer criteria can be used here.

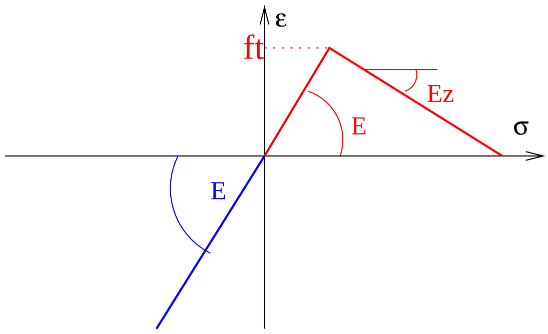

Figure 3: Equivalent one–directional constitutive law example

Use of smeared crack models often leads to results which are dependent on parameters of used finite element discretisation. This effect can be minimized with use of several techniques. The Bažants crack–band model is utilised here (Bažant and Planas, 1998).

When fixed crack are assumed, then the shear modullus must be also reduced during the crack propagation. There many different approaches can be used here. Figure 4 show some possible deformation—modullus reduction relations which are proposed by different authors.

3. Program codes

A constitutive model whis uses the approaches and methods mentioned here has been studied with help of in-house written computational code. These particular approaches were selected to obtain a partial compatibility with the much more complex Atena software (Vořechovský and Červenka, 2002) which was used for verification works. The use of in-house written software has been caused by needed ability of modification and detailed analysis of individual aspects of solution which is not so easy if a complex software code is used.

4. Examples

There are several selected examples to show some aspects of the non-linear constitutive modelling of concrete with used of the mentioned material model.

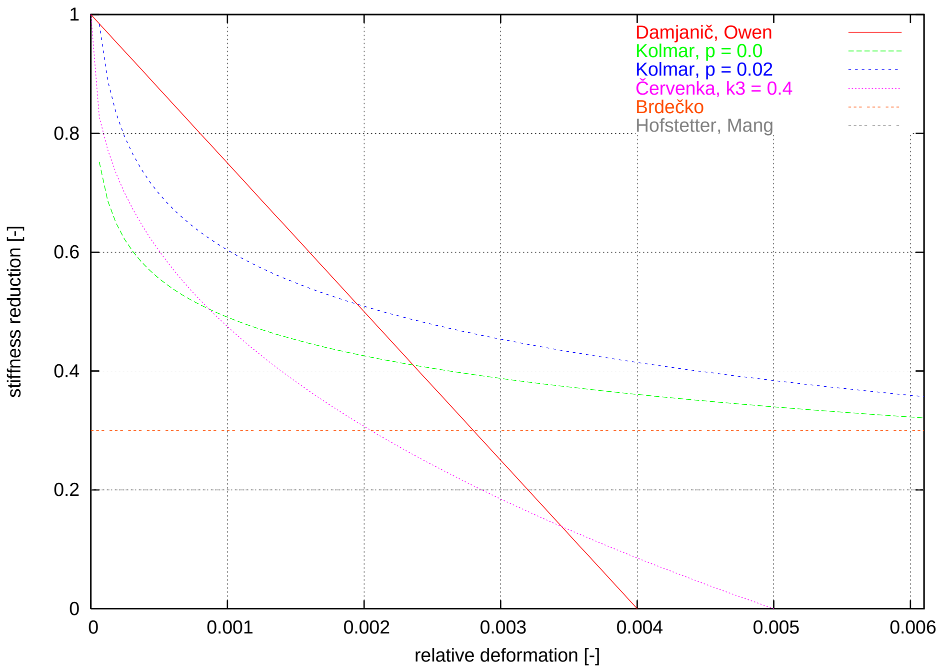

Figure 4: Comparison of shear modullus reduction approaches

The first example shows comparison of numerical simulation and experimental testing (the tests were done by Červenka et al (Červenka, 1985)). Thi case shown an ideal example with experimental model carefully prepared to minimize most of possible side effects. The model scheme is and the computational model are shown in Figure 5. The comparison of obtained results is shown in the form of load–displacement diagrams in the Figure 6.

Because the experimental data are relatively old there were some material properties of used concrete (such as fracture energy) unavailable. For this reason several different variants of material model were compared here.

The second example was used to study an influence of finite element mesh size on the results. It is prevented by Bažant's crack band model (Bažant and Planas, 1998) here.

It is need to be remembered that the Bažant's model affects only the problems caused by constitutive formulation. Finite element mesh shape and properties obviously can have influence on results in any case (including linear elastic problems).

A very simple model was created. One of studied meshes and example of results (normal stiffness reduction data) are shown in Figure 7.

This problem was studied with use of in-house writtnen software and also analysed in Atena (Vořechovský and Červenka, 2002). Figure 7 shows comparison of load–displacement diagrams which were obtained with use of in-house writtnen software for different mesh sizes.

5. Conclusions

There are mentioned in the paper selected approaches and issues which can have influence to results of numerical analysis of 2D concrete problems. Many of related issues were not adressed here like solution strategy selection (Riks, 1972) a algoritmical possibilities (Ritto–Corrêa & Carmotim, 2008) or load modelling problems.

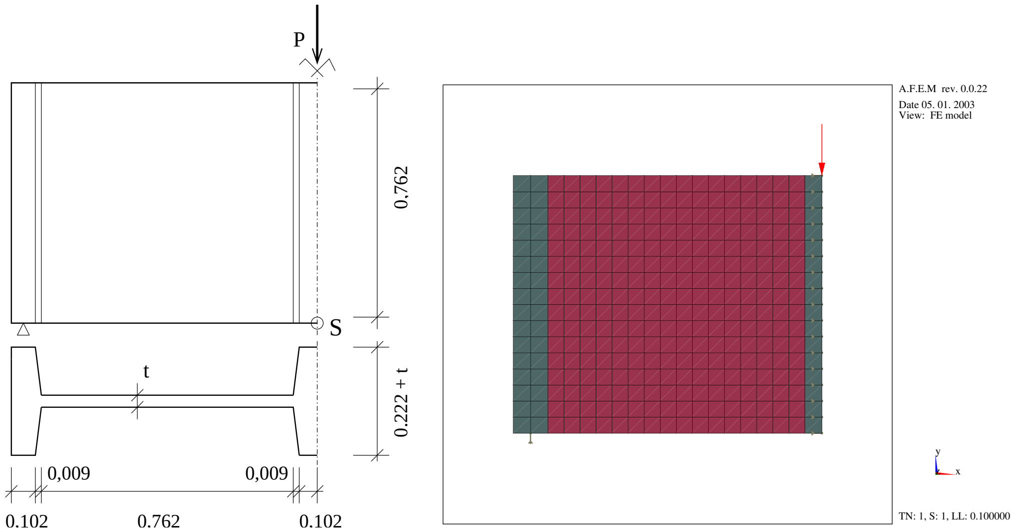

Figure 5: Experimental and numerical models for first example

6. Acknowledgement

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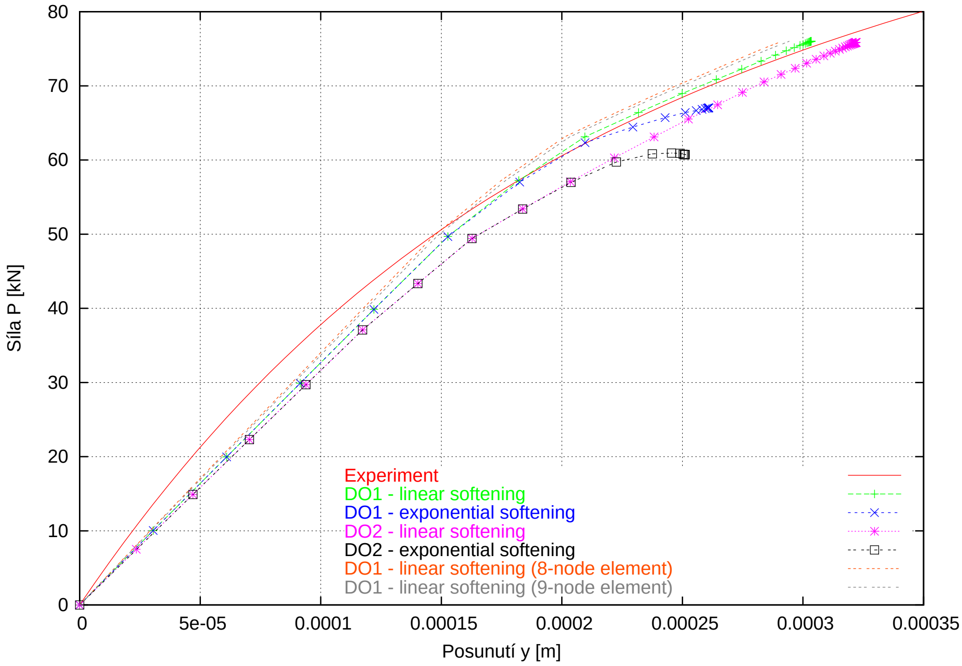

Figure 6: Comparison of load–displacement diagram for first example

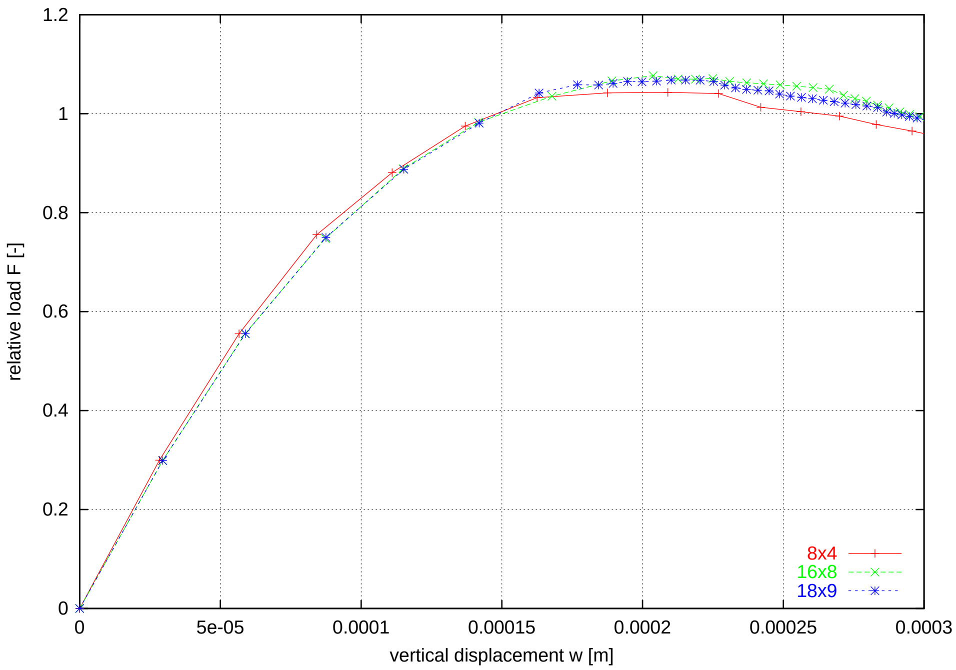

Figure 7: Results for different finite element meshes for second example