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# THE BOUNDARY CONDITION OF THE ADHESIVE FORCES EFFECT ON THE SOLID/LIQUID INTERFACE

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**Summary:** *In all areas of the technical creative work we do meet the design and planner engineers' tendency of lowering the device's power input in the power output conservation. This tendency is not only necessary but for the hydraulic systems also possible. There are two ways of solving: the first one uses for the hydraulic losses decreasing the shape optimization; the second one, more recently used, is based on the adhesive forces decreasing on the solid/liquid interface.*

## 1. INTRODUCTION

In consequence of the adhesive forces decreasing in the surface layer there does not operate the condition of cleaving. The liquid starts slipping on the surface and the hydraulic losses are lowering.

The wettability ratio (factor) of the liquid-solid interface depends on the surface tension. It is characterized by the contact angle $\Theta$, see Fig. 1, though its use it is the surface energy of the liquid drop established. The adhesive forces magnitude also depends on the slipping velocity of the liquid on the surface. In the following part of article we assume the hypothesis of the colinearity of the adhesive shear stress vector with the slipping velocity vector.

## 2. SURFACE ENERGY

Set the surface energy of the solid material directly is very complicated, that's why are used the indirect methods. One of the easiest methods - the contact angle of the liquid drop on the solid surface is defined by the drop mechanical equilibrium between the three surface tensions $\gamma$: solid/vapor, solid/liquid, liquid/vapor (Fig. 1)

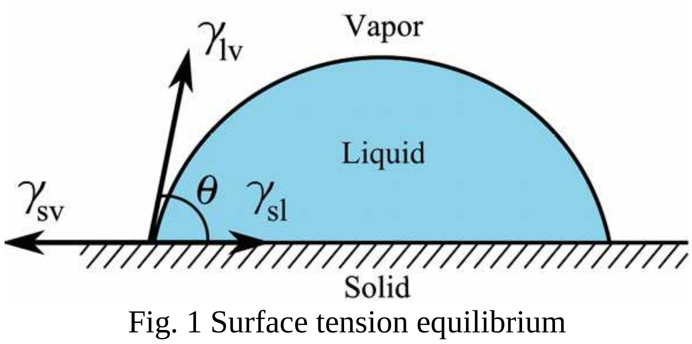

Fig. 1 Surface tension equilibrium

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The solid body surface energy determined from the contact angle depends on the equation set by Thomas Young in 1805. This equation is generally known as a Young equation:

$$\gamma_{sv} - \gamma_{sl} = \gamma_{lv} \cos\theta \tag{1}$$

where: *sv* (solid/vapor), *sl* (solid/liquid) and *lv* (liquid/vapor).

The mathematical models used in this work come out this equation. They are divided into one-, two-, three-liquid and regression model.

On such a principle were set the surface energies of five different surface types. Three of them were formed by a Teflon layer (black metallic paint TC 3072, normal black TC 1191, green TC 4111), next was formed by the synthetic dope (normally used for the airplane wings coating – white aero-plate), last of them was a glass.

| Specimen | 1 liquid | 2 liquids | 3 liquids | AB regression | OW regression |
|---|---|---|---|---|---|
| | $\gamma$ [mJ/m$^2$] | $\gamma$ [mJ/m$^2$] | $\gamma$ [mJ/m$^2$] | $\gamma$ [mJ/m$^2$] | $\gamma$ [mJ/m$^2$] |
| White plate | 43,3 | 46,4 | 49,7 | 43,3 | 47,8 |
| TC 1191 (black) | 39,6 | 41,8 | 42,3 | 39,6 | 42,6 |
| TC 3072 (metal black) | 29,4 | 27,8 | 31,7 | 24,3 | 28,6 |
| $TiO_2$ (specimen 1) | 45,9 | 47,6 | 43,8 | 42,7 | 53,4 |
| $TiO_2$ (specimen 2) | 45 | 46,7 | 43,4 | 43,6 | 51,7 |
| $TiO_2$ (specimen 3) | 44,4 | 48,3 | 54,5 | 50,4 | 48,8 |
| glass | 49 | 50,9 | 48,2 | 47,8 | 59,3 |
| TC 4111 (green) | 27 | 26,6 | 27,5 | 25,7 | 26,2 |

Fig. 2 Surface energy of different materials

There is visible the quantitative wettability difference between the white air-plate and Teflon plate TC 4111. See Fig. 3. Very low surface energy of the Teflon layer makes the water collection into the rope (in consequence of the liquid surface energy).

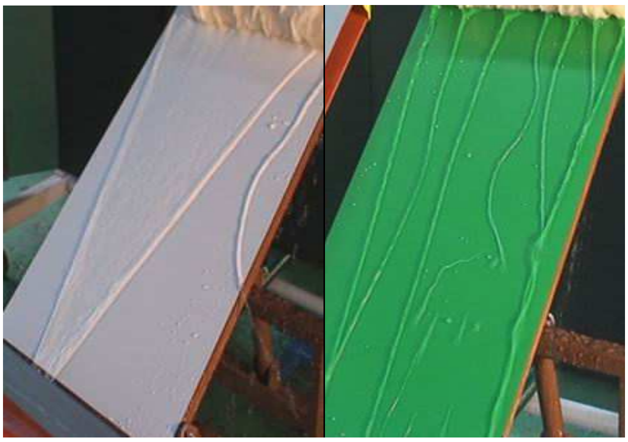

Fig. 3 The liquid layer running down on the inclined plane

## 3. THE BOUNDARY CONDITION AND THE STEADY ROTATIONAL LIQUID MOVEMENT

The generally curved surface, which is on the contact with the liquid, is supposed. It is assumed that the liquid doesn't stick to the surface and is moving with the velocity **c**. In the following we will bear on the hypothesis that the adhesive stress vector **σ** is oriented tangentially to the surface, which is formed by the velocity vector **c** and the base vector of the surface external normal **n** (see Fig. 4)**.**

For the adhesive shear stress vector $\boldsymbol{\sigma}_A$ that is placed in the tangential plane holds:

$$\boldsymbol{\sigma}_A = (\boldsymbol{\sigma} \times \mathbf{n}) \times \mathbf{n} = -k\mathbf{c} \tag{2}$$

As a remark, this hypothesis comes out from the assumption that the adhesive shear stress vector is **colinear** to the liquid velocity vector.

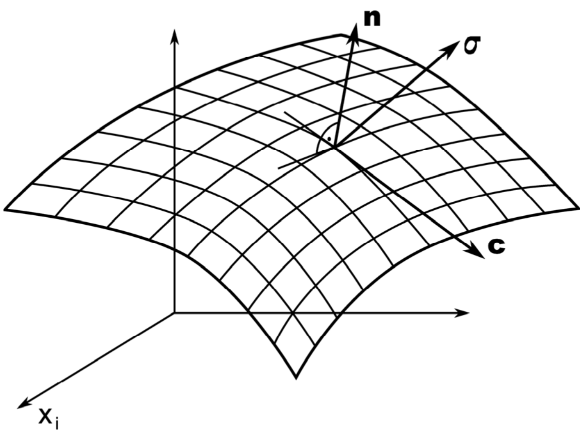


Fig.4 The shear stress on the curved surface

After performing the vector product and formulating the individual components, the boundary condition has form (because $\sigma_i = \Pi_{ij} n_j$ ):

$$\sigma_{Ai} = \Pi_{jk} n_k n_j n_i - \Pi_{ij} n_j = -kc_i \, . \tag{3}$$

The irreversible stress tensor is specified by the term (5). The term $\Pi_{jk} n_k n_j n_i$ represents the i$^{th}$ component of the adhesive shear stress that depends on the surface curvature. For the plane flow is the boundary condition (3) simplified to the form:

$$\Pi_{ij} n_j = kc_i \, , \tag{4}$$

where $$\Pi_{ij} = \eta \left( \frac{\partial c_i}{\partial x_j} + \frac{\partial c_j}{\partial x_i} \right) \tag{5}$$

The special plane flow case is the rotary movement of the liquid between the two coaxial cylinders of circular section. The inner cylinder radius is $R_1$, the outer cylinder radius is $R_2$, the inner cylinder rotates around the common rotation axis with constant angular speed $\omega_1$ (see Fig. 5).

The inner cylinder is partially wettable. Boundary condition (4) holds for its surface, which is for the cylindrical coordinate system in form:

$$r = R_1;\ \ n = 1;\ \ \eta\left(\left.\frac{\partial c}{\partial r}\right|_{r=R_1} - \frac{c}{r}\right) = k(c-u), \tag{6}$$

where $u$ means the circumferential velocity of the rotating cylinder

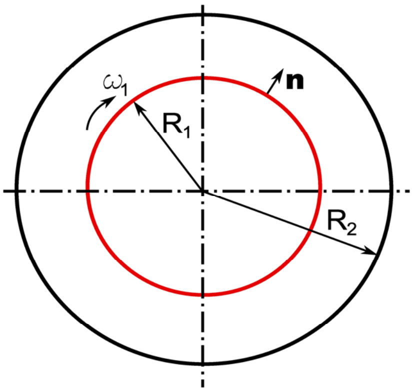


Fig. 5 Liquid movement between two coaxial cylinders

On the outer cylinder surface, which is wettable and immobile, there holds the boundary condition of liquid cleaving.

$$r = R_2;\ \ c = 0\,. \tag{7}$$

Providing the rotational and steady flow of the incompressible liquid, it is possible to adjust the Navier-Stokes equation in the cylindrical system of coordinates as:

$$\frac{\partial^2 c}{\partial r^2} + \frac{1}{r}\frac{\partial c}{\partial r} - \frac{c}{r^2} = 0\,, \tag{8}$$

That is the Euler type equation and its solution is (for example see [8]):

$$c = \frac{A}{r} + Br\,,\ \ \frac{\partial c}{\partial r} = -\frac{A}{r^2} + B\,. \tag{9}$$

The shear stress in the cylindrical coordinates for the above mentioned case:

$$\Pi_{r\varphi} = \eta\left(\frac{\partial c}{\partial r} - \frac{c}{r}\right). \tag{10}$$

After the integration constants setting, with the boundary condition consideration, we obtain the equations for the velocity and its differentiation with respect to $r$ [6].

$$c = \frac{uR_1^2}{R_1\left(R_1^2 - R_2^2\right) - 2\frac{\eta}{k}R_2^2}\left(r - \frac{R_2^2}{r}\right); \quad \frac{\partial c}{\partial r} = \frac{uR_1^2}{R_1\left(R_1^2 - R_2^2\right) - 2\frac{\eta}{k}R_2^2}\left(1 + \frac{R_2^2}{r^2}\right) \tag{11}$$

The total friction forces moment in the case of partially wettable inner cylinder surface will be in the form:

$$M_K = \int_0^{2\pi} \Pi_{r\varphi} r^2 d\varphi = -\frac{4\pi u \eta R_1^2 R_2^2}{R_1\left(R_2^2 - R_1^2\right) + 2\frac{\eta}{k}R_2^2}. \tag{12}$$

This equation will be used for the adhesive force coefficient $k$ identification.

In the $k$ magnitude dependence are changing the hydraulic losses and thereby also the hydraulic efficiency of hydraulic features. The following figure (Fig. 6) shows the hydraulic efficiency behavior for the very low velocity ratio pump. The inner part of this pump is coated by special, very low wettable material. The efficiency increasing within the whole range of the flow rate is obvious.

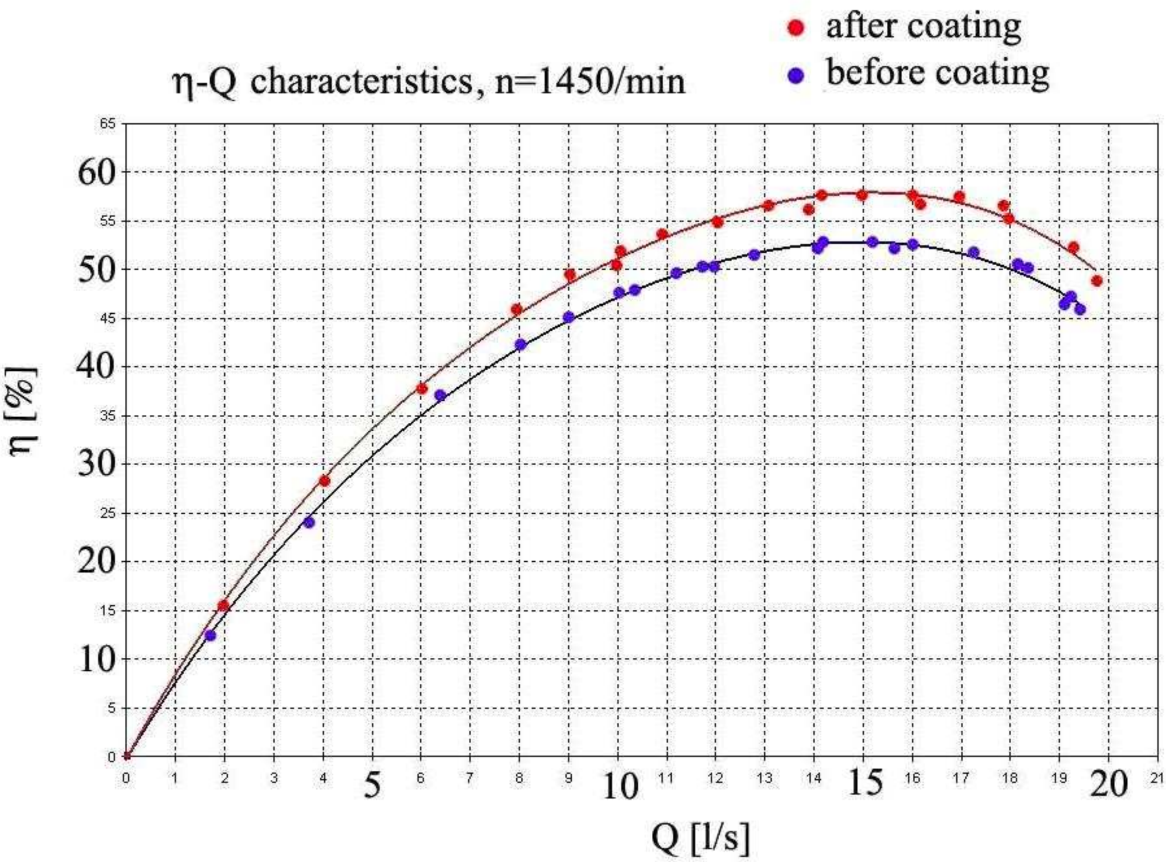


Fig. 6 Changes of the pump characteristics in the dependence of the various coating

## 5. ACKOWLEDGEMENT

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