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STOCHASTIC MODEL AND RELIABILITY ASSESMENT APPLIED FOR THE PLATINUM ORE DISINTEGRATION PROCESS

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Summary: *This paper focuses on a numerical analysis of the hard rock (ore) disintegration process. The bit moves into the ore (i.e. mechanical contact with friction) and subsequently disintegrates it. The disintegration (i.e. stress-strain relationship, reaction forces and fracture of the ore etc.) is solved via the FEM in combination with SBRA (Simulation-Based Reliability Assessment) Method (i.e. Monte Carlo simulations). The ore is disintegrated by deactivating the FE which satisfy the fracture condition. Material of the ore (i.e. yield stress, fracture limit, etc.), is given by bounded histograms (i.e. stochastic inputs). The results are compared with experiments. Application of the SBRA method in this area is a new and innovative trend. However, it takes a long time to solve this problem (due to material and structural nonlinearities, the large number of elements, many Monte Carlo simulations, etc.). Hence, parallel computers were used to handle the large computational needs. Finally, the probabilistic reliability assessment is proposed.*

1. Introduction

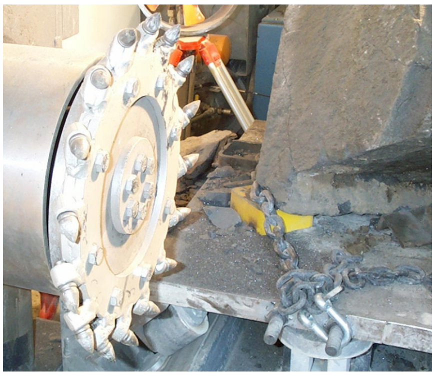

Figure 1 Typical Example of Mechanical Interaction between Bits and Hard Rock (Ore Disintegration Process).

Scientific and technical developments (in all areas of world-wide industry) are affected by the growing demand for basic raw materials and energy. The provision of sufficient quantities of raw materials and energy for the processing industry is the main limiting factor of further development. It is therefore very important to understand the ore disintegration process, including an analysis of the bit (i.e. excavation tool) used in mining operations. The main focus is on modelling of the mechanical contact between the bit and the ore and its evaluation (i.e. practical application in the mining technology), see Figure 1. However, material properties of the ore (as known from nature) have a large stochastic variability. Hence, the stochastic approach (i.e. Simulation-Based Reliability Assessment Method) in combination with FEM is applied.

2. Finite Element Model of the Ore Disintegration Process

FEM (i.e. MSC.Marc/Mentat software, see references Frydrýšek 2009, Frydrýšek 2008 and Frydrýšek & Gondek 2008) was used in modelling the ore disintegration process. Figure 2

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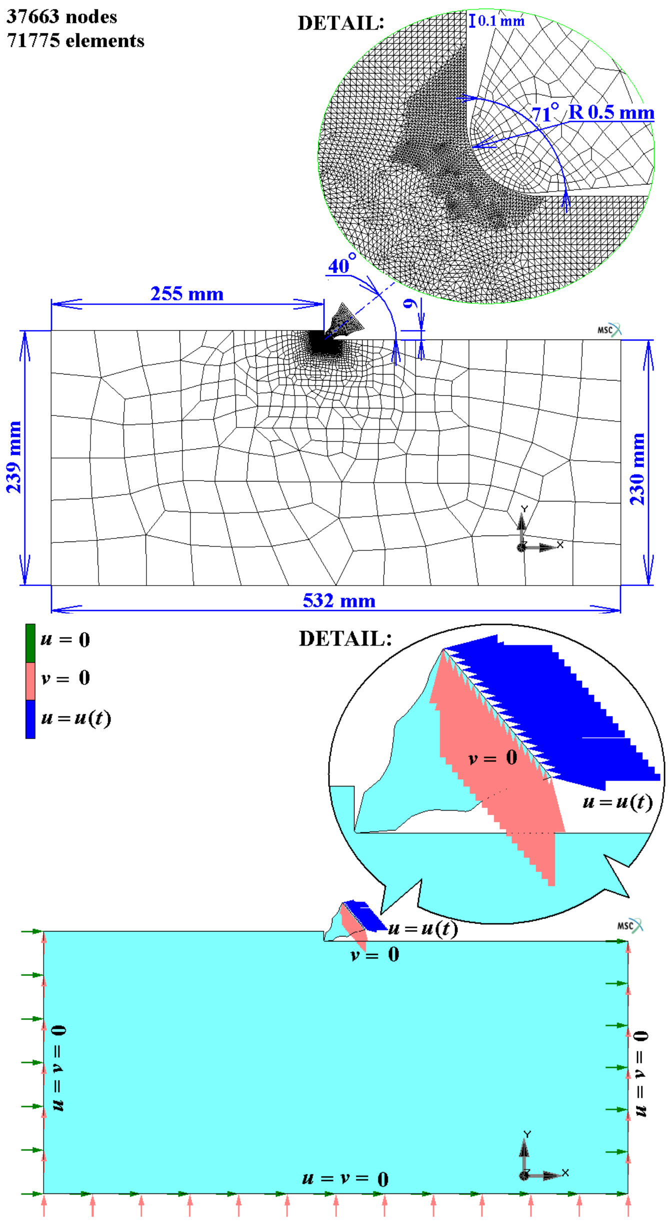

Figure 3 Boundary Conditions.

shows the basic scheme (quasi-static loading, plain strain formulation, mechanical contact with friction between the bit and platinum ore, boundary conditions, etc.).

Figure 3 shows that the bit moves into the ore with the prescribed time dependent function $u = u(t)$, and subsequently disintegrates it. When the bit moves into the ore (i.e. a mechanical contact occurs between the bit and the ore) the stresses σ_{HMH} (i.e. the equivalent von Mises stresses) in the ore increase.

When the situation $\sigma_{HMH} \geq R_m$ occurs (i.e. the equivalent stress is greater than the fracture limit) in some elements of the ore, then these elements break off (i.e. these elements are dead). Hence, a part of the ore disintegrates. In MSC.Marc/Mentat software, this is done by deactivating the elements that satisfy the condition $\sigma_{HMH} \geq R_m$. This deactivation of the elements was performed in every 5^{th} step of the solution, see Figure 4.

For further information see references Frydrýšek & Gondek 2008, Frydrýšek 2008 and Frydrýšek 2009.

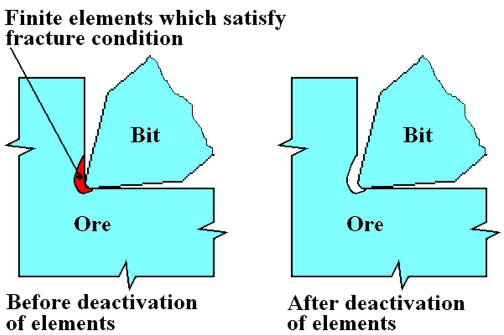

Figure 4. Disintegration of a Part of the Ore.

3. Probabilistic Inputs and SBRA (Simulation-Based Reliability Assessment) Method

A deterministic approach (i.e. all types of loading, dimensions and material parameters etc. are constant) provides an older but simple way to simulate mechanical systems. However, a deterministic approach cannot truly include the variability of all inputs (i.e. variability of material properties of the ore), because nature and the world are stochastic. Solution of the ore disintegration process via deterministic approach (i.e. basic simple solution) is shown in reference Frydrýšek & Gondek 2008. However, this problem is solved via probabilistic approaches which are based on statistics.

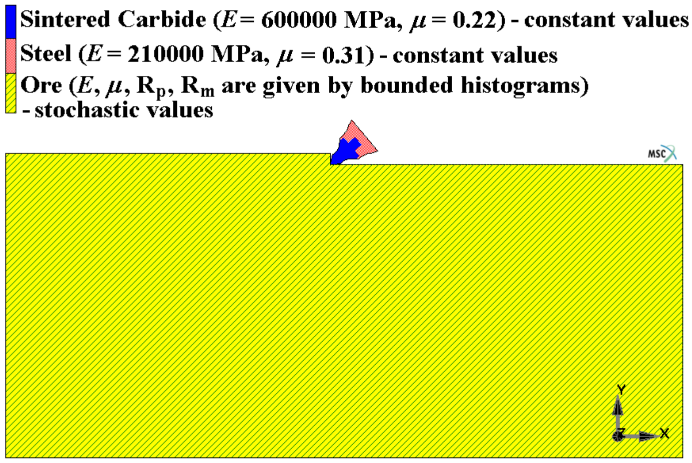

Figure 5 Material Properties (Whole Model).

Let us consider the "Simulation-Based Reliability Assessment" (SBRA) Method, a probabilistic direct Monte Carlo approach, in which all inputs are given by bounded (truncated) histograms. Bounded histograms include the real variability of the inputs. Application of the SBRA method (based on Monte Carlo simulations) is a modern and innovative trend in mechanics, for example see references Marek et all 1995 and 2003.

The material properties (i.e. isotropic and homogeneous materials) of the whole system are described in Figure 5, where E is Young's modulus of elasticity and μ is Poisson's ratio.

The bit is made of sintered carbide (sharp edge) and steel. Stochastic influences of material parameters of the bit can be neglected.

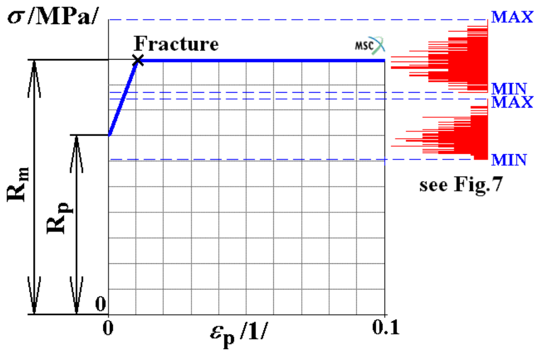

Figure 6 Stochastic Material Properties of the Ore (Stress σ vs. Plastic Strain ε_p).

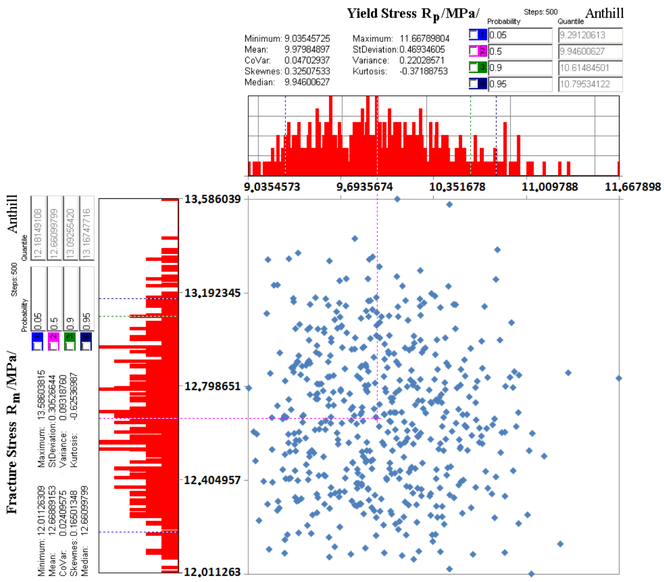

Figure 7 Stochastic Material Properties of the Ore (Histograms for Yield Stress and Fracture Stress).

The ore material is elasto-plastic with isotropic hardening rule. The plastic properties are described by yield stress $R_p = 9.946^{+1.722}_{-0.911}$ MPa and fracture stress $R_m = 12.661^{+0.925}_{-0.650}$ MPa, which are given by bounded histograms, see Figures 6 and 7.

The elastic properties of the ore are described by Hooke's law in the histograms ($E = 18513.8^{+2608.8}_{-2418.8}$ MPa and $\mu = 0.199^{+0.021}_{-0.019}$), see Figures 8 and 9.

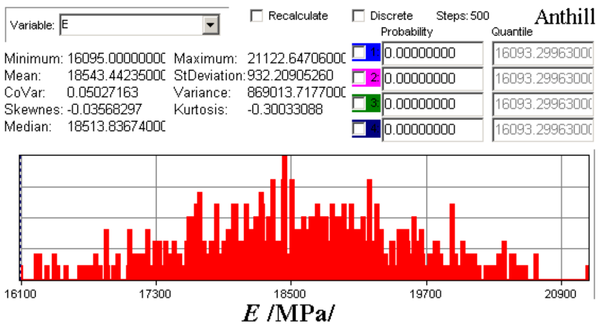

Figure 8 Stochastic Inputs for the Material of the Ore (Histogram of Young's Modulus).

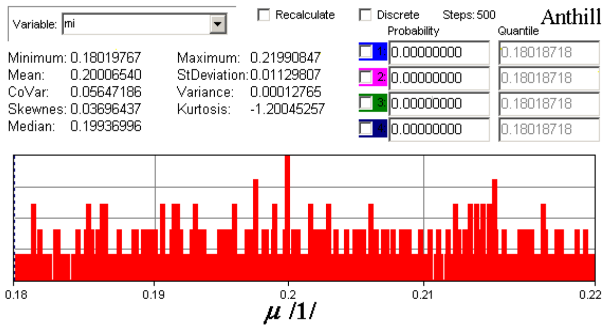

Figure 9 Stochastic Inputs for the Material of the Ore (Histogram of Poisson's Ratio).

The results (acquired by SBRA method in combination with FEM) were subsequently statistically evaluated, as shown in Figure 10. Anthill, MSC.Marc/Mentat and Mathcad software were used.

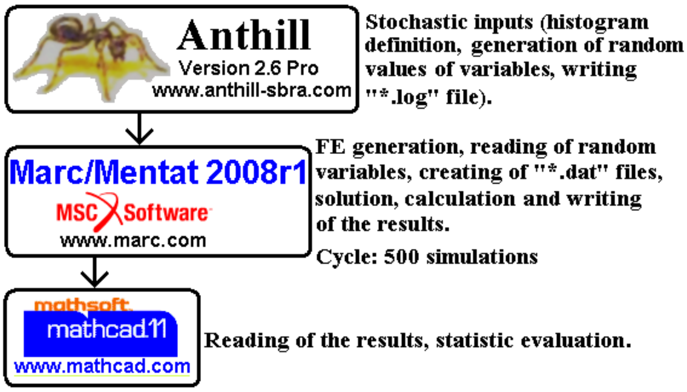

Figure 10 Computational Procedure - Application of the SBRA Method and FEM (Solution of the Ore Disintegration Process.

4. Solution - SBRA Method in Combination with FEM

Because of the material non-linearities, the mechanical contacts with friction, the large number of elements, many iteration steps, and the choice of 500 Monte Carlo simulations, four parallel computers (with 26 CPU) were used to handle the large computational requirements for this problem. The FETI Domain Decomposition Method (i.e. application of parallel computers) was used, see Figure 11.

The whole solution time for the non-linear solution (i.e. 1.04 s) was divided into 370 steps of variable length. The Full Newton-Raphson method was used for solving this non-linear problem. Solution of 500 Monte Carlo simulations (calculated simultaneously on four different parallel computers with 26 CPU) takes 70.4 hours. For more information see reference Frydrýšek 2008 and 2009.

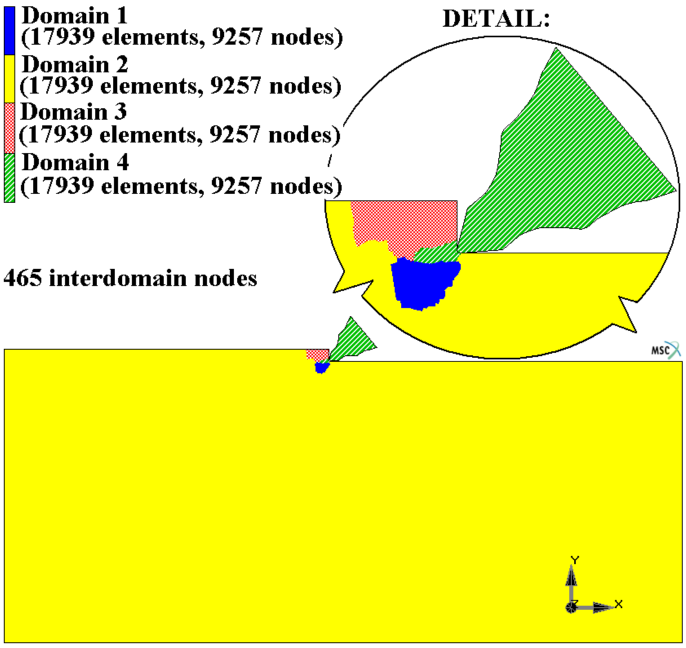

Figure 11 FETI Domain Decomposition Method Used for Application of 4 CPU.

5. Results and Stochastic Evaluation

Figures 12 to 20 show the equivalent stress (i.e. σ_{HMH} distributions) at some selected time t of the solution calculated for one of the chosen 500 Monte Carlo simulations (i.e. for one situation when the material of the ore is described by values $\mathrm{R_p} = 12\ \mathrm{MPa}$, $\mathrm{R_m} = 13.5\ \mathrm{MPa}$, $E = 20000\ \mathrm{MPa}$ and $\mu = 0.2$).

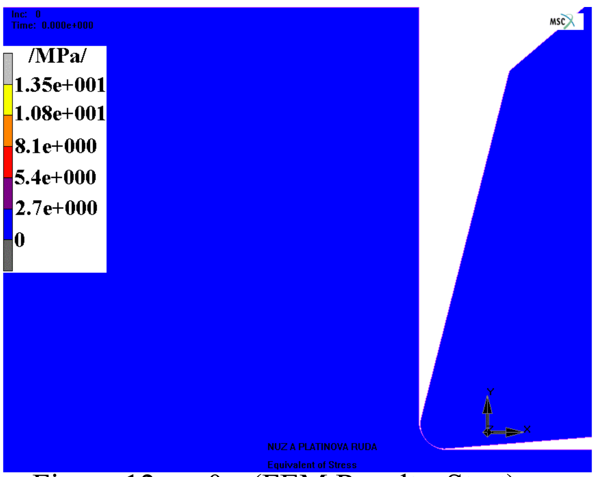

Figure 12 $t = 0$ s (FEM Results, Start).

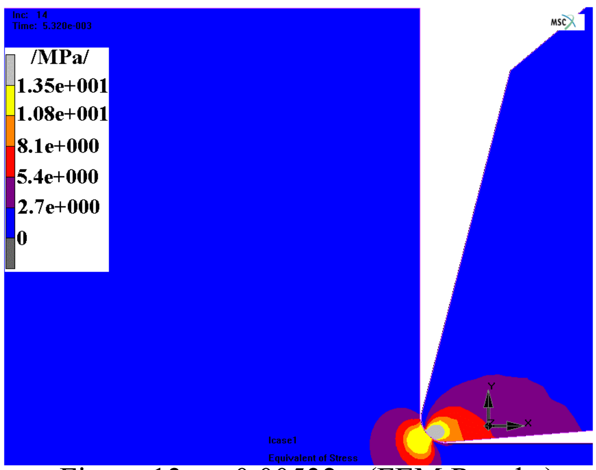

Figure 13 $t = 0.00532$ s (FEM Results).

The movement of the bit and also the subsequent disintegration of the ore caused by the cutting are shown.

Equivalent stress distribution which is shown in Figure 13 (loading below yield stress) is in good correlation with Hertzian contact theory.

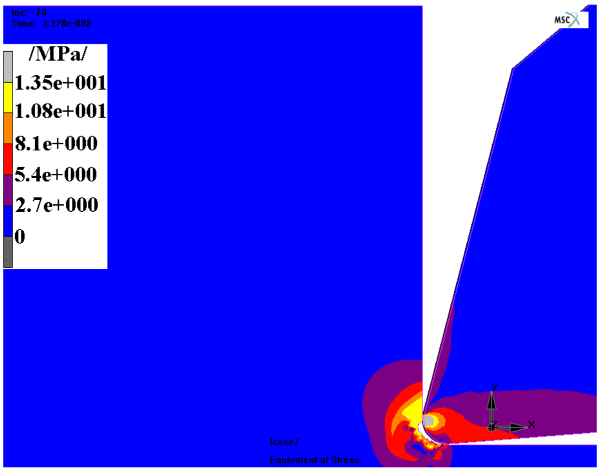

Figure 14 $t = 0.0337$ s (FEM Results).

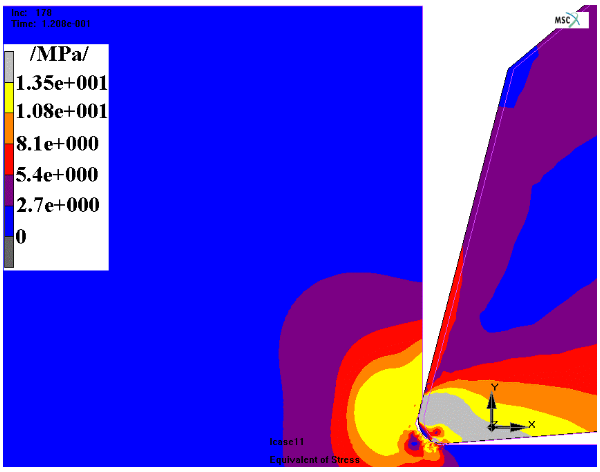

Figure 15 $t = 0.1208$ s (FEM Results).

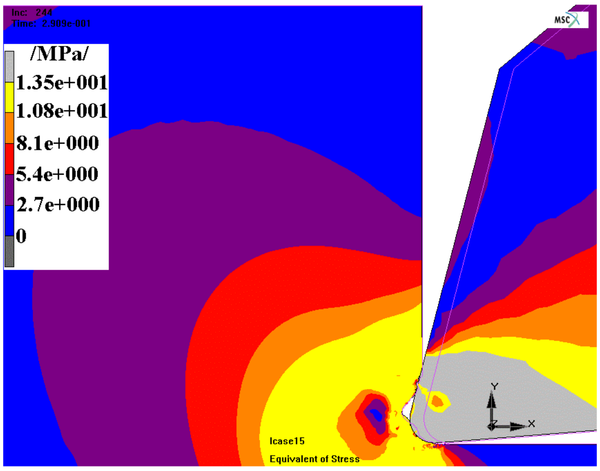

Figure 16 $t = 0.2909$ s (FEM Results).

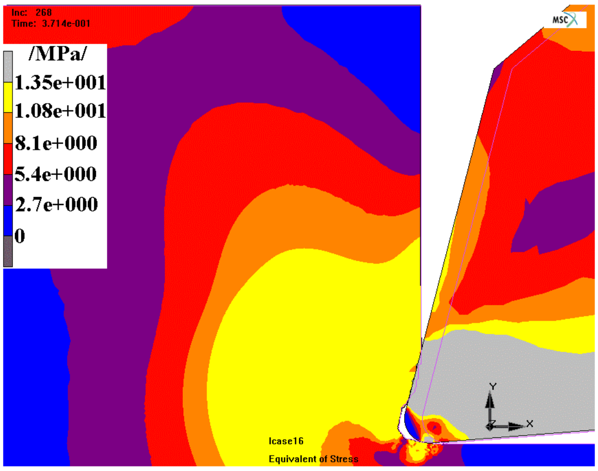

Figure 17 $t = 0.3714$ s (FEM Results).

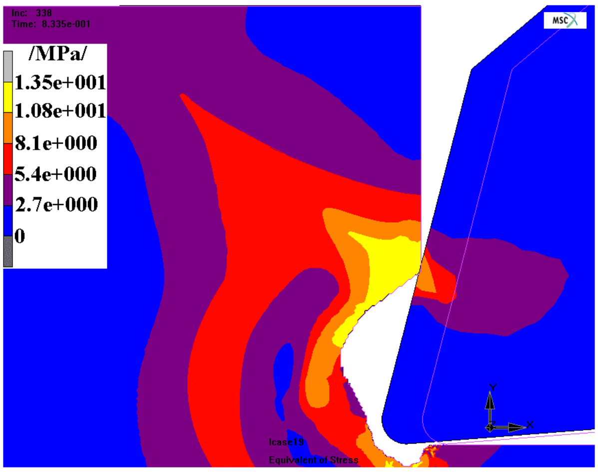

Figure 18. $t = 0.8335$ s (FEM Results).

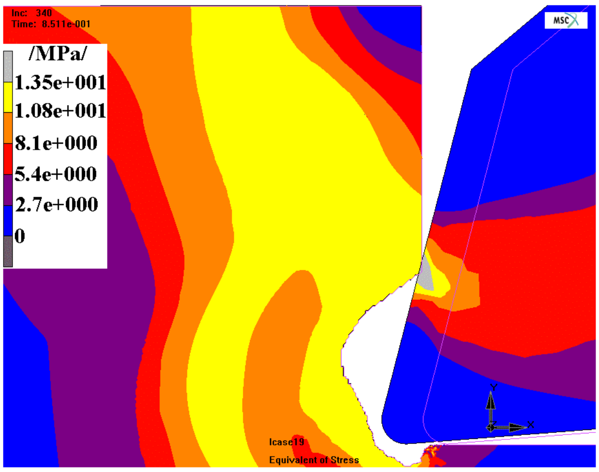

Figure 19. $t = 0.8511$ s (FEM Results).

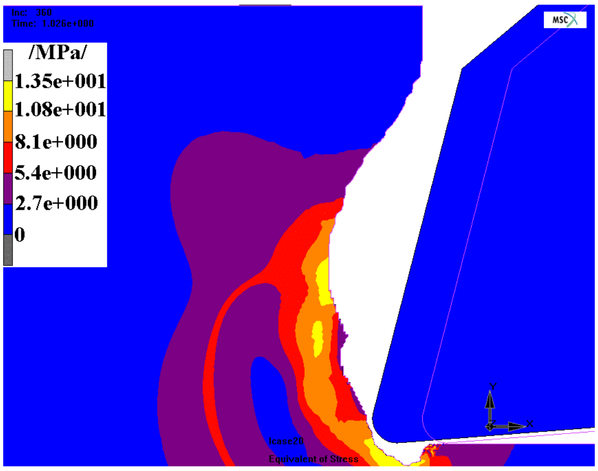

Figure 20 $t = 1.026$ s (FEM Results).

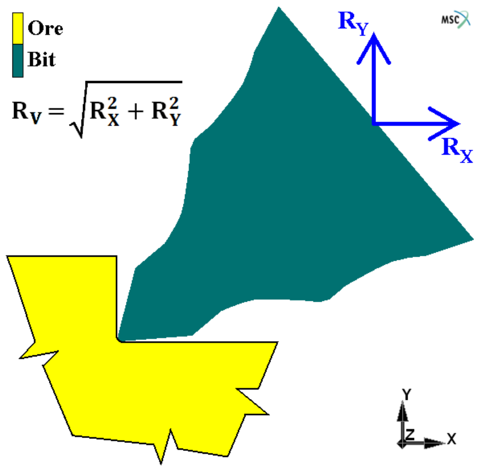

Figure 21 Reaction Forces in the Bit.

From the FEM results, the reaction forces R_X, R_Y and the total reaction force $R_V = \sqrt{R_X^2 + R_Y^2}$ can be calculated. These forces act in the bit, see Figure 21 and 22. Figure 22 is calculated for one simulation (i.e. for the situation when the material of the ore is described by values $R_p = 12$ MPa, $R_m = 13.5$ MPa, $E = 20000$ MPa and $\mu = 0.2$).

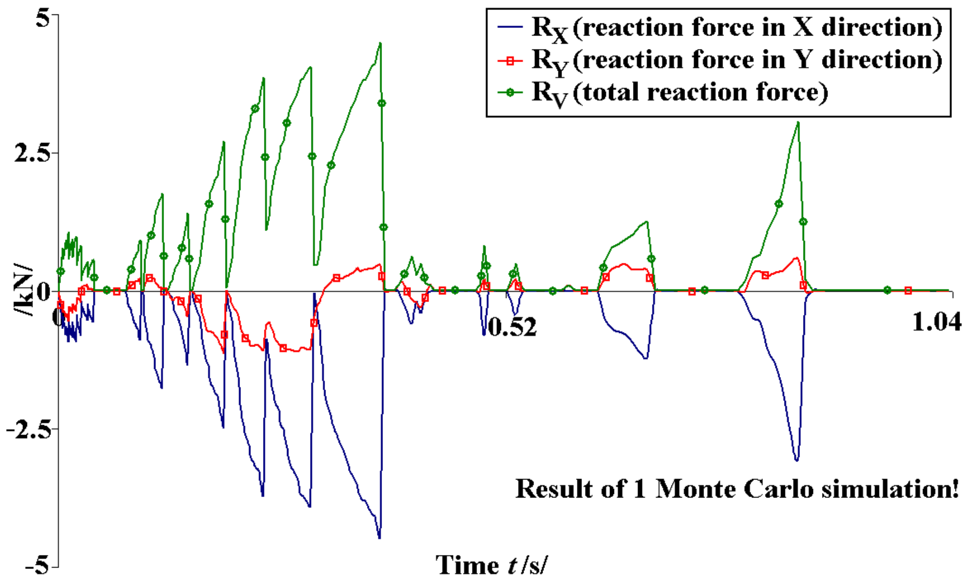

Figure 22. Reaction Forces in the Bit.

A distribution of the total reaction forces acquired from 500 Monte Carlo simulations (i.e. stochastic result) is shown in Figure 23.

The maximum total reaction force (acquired from 500 Monte Carlo simulation) is given by the histogram $R_{V\,MAX\,SBRA,\,FEM} = 5068^{+1098}_{-984}$ N, see Figure 24.

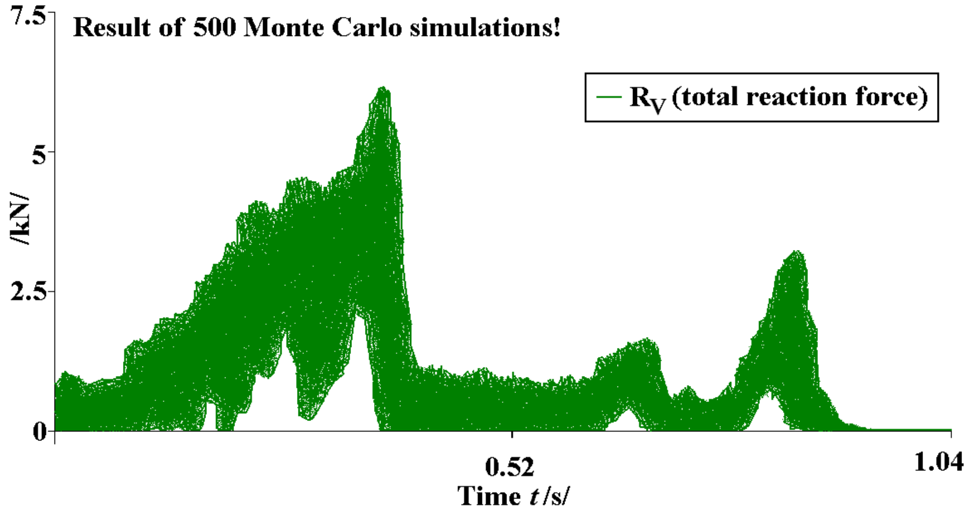

Figure 23 Reaction Forces in the Bit (SBRA-FEM Results).

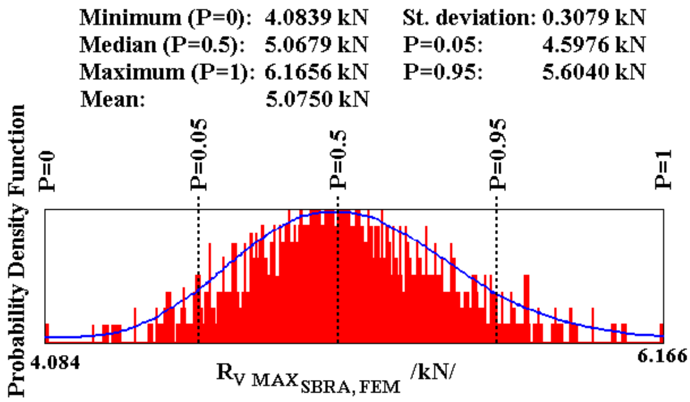

Figure 24 Maximum Total Reaction Forces in the Bit (SBRA-FEM Results of 500 Monte Carlo Simulations) and their Evaluation.

6. Comparison between Stochastic Simulations and Experimental Measurements

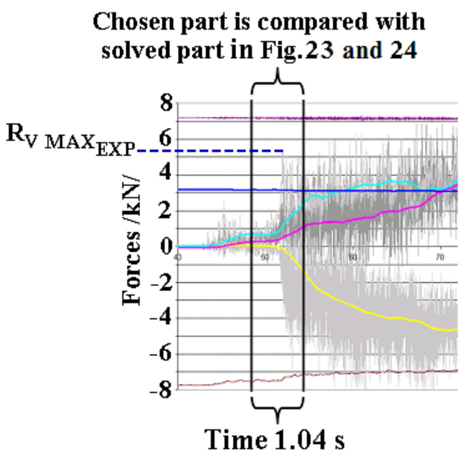

Figure 25 Experimental Measurement, compared with the SBRA-FEM Simulations.

The calculated maximum forces (i.e. SBRA-FEM solutions, see Figure 23 and 24) can be compared with the experimental measurements (i.e. compared with a part of Figure 25), see also references Frydrýšek & Gondek 2008, Frydrýšek 2008, Frydrýšek 2009 and Valíček 2007.

The evaluation of one force measurement (Figure 25) shows that the maximum force is $R_{MAX_{EXP}} = 5280\ \text{N}$. Hence, the relative error calculated for the acquired median value $R_{MAX_{SBRA,\,FEM-MED}} = 5068\ \text{N}$, see Figure 24, is:

$$\Delta_{R_{MAX}} = \frac{R_{MAX_{EXP}} - R_{V\,MAX_{SBRA,FEM-MED}}}{0.01 \times R_{MAX_{EXP}}} = 4.02\,\%.$$

The error of 4.02% is acceptable. However, the experimental results also have large variability due to the anisotropic and stochastic properties of the material and due to the large variability of the reaction forces, see Figures 24 and 25.

7. Proposition of Fully Probabilistic Assessment

Reliability function RF , see references Marek et all 1995 and 2003, can be defined by:

$$RF = R_{V\,MAX_{ALLOWABLE}} - R_{V\,MAX_{SBRA,FEM}},$$

where $R_{V\,MAX_{ALLOWABLE}}$ is the allowable reaction force in the cutting bit, which can be acquired from the real capacity of the whole cutter-loader system in the mine. If situations when $RF \le 0$ occur, then the cutter-loader system is overloaded. Else if $RF > 0$, then safe situations of loading occurs.

Hence, fully probabilistic assessment can be calculated by comparing of probabilities:

$$P(RF \le 0) \le P_{ALLOWABLE},$$

where, $P_{ALLOWABLE}$ is the acceptable probability of overloading of the cutting-loader system. This overloading sometimes really occurs in the mine. Value of $P_{ALLOWABLE}$ can be given by chosen performance requirements of the client (i.e. investor), see Figure 26.

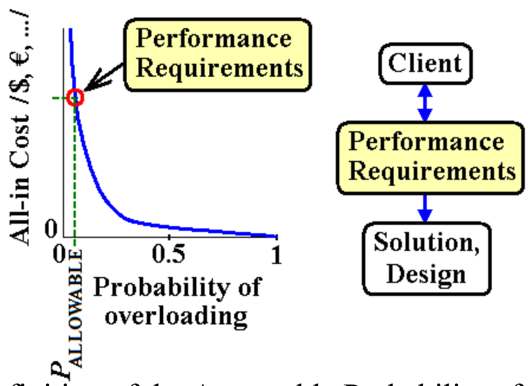

Figure 26 Definition of the Acceptable Probability of Overloading.

8. Conclusion

This paper combines the SBRA (Simulation Based Reliability Assessment) Method and FEM as a suitable tool for simulating the hard rock (platinum ore) disintegration process. All basic factors have been explained (i.e. 2D boundary conditions, material nonlinearities, mechanical contacts and friction between the cutting bit and the ore, the methodology for deactivating the finite elements during the ore disintegration process, application of parallel computers). The use of finite element deactivation during the ore disintegration process (as a way of expanding the crack) is a modern and innovative way of solving problems of this type.

The error of the SBRA-FEM results (i.e. in comparison with the experiments) is acceptable. Hence, SBRA and FEM can be a useful tool for simulating the ore disintegration process.

Because the real material of the ore (i.e. yield limit, fracture limit, Young's modulus, Poisson's ratio etc.) is extremely variable, stochastic theory and probability theory were applied (i.e. application of the SBRA Method).

The SBRA Method, which is based on Monte Carlo simulations, can include all stochastic (real) inputs and then all results are also of stochastic quantities. However, for better application of the SBRA method (for simulating this large problem of mechanics), it is necessary to use superfast parallel computers. Instead of 500 Monte Carlo simulations (wall time cca 70.4 hours, as presented in this article), it is necessary to calculate $>10^4$ simulations (wall time cca 58 days or more). Our department will be able to make these calculations when new and faster parallel computers become available.

Hence, the fully probabilistic assessment was proposed according to the acceptable probability of overloading of the whole cutting-loader system.

All the results presented here were applied for optimizing and redesigning of the cutting bit (excavation tool), see Figure 27 and reference Valíček 2007.

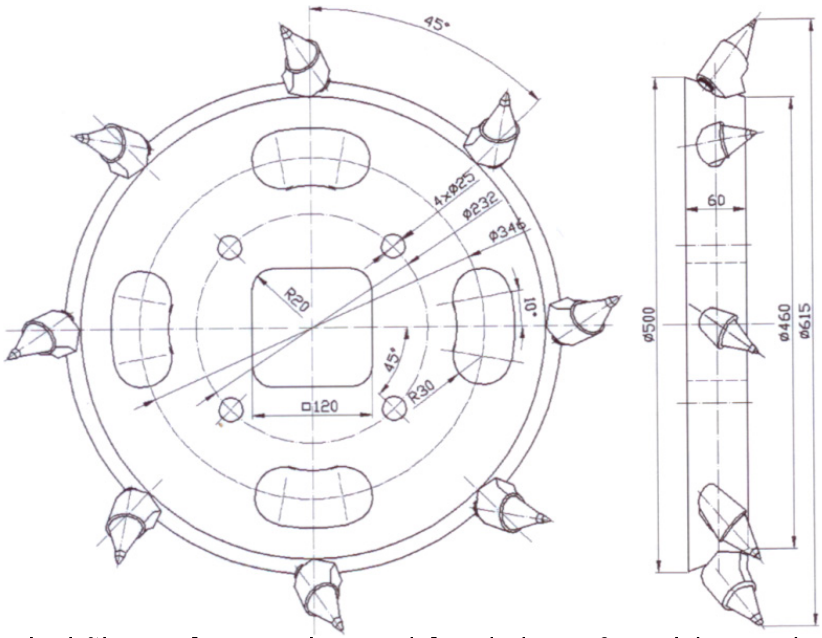

Figure 27 Final Shape of Excavation Tool for Platinum Ore Disintegration Process.

In the future, 3D FE models (instead of 2D plane strain formulation) will be applied for greater accuracy.

9. Acknowledgement

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10. References

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