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STABILIZING PERIODIC ORBITS OF CHAOTIC REGIMES IN A KELVIN TYPE GYROSTAT SATELLITE

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Summary: *In this paper, we present a study of the dynamical behavior in a Kelvin type gyrostat satellite. We firstly obtain the Hamiltonian equations of our model by using Cardan angles as generalized coordinates. Then, we make this Hamiltonian dimensionless and calculate motion equations for this dimensionless system. The study of the Poincare's sections of this system shows us that chaotic motion regimes are present for specific parameter values. The main goal of this work is the finding of stabilizing orbits by using a control technique, the fuzzy control of Poincare map method, so that it can be applied to stabilize special periodic orbits in this system. Finally, we expect that the technique can be useful for a better understanding of control theory and their applications in gyrostat problems.*

1. INTRODUCTION

A Kelvin type satellite consists of two rigid parts, an axi-symmetric rotor, R inside a bigger platform, P. We assume that the center of mass of our satellite is rotating on a circular orbit around a central mass which can be the Earth and that the rotor angular velocity is very high. Also, the platform can rotate slowly in comparison to rotor's velocity. These satellites are known as gyrostat satellites [1].

Chaotic motions in nonlinear systems arise in many real problems. Investigating chaos in satellite dynamic was started in the works by Liu et al. [2, 3]. The authors have shown that chaotic motion is possible in different kinds of satellites such as satellite in circular orbits [4] and also gyrostats in a central gravitational field [5-9].Since the pioneering work on controlling chaos due to Ott, Grebogi, and Yorke [10], named OGY, different control schemes have been proposed that allow one to obtain a desired response from a dynamical system by applying some small but accurately chosen perturbations [11,12].

The methods stated to control chaos can be classified in feedback and non-feedback methods [13,14], depending on how they interact with the system. Feedback methods of chaos control, as the celebrated OGY [10], stabilize one of the unstable orbits that lie in the chaotic attractor by using small state-dependent perturbations into the system. However, in experimental implementations, the fast response that these methods require cannot usually be provided. For these situations, non-feedback methods are more useful. Non-feedback methods have been mainly used to suppress chaos in periodically driven dynamical systems.

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Also in the period of time of OGY work, the Pyragas method, based on delayed feedback control was presented [12, 15]. In recent years, some chaos controller based on fuzzy systems, have been proposed [16-18]. In [16] the idea of chaos control by fuzzy systems is introduced and the Chua's circuit was controlled via fuzzy systems. The fuzzy estimation of OGY and Pyragas controllers are also used for chaos control and is applied to a Bonhoeffer-Van der Pol oscillators as shown in Ref. [18]. In Ref. [19] the author considered the fuzzy control of Poincaré maps, and two algorithms for chaos control based on fuzzy systems are proposed for stabilizing the fixed points or unstable periodic orbits. The first algorithm provides a fuzzy system for the controller using the clustering technique and the second one design the controller by fuzzy table look up method. The advantage of the proposed algorithms is that only the state variables of the system on a Poincaré section are used for chaos control and there is no need to know the mathematical model of the system and its Poincaré map. Because these controllers are constructed on the Poincaré sections, the method of this paper can be used for both discrete and continuous systems.

In this paper, nonlinear governing equations of a gyrostat without any restriction to small angles or perturbations are adapted from the work described in Ref. [1]. Later, attitude dynamic of this system is investigated in Poincare map. The fixed points in these Poincare maps are found using a recursive method and stabilized using the fuzzy control method presented in [19].

This paper is organized as follows. In Sec. II we present a complete description of our model, the Kelvin type satellite. Section III presents a complete estimation of the parameters of our model. Sec. IV provides a full description of the control method used for the stabilization of our system, namely Fuzzy Control. Numerical evidence of the robustness of the Fuzzy Control technique is given in Sec. V. Conclusions and discussions of the main results of this paper are presented in Sec.VI.

2. MODEL DESCRIPTION

We now introduce our prototype model, the Kelvin type Gyrostat Satellite. In their equations, we assume that both gyrostat and rotors are rotating about axis z. A Cxyz coordinate system is fixed to the gyrostat. The center of mass, C, rotates around the Earth in a circular orbit. θ_2, θ_1 and θ_3 are three Cardan angels about axis y_1, x_0 and z, respectively. Kinematic and potential energy of this system can be calculated as

$$T = \frac{A}{2}[\dot{\theta}_1^2 \cos^2\theta_2 + \dot{\theta}_2^2 + \Omega^2(\sin^2\theta_1 + \cos^2\theta_1 \sin^2\theta_2) - 2\dot{\theta}_1\Omega\cos\theta_1\sin\theta_2\cos\theta_2 + 2\dot{\theta}_2\Omega\sin\theta_1] + \frac{C^P}{2}\left(\omega_z^P\right)^2 + \frac{C^R}{2}\left(\omega_z^R\right)^2 \tag{1}$$

and

$$V = \frac{3}{2}(C - A)\Omega^2\sin^2\theta_2 \tag{2}$$

In these relations, $A = A^P + A^R, C = C^P + C^R$ are moments of inertia of satellite and ω_z^R, ω_z^P are angular velocities of rotor and platform about z axis, respectively. $\Omega = \sqrt{\frac{\mu}{Q^3}}$ is the orbital angular velocity of the whole system, where $\mu = GM$ and Q is the radius of the satellite orbit around the Earth, and θ_3 is a cyclic coordinate. So, we can conclude that P_3 is constant during the motion. Now, if we assume that the angular momentum of the rotor is constant too we can write

$$\begin{aligned}\omega_z^P &= \dot{\theta}_1 \sin\theta_2 + \dot{\theta}_3 + \Omega\cos\theta_1 \sin\theta_2 \\ &= \omega_0 = const\end{aligned} \tag{3}$$

Using this relation, the Hamiltonian of this system can be obtained as

$$\begin{aligned}H = P_1\dot{\theta}_1 + P_2\dot{\theta}_2 + P_3\dot{\theta}_3 - L = \frac{1}{2A}\left(\frac{P_1^2}{\cos^2\theta_2} + P_2^2\right) + P_1\Omega\cos\theta_1\tan\theta_2 \\ -\frac{P_1P_3\sin\theta_2}{A\cos^2\theta_2} - P_2\Omega\sin\theta_1 - P_3\Omega\frac{\cos\theta_1}{\cos\theta_2} + \frac{P_3^2}{2A}\tan^2\theta_2 + \frac{3}{2}(C-A)\Omega^2\sin^2\theta_2\end{aligned} \tag{4}$$

where

$$\begin{aligned}P_1 &= A\left(\dot{\theta}_1\cos^2\theta_2 - \Omega\cos\theta_1\sin\theta_2\cos\theta_2\right) + P_3\sin\theta_2 \\ P_2 &= A\left(\dot{\theta}_2 + \Omega\sin\theta_1\right) \\ P_3 &= C^P\omega_z^P + C^R\omega_z^R = const\end{aligned} \tag{5}$$

Now, by introducing dimensionless variables $\tau = t\sqrt{H/A}$ and $p_i = \frac{P_i}{\sqrt{HA}}$ we will have

$$\begin{aligned}h = \frac{1}{2}\left(\frac{p_1^2}{\cos^2\theta_2} + p_2^2\right) + \sqrt{\gamma}\,p_1\Omega\cos\theta_1\tan\theta_2 - \sqrt{\gamma}\lambda_2\lambda_3 p_1\frac{\sin\theta_2}{\cos^2\theta_2} \\ -\sqrt{\gamma}\,p_2\sin\theta_1 - \gamma\lambda_2\lambda_3\frac{\cos\theta_1}{\cos\theta_2} + \frac{\gamma}{2}\lambda_2^2\lambda_3^2\tan^2\theta_2 + \frac{3}{2}\gamma(\lambda_1 - 1)\sin^2\theta_2,\end{aligned} \tag{6}$$

where h is the dimensionless Hamiltonian equation. In this equation, $\lambda_1 = C/A$, $\lambda_2 = \omega_0/\Omega$ and $\lambda_3 = C^*/A = \frac{C}{A} + \left(\frac{\omega_z^R}{\omega_z^P} - 1\right)\frac{C^R}{A}$ are control parameters of the system. Also, $\gamma = A\Omega^2/H$ is the ratio of gravitational energy of the rotational energy of the satellite.

By letting $h = 1$, the equations of motion will be obtained as

$$\begin{aligned}\dot{\theta}_1 &= \frac{p_1}{\cos^2\theta_2} + \sqrt{\gamma}\cos\theta_1\tan\theta_2 - \sqrt{\gamma}\lambda_2\lambda_3\frac{\sin\theta_2}{\cos^2\theta_2} \\ \dot{p}_1 &= \sqrt{\gamma}\,p_1\sin\theta_1\tan\theta_2 + \sqrt{\gamma}\,p_2\cos\theta_1 - \gamma\lambda_2\lambda_3\frac{\sin\theta_1}{\cos\theta_2} \\ \dot{\theta}_2 &= p_2 - \sqrt{\gamma}\sin\theta_1 \\ \dot{p}_2 &= -p_1^2\frac{\sin\theta_2}{\cos^3\theta_2} - \sqrt{\gamma}\,p_1\frac{\cos\theta_1}{\cos^2\theta_2} + \sqrt{\gamma}\lambda_2\lambda_3 p_1\frac{1+\sin^2\theta_2}{\cos^3\theta_2} \\ &\quad -\gamma\lambda_2^2\lambda_3^2\frac{\sin\theta_2}{\cos^3\theta_2} + \gamma\lambda_2\lambda_3\cos\theta_1\frac{\sin\theta_2}{\cos^2\theta_2} - 3\gamma(\lambda_1 - 1)\sin\theta_2\cos\theta_2\end{aligned} \tag{7}$$

Studying the behaviour of chaotic systems is much simpler when discretized. The idea of reducing the study of continuous time systems to the study of an associated discrete time system is due to Poincaré (1899). As a matter of fact, associated to an ordinary differential equation we can construct a discrete time dynamical system which is called a Poincaré map [20]

$$\vec{X}(n+1) = P\left(\vec{X}(n), u(n)\right), \tag{8}$$

where $P(.,.)$ is the Poincaré map, $\vec{X}(n)$ is the state vector on the Poincaré section in which the Poincaré map is defined, and $u(n)$ is the controlling action. The fixed point for a chaotic system is defined as the state which maps into itself through the Poincaré map. In other words, this specific trajectory of the system, beginning from a fixed point, returns to this point after a specific time named period. In Fig. 1 and Fig. 2 Poincaré map of this equations for different values of parameters are shown. As it is easily observed, in each of them there are different types of orbits (periodic, chaotic, etc) that can be stabilized by implementing our fuzzy control scheme we describe in the next section. We also observe KAM islands which are typical in Hamiltonian systems.

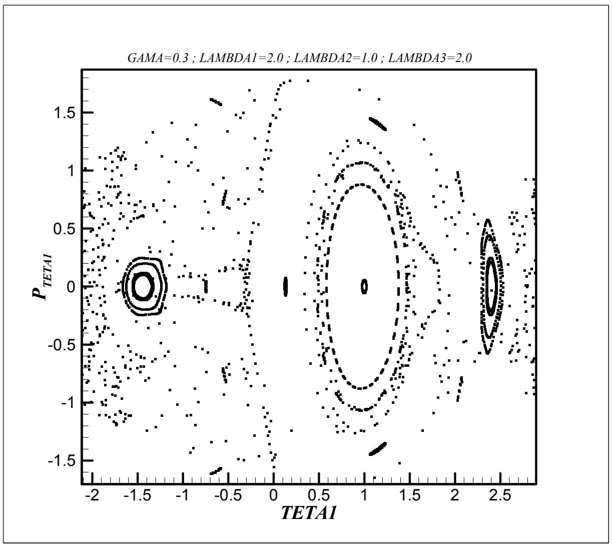

Figure 1 Poincaré map of our Kelvin type Gyrostat satellite for parameter values as follows: $\gamma = 0.3$ and $\lambda_1 = 2.0$, $\lambda_2 = 1.0$, $\lambda_3 = 2.0$. Different dynamical behaviors are observed as periodic, chaotic, etc.

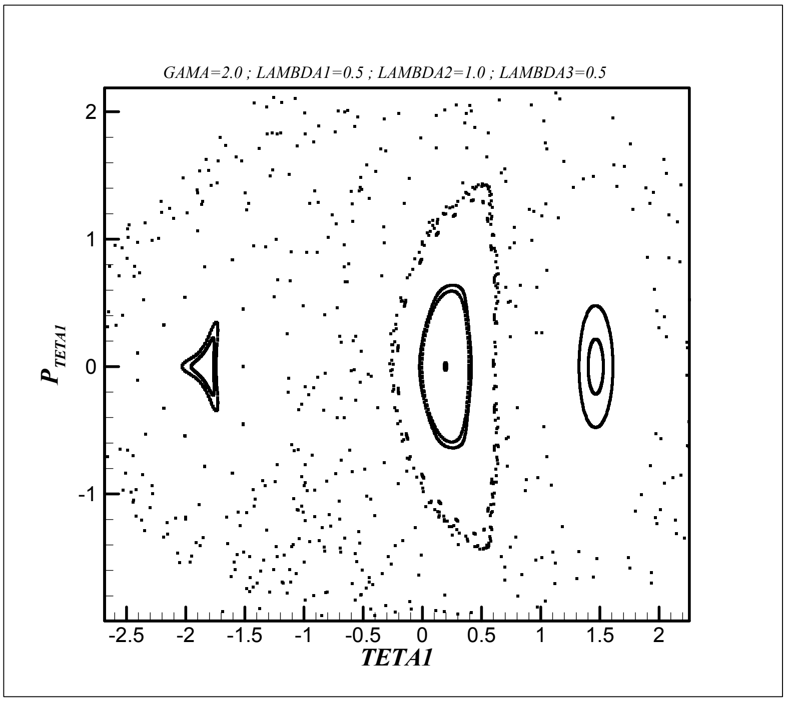

Figure 2 Poincaré map of our Kelvin type Gyrostat satellite for parameter values as follows $\gamma = 2.0$ and $\lambda_1 = 0.5, \lambda_2 = 1.0, \lambda_3 = 0.5$. As in Fig. 1 different dynamical behaviors can be observed.

The chaotic system starting from an arbitrary point exhibits erratic behavior in the Poincaré map which fills specific areas in this surface. With an appropriate control of $u(n)$, the system can be forced towards its fixed point which is the desired state in most cases. In this case the chaotic behaviour of the system transforms into a periodic behaviour. It is assumed that the dynamic equation of the system is unknown, but the state vector, $\vec{X}(n)$, is obtainable, then the main goal is to design an identifier/controller scheme to stabilize the unstable fixed points of the system as the authors stated in Ref. [19].

3. Parameter Estimation

The parameter $\lambda_1 = C/A$ defined previously is the degree of oblateness of the satellite. For typical geometrical shapes we can assume that this degree is of order 1. As an example, in a cube this degree is exactly equal to 1 and in a cylinder is $\lambda_1 = \dfrac{6r^2}{3r^2 + h^2}$, where r is the radius of the cylinder, and h is its height. We can simply assume that this parameter is constant and for example, equal to 2. We are not going to use this parameter as action control in our fuzzy system. The change of moment of inertia has some other effects on the equations of motions and also other parameters. In fact, we can not look at this parameter as a free variable and change it without considering these effects on the whole system.

For a satellite rotating around the Earth it is well known that $\mu = 3.986005 \times 10^{14}\ \mathrm{m^3/s^2}$ and $6800 \le Q \le 7500\,km$. Therefore, angular velocity of the satellite around the Earth is small, and of order $\Omega \propto O\left(e^{-3}\right)$ and the angular velocity of the platform can have different values. In especial flight missions, it is necessary for spacecraft to keep its orientation to the Earth. Hence, the angular velocity of the satellite, around its major axis, should be equal to its orbital angular velocity and therefore, $\lambda_2 = \omega_0/\Omega \approx 1$. Control of this value and keeping $\lambda_2 = 1$ is a goal of control systems in these satellites. In some other types which the orientation of the satellite is not a control goal of the system, we can assume it as an action control for obtaining especial maneuvers and orientations. The range of variation of λ_2 in theses spacecrafts could be very wide. If we assume angular velocity of satellite around its major axis, ω_0 equal to unity we will have $\lambda_2 = \omega_0/\Omega \approx 1000$. Using the relation stated in Eq. 5(c) and by changing the angular velocity of the rotor we can change λ_2 in the range of $-1000 \le \lambda_2 \le 1000$. Variation in this parameter is a result of variations in the angular velocity of rotor. On the other hand, the angular velocity of the rotor could not exceed especial values depending on the rotor characteristics. So, this range of variation for λ_2 can not be obtained in real cases.

The last control parameter in this system is λ_3. If the rotor, R, contains only 5 % of the whole satellite weight we will have, $C^R/A = 0.1$. Also, we know that $\omega_z^R >> \omega_z^P$. Now, if we let $-50 \le \frac{\omega_z^R}{\omega_z^P} \le 50$, the range of variation of λ_3 will be $-4 \le \lambda_3 \le 6$.

By changing ω_z^R, both λ_2 and λ_3 will be change. Also, these two parameters always appear in equations of motions together. So, we can let $\lambda = \lambda_2\lambda_3$ and choose it as the control input of control system. Range of practical control inputs in this system is so that the acceptable ranges for λ_2 and λ_3 are satisfied.

Now, by using the fuzzy control method developed in Ref. [19] we can design a fuzzy controller to stabilize periodic orbits in the Poincaré map.

4. FUZZY CONTROL METHOD

One of the methods available for constructing a fuzzy model from input-output data pairs is the fuzzy clustering method. This method is especially useful when the number of input-output pairs is limited. The basic idea is to group the input-output pairs into clusters and use a specific rule for each cluster, in the form of

$$\text{IF } x \text{ in } A[x_c^l], \text{ THEN } y \text{ in } B[y_c^l], \qquad (9)$$

where $A[x_c^l]$ and $B[y_c^l]$ are input and output fuzzy sets with centers at x_c^l and y_c^l, respectively and l is the number of cluster. There are several algorithms to make a fuzzy system based on the clustering. One of the simplest methods is the nearest neighborhood algorithm. This method is explained extensively in many fuzzy control books such as shown in Ref. [22]. The designed fuzzy system using singleton fuzzifier, products inference engine and center average deffuzifier based on *k* input-output pair clustered in this method can be written as follows:

$$f_k(x)=\frac{\sum_{l=1}^{M}a^l(k)\exp\left(-\frac{\left|x-x_c^l\right|^2}{\sigma^2}\right)}{\sum_{l=1}^{M}b^l(k)\exp\left(-\frac{\left|x-x_c^l\right|^2}{\sigma^2}\right)} \qquad (10)$$

where M is the number of clusters constructed, x_c^l denotes the center of the l^{th} cluster, $a^l(k)$ is the summation of all output data gathered in cluster l and $b^l(k)$ is the number of data points gathered in cluster l after examining k data points. In the above equation σ is a smoothing parameter. The smaller the σ the smaller the matching error becomes, but the less smooth the $f_k(x)$. The matching error is the difference between the actual output and the one obtained from fuzzy model. It should be noted that the number of clusters depends on the distribution of input points and the radius r.

The following algorithm is adapted from Ref. [19] to construct a suitable controller for this system. For further studies you can return to the original paper. Now, we provide a complete description of the algorithm implemented in order to stabilize the orbits in our model.

5. I Algorithm:

The Fuzzy algorithm, can be built according to the following steps:

STEP 1: Let $\vec{X}(1)$ be an arbitrary point in the domain of Poincaré map, then choose a random value for $u(1)$ in its prescribed domain, and measure $\vec{X}(2)$ due to problem assumptions.

STEP 2: Repeat step 1, by setting $\vec{X}(2)$ as an starting point to generate $\vec{X}(3)$ by using a random value for $u(2)$.

STEP 3: By iterating step 2, a set of $\Gamma=\left\{\left(\vec{X}(k+1),\vec{X}(k),u(k)\right),k=1,2,...,N\right\}$ for a large N, is generated.

STEP 4: The input-output data pairs for the fuzzy clustering algorithm are obtained according to:

STEP *4.1:* Let $j=0$ and $k=1$.

STEP *4.2:* Consider $\left(\vec{X}(k+1),\vec{X}(k),u(k)\right)$, then examine the following condition

$$\left|\vec{X}(k+1)-\vec{X}_F\right|\leq m\left|\vec{X}(k)-\vec{X}_F\right|, \qquad (11)$$

where $0<m<1$ is selected arbitrarily and called the approaching factor. If the above condition is satisfied then let $j=j+1$, $\vec{X}_0^j=\vec{X}(k)$, $u_0^j=u(k)$. The j^{th} input-output data pair is let $\left(\vec{X}_0^j;u_0^j\right)$.

STEP 4.3: Iterate step 4.2. by $k=k+1$.

STEP 5: Now the clustering algorithm is applied on $\left(\vec{X}_0^j;u_0^j\right)$ to obtain $U(.)$.

6. NUMERICAL RESULTS

The periodic orbits of the Poincaré map in Fig. 1 and Fig. 2 can be found by using a recursive method. The central periodic orbit shown in Fig. 1 takes place for:

$$\begin{aligned} \theta_1 &= 1.00001 \\ p_{\theta_1} &= 0 \\ \theta_2 &= 0 \\ p_{\theta_2} &= 2.152278 \end{aligned} \tag{9}$$

Results which are obtained from fuzzy control are shown in Figs. (3-4). Figure 3 and figure 4 show convergence to the fixed point of the Fig. 1 once our fuzzy control is implemented. After, approximately, 100 iterations in our control scheme, our orbit trend to a fixed point.

Finally, Fig. 5 shows the phase space of our model after applying our control method in which we easily observe the convergence to a fixed point for which the stabilization is completely achieved.

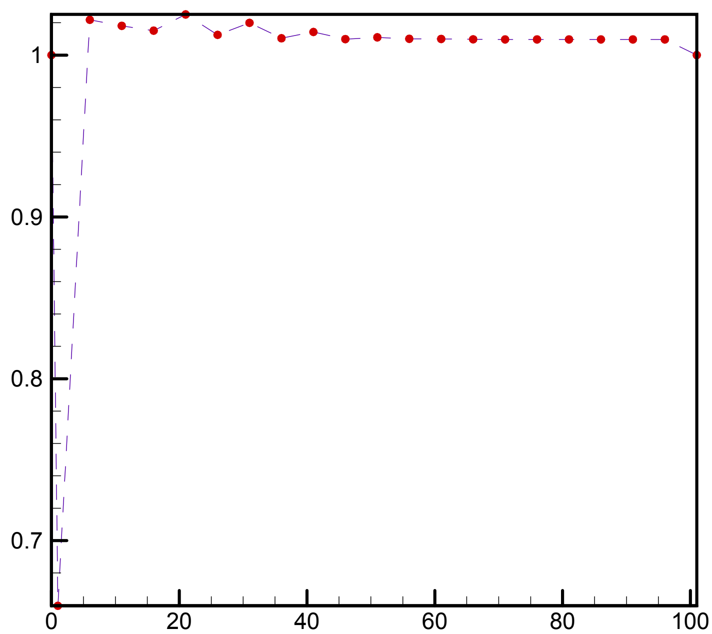

Figure 3 Trend of θ_1 convergence to the fixed point on Poincaré map in which horizontal axis shows number of iterations.

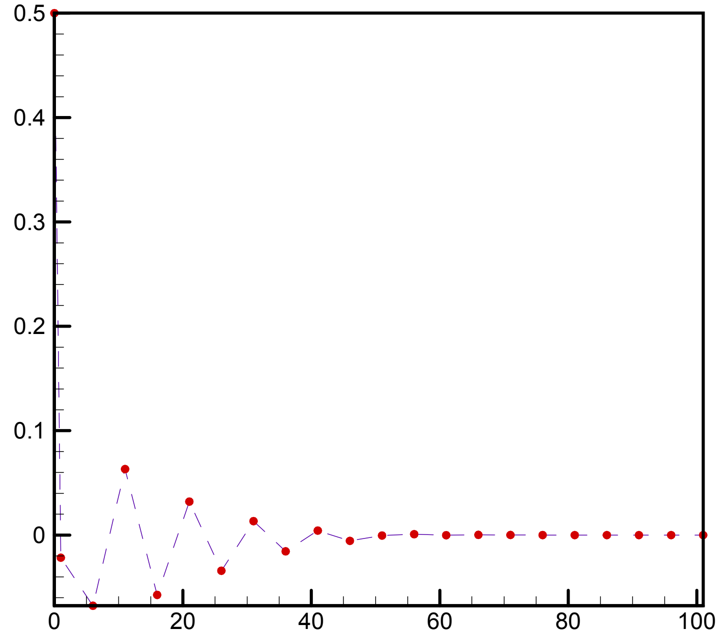

Figure 4 Trend of p_{θ_1} convergence to the fixed point on Poincare map, in which horizontal axis shows number of iterations.

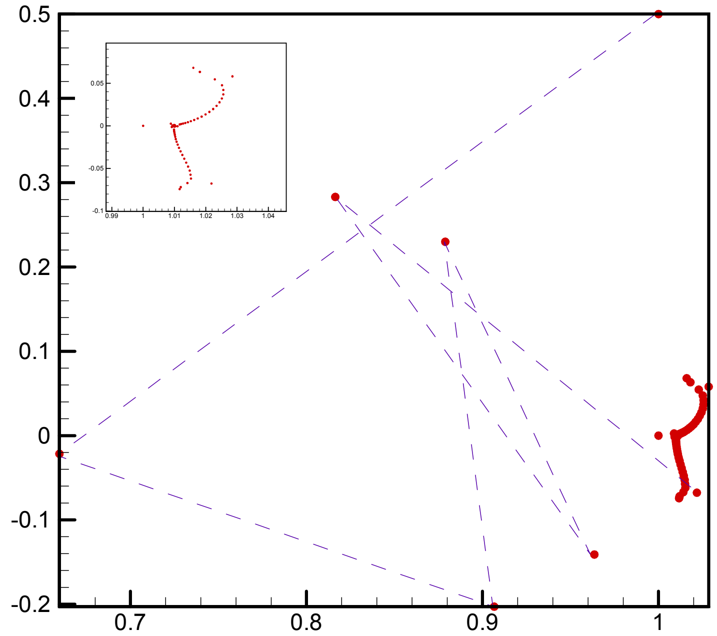

Figure 5 Convergence of points on $\theta_1 - p_{\theta_1}$ in the Poincaré map. The stabilization in a periodic orbit is obtained.

7. CONCLUDING REMARKS

Summarizing, we have implemented a control technique to stabilize orbits in chaotic or periodic regimes in a Kelvin type gyrostat satellite. The Hamiltonian equations of our model are obtained by using Cardan angles as generalized coordinates. Our control scheme, named Fuzzy control, is fully described and applied to stabilize periodic or chaotic orbits. This technique is successfully applied for special orbits found in phase space. Finally, we expect our technique can be applied in other gyrostat models and different physical situations where control techniques are required.

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