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THE SECOND VISCOSITY OF FLUIDS

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Summary: *This paper is focused on the study of the second viscosity of the fluids with the emphasis on the pressure pulsations solution in the hydraulic systems. The methodic comes out from the nonequilibrium thermodynamics.*

1. INTRODUCTION

Using its laws the irreversible stress tensor and two viscosity coefficients, which depend on the rate-of-strain tensor and the spherical deformation velocity tensor will be derived. Bulk viscosity coefficient will be determined on the pressure wave's basis for the water.

2. IRREVERSIBLE STRESS TENSOR

It is possible to express the generalized equation for the irreversible stress tensor Π_{ij} according to Stokes as rate-of-strain tensor dependence:

$$\Pi_{ij} = 2\eta c_{ij} \tag{1}$$

where η represents the shear viscosity. In the experiments with the compressible liquids there was shown, that the absorption capacity of the sound waves, respectively of the pressure waves in liquids, is higher than corresponds to the dynamic viscosity effect. Stokes has already assumed that it will be necessary to extend the equation (1) by the so called second viscosity λ implementation:

$$\Pi_{ij} = 2\eta c_{ij} + \lambda \delta_{ij} c_{kk} \tag{2}$$

This equation is founded on the nonequilibrium thermodynamic methods, using the Onsager reciprocity relations [10] between the irreversible flows J_i and generalized thermodynamic forces X_j in the form: $J_i = L_{ij} X_j$, where L_{ij} are so called phenomenological coefficients of the thermodynamical force transfer.

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If we confine oneself to the isotropic medium and use the Currie [6] principle (the generalized thermodynamic force of certain tensor size can evoke the flow of the same tensor size) it is possible to write the deviator of the irreversible shear stress tensor D_{ij} as: $D_{ij} = 2\eta d_{ij}$, where d_{ij} is deviator of the deformation velocity tensor (2).

For the spherical stress tensor (scalar flow) it holds:

$$\frac{1}{3}\Pi_{kk} = \xi\, c_{kk}\ ,\ \xi\rangle 0 \text{ is so called volumetric viscosity, dependent on } \eta . \tag{3}$$

So:

$$\begin{aligned} \Pi_{ij} &= D_{ij} + \frac{1}{3}\delta_{ij}\Pi_{kk} = 2\eta d_{ij} + \xi\delta_{ij}c_{kk} \\ \Pi_{ij} &= 2\eta c_{ij} + \left(\xi - \frac{2}{3}\eta\right)\delta_{ij}c_{kk} \end{aligned} \tag{4}$$

If we compare (2)&(4), we can find the relationship between the second λ and volumetric ξ viscosity in form:

$$\lambda = \xi - \frac{2}{3}\eta \tag{5}$$

In contemporary literature and the most up-to-date software packages it is assumed, that $\lambda = -\frac{2}{3}\eta$, even if it is known that this term holds just for the monoatomic gas and providing that the pressure equals to the summation average of the main tensions [9]. But this assumption excepts the existence of the volumetric viscosity, which is split between the results of nonequilibrium thermomechanics. See for example S. de Groot, P. Mazur [4]

3. SOLUTION METHOD

The dependence of the irreversible stress tensor on the volume viscosity is given by the Onsager relations between the irreversible flows J_i and generalized thermodynamic forces X_j in the form:

$$J_i = L_{ij} X_j \tag{6}$$

From the following term implies that the thermodynamic forces X_j influence is developed to the irreversible thermodynamic flows in the same time. L_{ij} are the coefficients of the thermodynamic forces transfer and present the **material constants independent on time**.

So:

$$J_i(t) = L_{ij} X_j(t). \tag{7}$$

For the viscoelastic materials and liquids this assumption was not generally proved. On the experimental basis is shown that the flow depends on the whole history of thermodynamic forces changes.

If there are the flows J_i dependent on the whole forces history $X_j(t)$, is it possible to write the last equation in the form:

$$J_i(t) = \int_0^t L_{ij}(t-\tau)\; X_j(\tau)\, dT \tag{8}$$

where $L_{ij}(t)$ is possible to understand as a liquid memory. In the concrete for the scalar flow of the irreversible spherical stress tensor holds:

$$\frac{1}{3}\Pi_{kk} = \int_0^t \xi(t-\tau)\; C_{kk}(\tau)\, dT \tag{9}$$

where $\xi(t)$ is the volumetric liquid memory

The dependence on the frequency is possible to express from the Laplace image of the last equation:

$$\frac{1}{3} L\{\Pi_{kk}\} = \frac{1}{3} H_{KK}(s) = L\{\xi(t)\} L\{C_{kk}\} \tag{10}$$

If we specify:

$$L\{\xi(t)\} = \kappa(s) \tag{11a}$$

$$L\{C_{kk}\} = W_{kk}(s) \tag{11b}$$

For $s = i\Omega$ it holds:

$$\frac{1}{3} H_{kk}(i\Omega) = \kappa(i\Omega) W_{kk}(i\Omega). \tag{12}$$

The κ magnitude is possible to identify by the pressure wave dampening for the exactly known boundary conditions, for example in the pipe with closed endings.

4. SECOND VISCOSITY APPROXIMATE IDENTIFICATION

The second viscosity identification is based on the pressure wave eigen shape study in the circle cross section pipe with closed endings.

For this case is better to come out of the Navier-Stokes equation in the form:

$$\rho \cdot \frac{\partial c_i}{\partial t} - \frac{\partial \Pi_{ij}}{\partial x_i} + \frac{\partial p}{\partial x_i} = 0 \tag{13}$$

And the continuity equation:
$$\frac{\partial p}{\partial t} + \rho \cdot v^2 \cdot \frac{\partial c_i}{\partial x} = 0 \tag{14}$$

where c_i is the liquid velocity coordinate, p is the pressure and ρ is the density

In as much as we will study the liquid oscillation (the pressure wave propagation) in the pipe axis direction, in the equations (6) and (7) are **vanished the convective** terms.

In the continuity equation means the symbol v the pressure wave propagation velocity (the sound velocity in the liquid) and will also be identified.

By the equation (6) and (7) unification is possible to write the **wave equation** for the pressure wave propagation in the form:

$$\frac{\partial^2 p}{\partial t^2} - \frac{2 \cdot \eta + \lambda}{\rho} \cdot \frac{\partial^3 p}{\partial t \partial x_i^{\,2}} - v^2 \cdot \frac{\partial^2 p}{\partial x_i^{\,2}} = 0 \tag{15}$$

The magnitudes of λ and v will be set using the eigen oscillation analysis of the pressure function. The eigen shapes of the pressure wave $h(x_i, s)$ are defined by the equation:

$$s^2 \cdot h - \frac{2 \cdot \eta + \lambda}{\rho} \cdot s \cdot \Delta h - v^2 \cdot \Delta h = 0 \tag{16}$$

where Δ is the Laplace operator and s is the eigen number.

Now let’s do the scalar product of the equation (16) by the function h^*, that is conjugate imaginary function to the h:

$$s^2 \cdot \int_V h \cdot h^* \cdot dV + \frac{2 \cdot \eta + \lambda}{\rho} \cdot s \cdot \int_V \Delta h \cdot h^* \cdot dV + v^2 \cdot \int_V \Delta h \cdot h^* \cdot dV = 0 \tag{17}$$

It is necessary to add the boundary conditions for the equations (15), (16). So assume the blanking of the pipe ends as it holds the boundary condition (18).

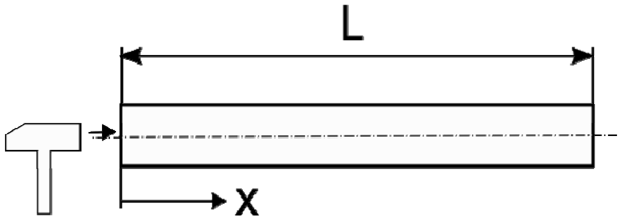

Fig. 1 The experiment schema

$$x = 0: \quad \frac{\partial p}{\partial x} = 0$$
$$x = L: \quad \frac{\partial p}{\partial x} = 0 \qquad (18)$$

Using these boundary conditions is possible the equation (17) rewrite in the form:

$$s^2 + \frac{2 \cdot \eta + \lambda}{\rho} \cdot s \cdot \frac{\int_V \Delta h \cdot h^* \cdot dV}{\int_V h \cdot h^* \cdot dV} + v^2 \cdot \frac{\int_V \Delta h \cdot h^* \cdot dV}{\int_V h \cdot h^* \cdot dV} = 0 \qquad (19)$$

In the equation means:

$$k = \left(\frac{\int_V \frac{\partial h}{\partial x} \cdot \frac{\partial h^*}{\partial x} \cdot dV}{\int_V h \cdot h^* \cdot dV} \right)^{\frac{1}{2}} ; \; s = \alpha + i\omega \qquad \alpha, \omega \in R \qquad (20)$$

k – is the magnitude of the wave vector, so:

$$k = \frac{2 \cdot \pi}{\Lambda}, \; \Lambda \text{ - wave length} \qquad (21)$$

For the first wave shape it holds: $\Lambda = 2L$

The λ and ν magnitudes are derived from the wave eigen values and eigen frequencies analysis:

$$\lambda = \frac{-2 \cdot \alpha \cdot \rho \cdot L^2}{\pi^2} - 2 \cdot \eta, \; v = \frac{L}{\pi} \cdot \sqrt{\alpha^2 + \omega^2}, \quad \alpha < 0 \qquad (22)$$

Above mentioned equations is possible to use just for the cases of wide difference between the eigen frequency of the pipe and liquid.

Out of the executed experiments it is confirmed, that the equations are valid for cases of very small centrifugal mass of both pipe covers. In other case it is necessary to assume the pipe/liquid interaction, because the wave length is changing by the pipe oscillation. The pipe oscillation causes the change of the node points of the pressure wave eigen shape.

5. THE VERIFIED MODEL OF THE SECOND VISCOSITY IDENTIFICATION

Fig. 2 shows the principal schema of the experiment that is based on the shock wave creation and the following liquid oscillation at the pipe interaction.

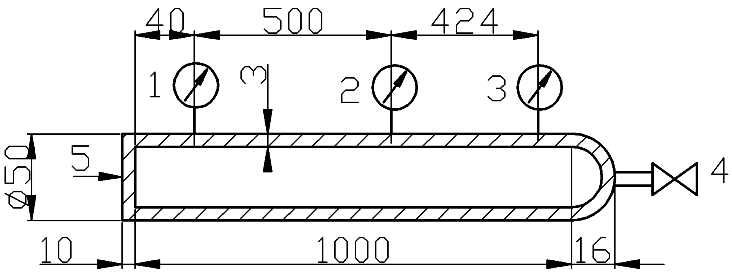

Fig 2 The schema of the experimental pipe

The shock wave is created by the direct stroke of the hammer to the pipe cover in the pipe axis direction, see Fig. 1. The pipe includes three sensors for the pressure measurement, as is shown in the Fig. 2. The liquid is deaerated before the experiment. The measurement was done for the different values of the static pressure up to 100MPa.

The photo of the measuring device is in the Fig. 3. Fig. 4 shows the pressure dependence on time (the sensor 1).

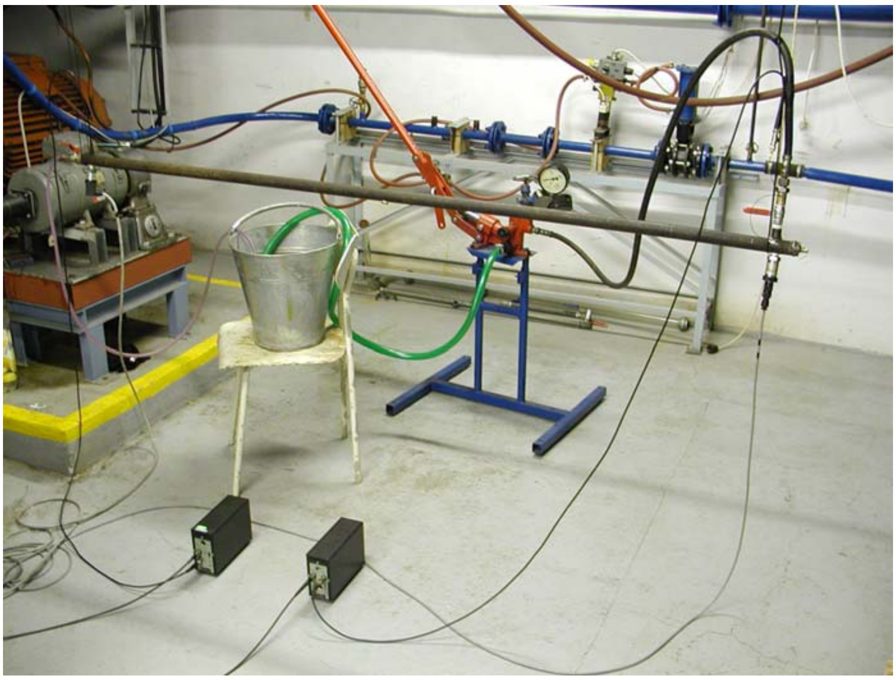

Fig. 3 The measuring device

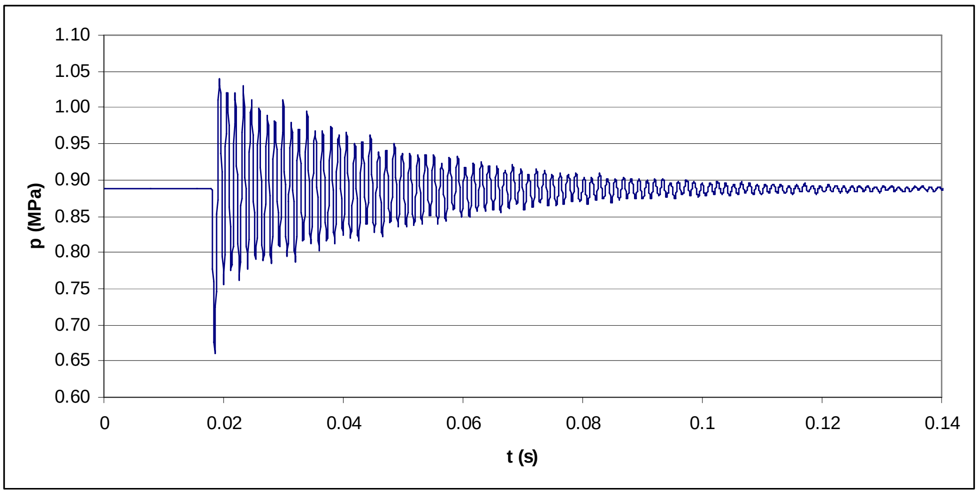

Fig. 4 The measured pressure in the time dependence

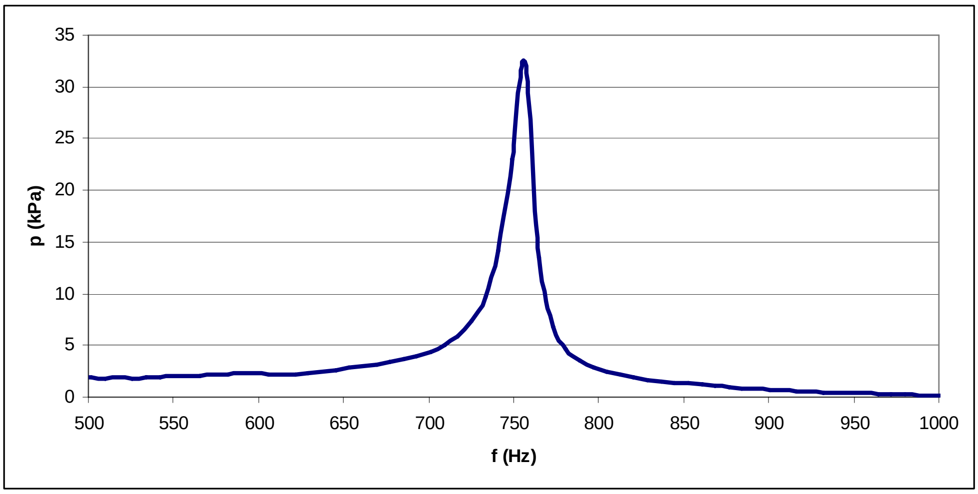

Fig. 5 The measured pressure after DFT

The signal was processed by the fast discrete Fourier transform, see Fig. 5. From the notification were set the real (α) and imaginary Ω part of the eigen number *s* for the first pressure wave shape:

$$s = \frac{\pi}{\sqrt{3}} \cdot \Delta f \pm i\Omega \tag{23}$$

The interacting oscillation of the liquid and the pipe was solved in the linear area using the transfer matrix of the liquid **P**, pipe $\mathbf{P_e}$ and the boundary conditions (the pipe covers mass). The boundary condition influence is given by the matrix $\mathbf{P_0}$.

In the transfer matrix is also implied the second viscosity effect, that is derived from the hypothesis of the zero determinant value.

$$\det\begin{Vmatrix} \mathbf{P_0} & -\mathbf{E} \\ \mathbf{P_C} & -\mathbf{P} \end{Vmatrix} = 0, \quad \mathbf{P_C} = \mathbf{P_0} \cdot \mathbf{P_T} \tag{24}$$

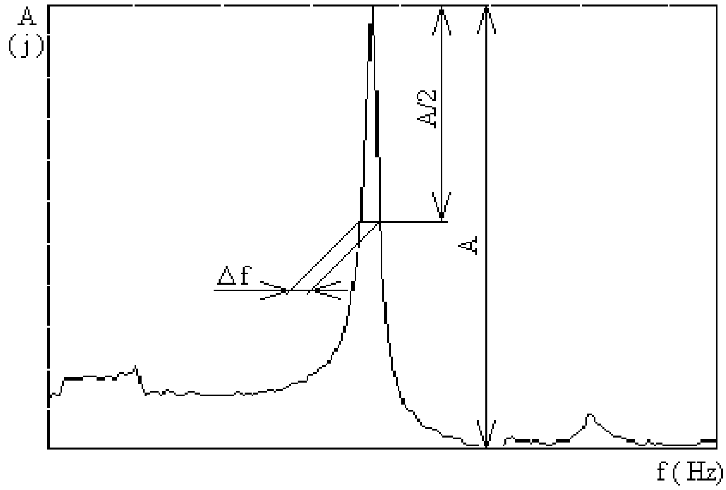

Fig. 6 Amplitude-frequency characteristic

By the eigen value (23) establishing into the equation (24) is possible to analyze the second viscosity of the liquid. Figs. 7-10 show the experimental results.

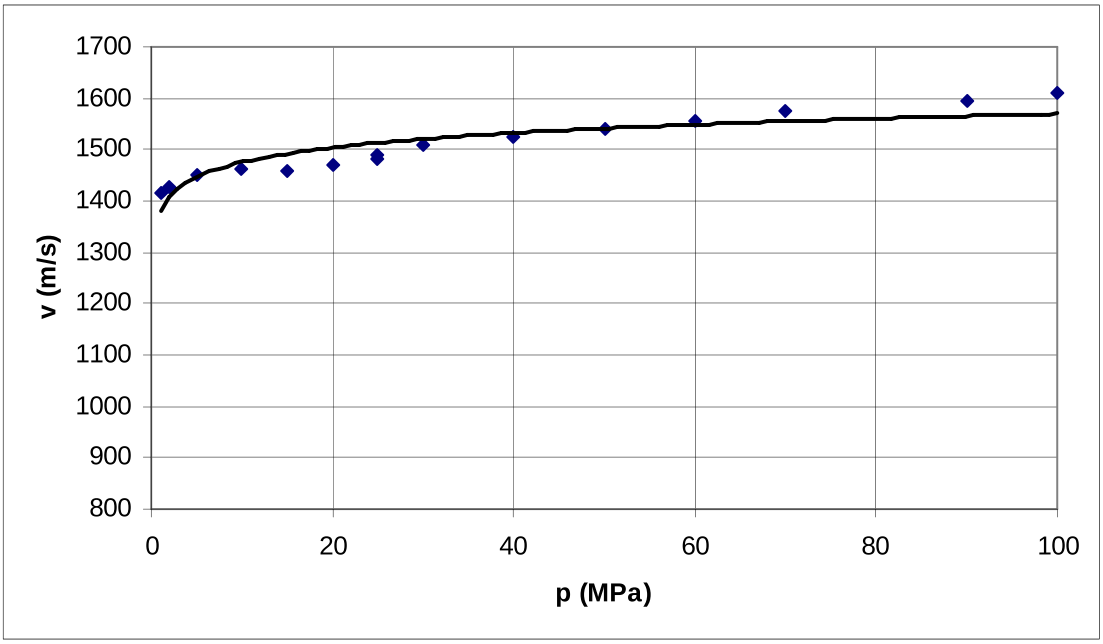

Fig. 7 The sound velocity in the static pressure dependence

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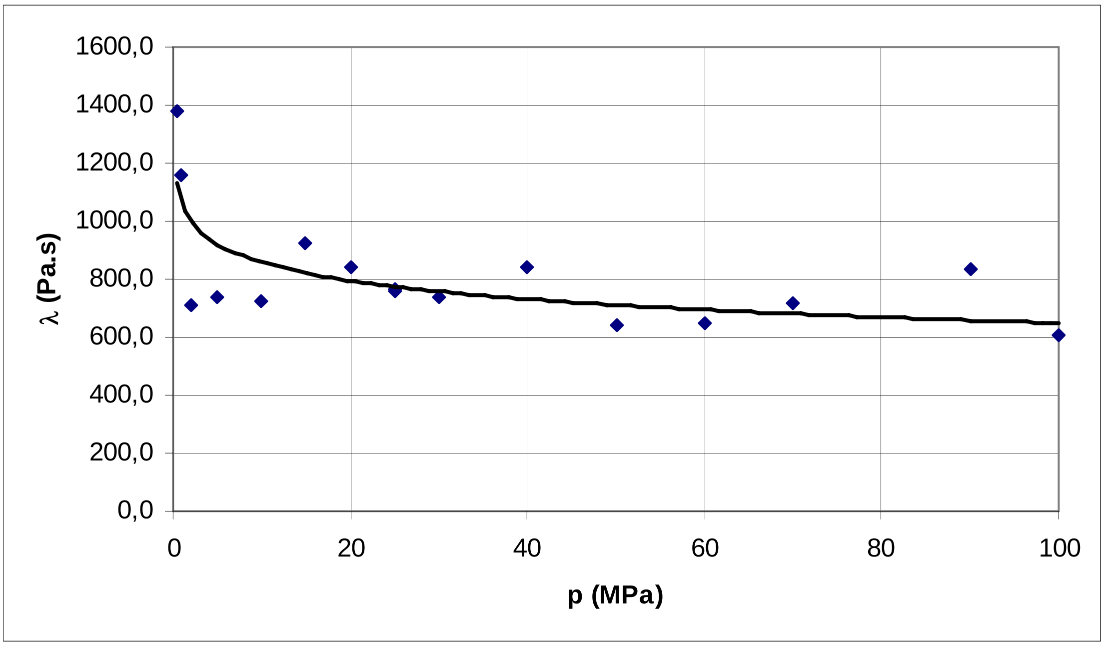

Fig. 8 The second viscosity in the static pressure dependence, frequency 6kHz

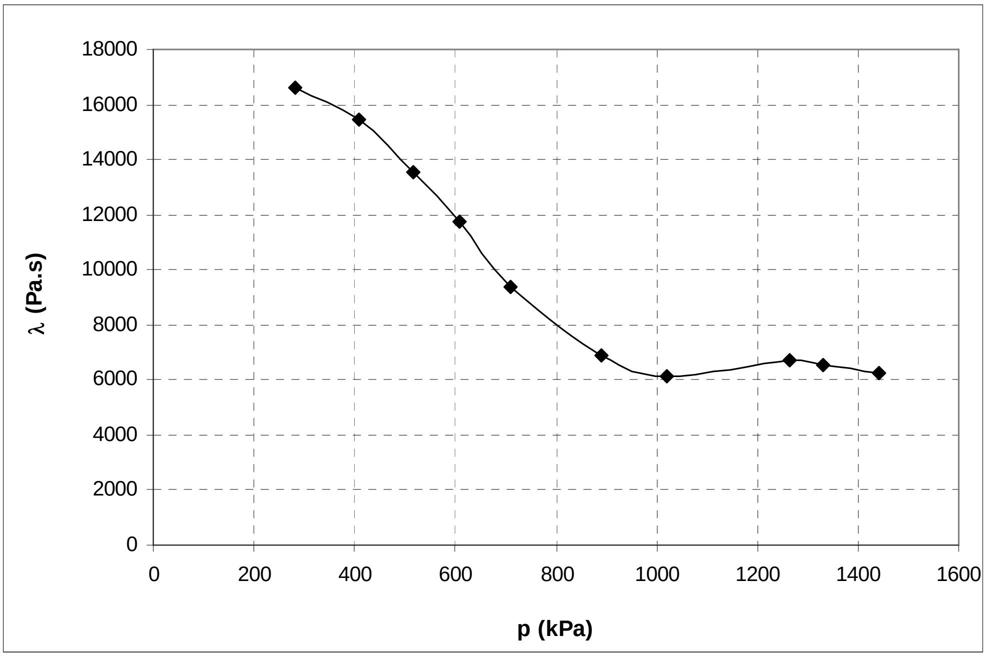

Fig. 9 The second viscosity in the static pressure dependence, frequency 700Hz

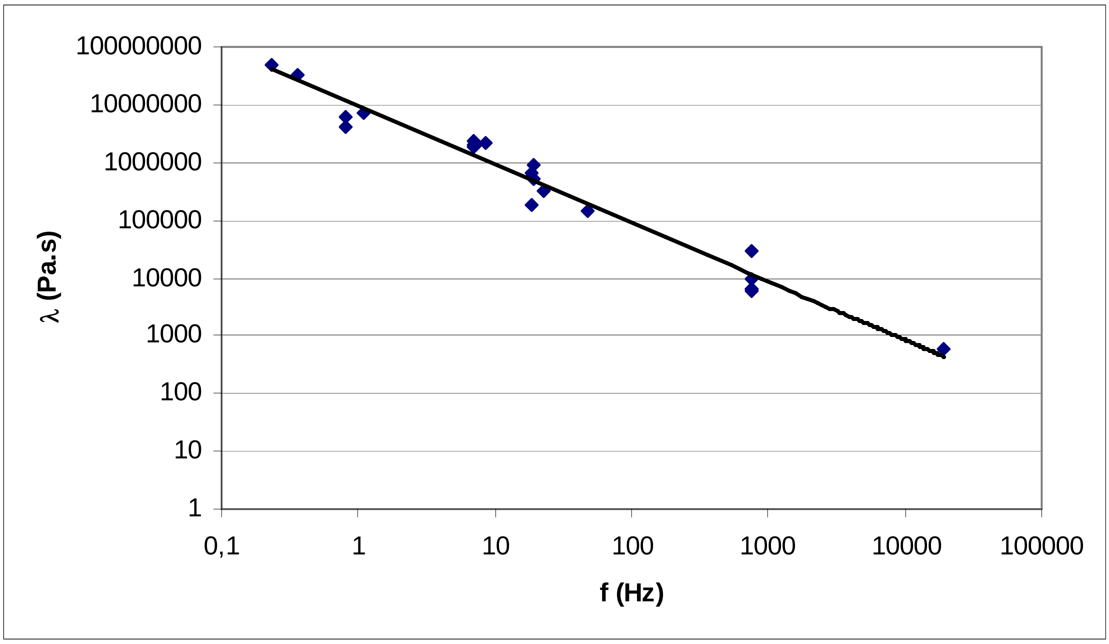

Fig. 10 The second viscosity in the frequency dependence

On the experimental results bases is possible to express the second viscosity-frequency dependence in the $f \in \langle 0.15\,,30000 \rangle$ interval as:

$$\lambda = \left(9{,}4850 \cdot 10^{6}\right) f^{(-1{,}0123)} \tag{25}$$

6. CONCLUSION

Out of the results it is visible that for the low pressures the absorbed air effects the results, but for the higher pressures became the effect insignificant.

Very important is especially the Fig. 10, where is shown the second viscosity dependence on the frequency, which is very significant.

Out of this dependence is clearly visible, that the second viscosity affects the memory interpretation. So the further solution if this problematic is focused on this function searching.

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8. ACKNOWLEDGEMENT

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