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TRANSITION FROM THE SALTATION MODE OF BED LOAD TRANSPORT TO THE ROLLING MODE

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Summary: *The paper deals with the sediment transport of sand particles conveyed by water in an open channel. A 3D numerical model developed by the authors was used for simulation of the sand particles movement in the channel with rough bed. Similarly to Shields theory of sediment transport incipient motion of the bed load particles the shear stress for the transition process from saltation mode to rolling mode can be defined. The shear stress value can be determined as a function of two dimensionless parameters, particle parameter and equivalent bed roughness. The particle parameter is a function of the particle size and density, and the carrier liquid density and viscosity. The presented numerical solution reveals that the shear stress for the transition process from saltation mode to rolling mode is less than the shear stress for the incipient sediment motion and depends on channel bed roughness.*

1. Introduction

Three consecutive modes of the sediment transport of solid particles can be determined: rolling and sliding (which are usually not distinguished), saltation, and suspension, depending on the particle size, density and flow conditions. Saltation is the main regime of transport of solid particles transported by water in channel with the rough bed, which very often consists of similar particles. During saltation the particles move by periodic jumps.

In many investigations the attention was given to the process of the beginning of motion of sediment in water stream. Shields (1936) was the first who got experimental results on flow shear stress for the beginning of motion and presented his well known diagram for the inception of motion of sand particles in a channel. On the diagram it is presented a dependence of a dimensionless bed shear stress (tractive force coefficient, in original) $\tau_* = u_*^2/(gRd)$ on the particle Reynolds number $Re = \dfrac{u_* d}{\nu}$ (u_* is the shear stress velocity, g is the gravity acceleration, $R = (\rho - \rho_f)/\rho_f$ is the particle submerged apparent density, ρ is the density of particle, ρ_f is the density of water, d is the particle diameter, ν is the kinematic viscosity of water). The dependence corresponds to the beginning of particle motion. One of the extensive studies

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of bed load transport was done by L. van Rijn (1984). Among other, he has redrawn the Shields diagram as a dependence of the bed shear stress τ_* on the particle parameter $D_* = d(Rg/v^2)^{1/3}$. He also found an approximation for the dependence (see Figure 1).

Fenton and Abbott (1977) conducted an experimental work on the start of sediment motion and found that the Shields dependence of the dimension threshold stress on particle Reynolds number implicitly contains the variation with relative protrusion of a bed particle above bed. Ikeda (1982) presented a theoretical and experimental work on the initiation of sediment motion and reported the dependence describing the inception of motion which is very close to the Shields curve.

Chi-Hai Ling (1995) conducted a theoretical work in which he presented equations for the inception of the rolling mode and inception of saltating mode. In the derivation of starting of rolling they considered the balance of the drag, lift and gravitational force acting on a particle resting on the bed. For the saltation start they considered the balance of gravitational and lift forces. The bed shear stress of the rolling threshold was found lower than that on the Shields curve, and the saltating threshold was higher.

Ancey et al. (2002) studied saltation, the beginning of rolling mode and the beginning of saltation mode of sediment transport experimentally and numerically. They obtained that Shields stress for the beginning of the rolling equaled 0.005 and for saltating mode it was 0.03. Also they analyzed the process of transition from rolling to saltation and developed a criterion for distinguishing of these modes.

Papanicolaou et al. (2002) presented a probabilistic approach to the initiation of sediment motion. They focused on stochastic aspects of the problem, i.e. the turbulence fluctuations, bed structure and the probability of initiation of motion of a grain. Yalin and Karahan (1979) conducted an experimental work, which confirms the results of Shields.

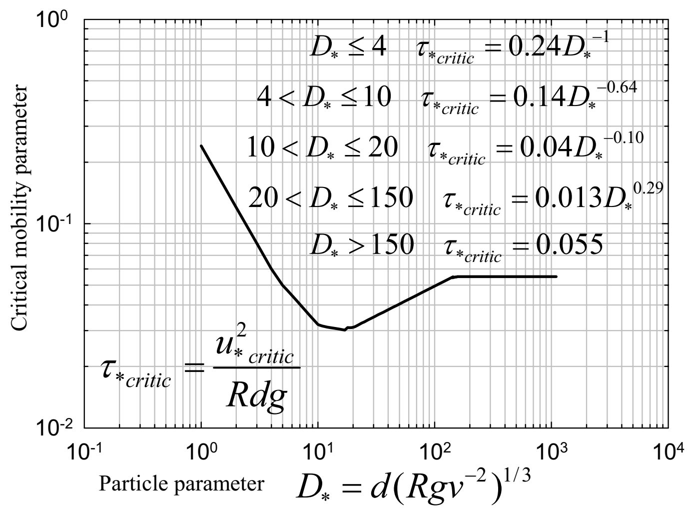

Figure 1 Initiation of motion according to Shields, in L. van Rijn (1984) variables.

The present work investigates the stop of saltation mode of bed load transport, namely the transition from the saltation to rolling mode. This numerical study is based on a 3-dimensional model of particle saltating motion in a turbulent flow, described in details in

Lukerchenko et al. (2009). As a result, there will be calculated a dependence of the dimensionless shear stress for the transition process from saltation mode to rolling mode on the dimensionless particle parameter and dimensionless bed roughness (Van Rijn notation).

2. The statement of the problem, dimensional analysis

Let us regard a solid spherical particle transported by water in a channel with rough bed. The particle moves near the bed. The turbulent flow in boundary-layer has the logarithmic f velocity profile: $v_f\left(y\right)=\frac{u_*}{k}\ln\left(\frac{y}{y_0}\right)$, where u_* is the shear velocity, $k=0.4$ is the Karmans‘ constant, y is vertical coordinate, $y_0=0.11\left(\nu/u_*\right)+0.033k_s$ is the height where velocity is zero, (Nikuradse, 1933), ν is the kinematic viscosity of water, k_s is the equivalent bed roughness (Nikuradse roughness).

Generally the value k_s is defined as a magnitude of average height of the jut of bed roughness. In case that a bed consists of equal particles packed densely at one level (an artificial bed), the value of bed roughness is equal to the particle diameter (Nikuradse, 1933). In case of a natural bed the equivalent bed roughness k_s depends on a shape, a dimension, and an arrangement of bed juts. And in that case the bed roughness can be estimated by analysis of individual velocity profile (for instance Ikeda, 1982).

The particle moves in a saltation mode. When the shear velocity decreases, the length of particle jump also decreases. The particle ceases saltating and starts rolling after decreasing u_* to a critical value u_{*sr} (subscript “sr” refers to “transition from saltation to rolling”i). Let us find the value u_{*sr}. The magnitude u_{*sr} is a function of the following dimensional parameters:

$$u_{*sr}=u_{*sr}(d,\rho,\rho_f,\nu,k_s,g)\,, \tag{1}$$

With help of dimensional theory the equation (1) is transformed to form:

$$\tau_{*sr}=\tau_{*sr}(R,D_*,k_s')\,, \tag{2}$$

where $k_s'=k_s/d$ is the dimensionless equivalent bed roughness. The submerged particle apparent density is considered constant (corresponding to that of water and sand $R=1.65$), thus (2) is reduced to

$$\tau_{*sr}=\tau_{*sr}(D_*,k_s')\,. \tag{3}$$

3. Numerical method

We use the following condition for the transition of saltation mode to rolling mode: the average length of saltation jump becomes as small as the diameter of the particle

$$L=d\,. \tag{4}$$

This condition is a simple geometrical restriction of saltation and is close to that used by Ancey et al. (2002) $L=d/2$ as a criterion to distinguish the saltation mode from the rolling mode. In their experiments they observed that the transition from rolling to saltating is fuzzy and for some flow conditions a particle can move partially in rolling mode, partially in saltation mode. They offered the following division of rolling and saltation. The “rolling motion” refers to the motion of the particle in sustained contact with the bed; when colliding with a bed particle, the moving particle sometimes underwent a micro-leap, the typical length of which was less than the particle radius r.

In order to find the value of the shear stress for the transition process from saltation mode to rolling mode τ_{*sr} an iterative process was done that will be described in what follows. In frames of these calculations it is not possible to predefine the average saltation length equal to the diameter of saltating particle, and the average saltation length is calculated numerically by the model (Lukerchenko et al., 2009). By changing the shear velocity we can alter the average saltating length. To satisfy the condition (4), firstly two values of shear velocity were chosen, one providing the saltation length definitely longer than the particle diameter *d*, and the other providing saltation length definitely shorter than the diameter. The two values of shear velocity define an interval in which the sought value of the shear velocity that satisfies the condition (4) lays. The saltation length was calculated for the value of shear velocity that lies in the middle of the interval. Depending on whether the calculated saltation length was higher or lower than the diameter, the new interval containing the sought shear velocity and having the middle of previous interval as a boundary point was defined. After the new interval was found the process was repeated. The iterations were repeated and finally the condition (4) was satisfied with 1% accuracy. The dimensionless bed shear stress satisfying (4) was calculated for different values of k_s' and D_*, according to (3).

4. Results and analysis

The provided calculations were performed for values of the dimensionless equivalent bed roughness k_s' ranking from 0.5 to 2.0 and the dimensionless particle parameter D_* from 1 to 1300 (corresponding to main region of Shields curve in L. Van Rijn, 1984 variables, see Figure 1).

The calculated dependences of the shear stress for the transition process from saltation mode to rolling mode τ_{*sr} on the dimensionless particle parameter D_* for different bed roughness k_s' are presented in Figure 2. The 3D surface plot of the τ_{*sr} dependence on D_* and k_s' is presented in Figure 3.

As can be seen in Figure 2 the values of τ_{*sr} that correspond to the transition from saltation to rolling mode are lower than the values corresponding to the beginning of sediment motion (Shields curve).

The results can be also compared with the experimental results of Ancey et al. (2002) on transition from rolling to saltation. In their experiments they had the following conditions: $D_* = 74$ at $k_s' = 0.7 \div 1.8$ and $D_* = 243$ at $k_s' = 0.3 \div 0.9$. They reported that at $\tau_* \geq 0.03$ particles were in saltation most of the time. The present results are not in contradiction with the results of Ancey et al. (2002), since the saltation stop occurs at lower τ_* than the start of saltation.

The dependence $\tau_{*sr}(D_*, k_s')$ was fitted with the following function:

$$\tau_{*sr} = a_1 + a_2 k'_s + \frac{b_1 + b_2 k'_s}{D_* + c D_*^2}, \tag{5}$$

where

$$\begin{aligned} a_1 &= 0.0044, \quad a_2 = 0.0126 \\ b_1 &= 0.47 \quad b_2 = 0.037 \quad c = 2.1 \end{aligned} \tag{6}$$

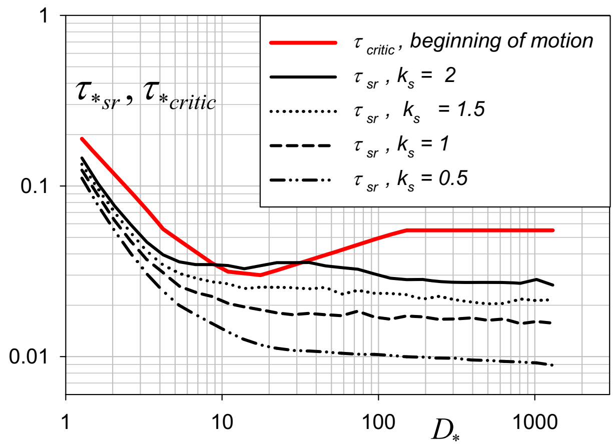

Figure 2 Dependences of the dimensionless critical shear stress τ_{*sr} on particle dimensionless parameter D_* for different dimensionless equivalent bed roughness k'_s.

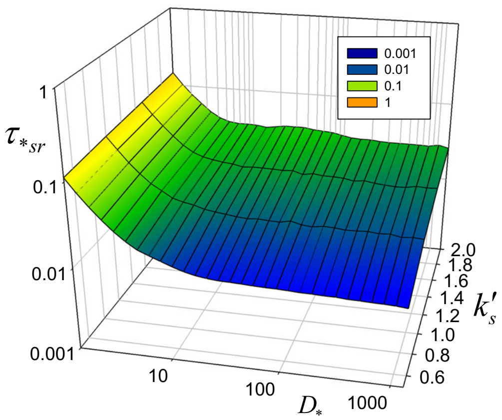

Figure 3 3D graph of the dimensionless critical shear stress dependence τ_{*sr} on particle dimensionless parameter D_* and dimensionless equivalent bed roughness k'_s.

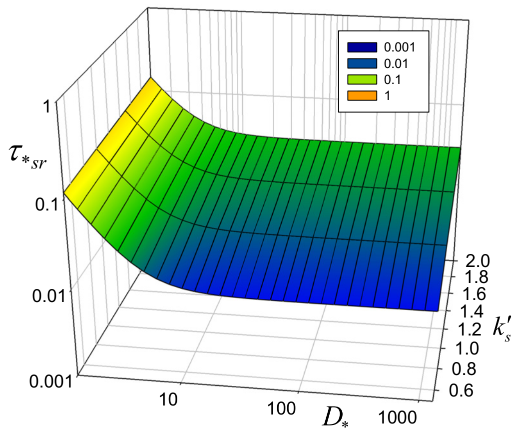

Figure 4 3D graph of the dimensionless critical shear stress dependence τ_{*sr} on particle dimensionless parameter D_* and dimensionless equivalent bed roughness k'_s, equation (5).

5. Conclusion

The process of transition from the saltation mode of bed-load transport to the rolling mode in channel with rough bed was investigated theoretically. The transition was studied for the following dimensionless parameters: the apparent density of particles $R = 1.65$ (corresponding to sand and water), the dimensionless particle parameter D_* ranking from 1 to 1300 and the equivalent bed roughness k_s' ranking from 0.5 to 2.0. The dependence of the shear stress for the transition process from saltation mode to rolling mode τ_{*sr} on particle parameter D_* and equivalent bed roughness k_s', see Figure 2-4 was found.

The main problem was to define a criterion of transition from saltation to rolling mode. Finally, the following criterion of the transition of saltation mode to rolling mode was chosen: the average length of saltation jump becomes as small as the diameter of the particle. It provides a simple geometrical restriction of the saltation process.

The presented numerical solution reveals that the critical value of the shear stress for the transition from saltation to rolling mode is less than that corresponding to initiation of saltation and less than that corresponding to start of sediment motion. The calculated critical bed shear stress depends nearly linearly on channel bed roughness.

6. Acknowledgement

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