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CONTROL OF CHAOS IN THE DYNAMIC SYSTEMS

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Summary: *Chaos is present in many aspect of life. Physics is usually the field where chaos control became a paradigma and discipline itself. It is very difficult to detect and control chaotic behavior in nonlinear engineering dynamical systems. This contribution introduces some basic concepts for controlling chaos and describes some mathematical methods for controlling chaos in dynamic systems*

1. Introduction

In mechanical engineering the application of chaos usually started as experimental demos for educational purposes, for instance as the control of pendulum, beams and plates. These demos were quickly developed in more realistic applications such as the control of vibroformers, stabilization of crane oscillations, spacecraft, satellite and others. A recent review by Andrievskii & Fradkov (2004) summarized the main applications of chaos control.

Mathematical methods can by used for controlling of chaos in dynamical systems. In these methods, used to control a real dynamic systems, however, due to efficiency and accuracy requirements, it was necessary to use fuzzy logic to model the uncertainty, which is present when numerical simulations are performed.

2. Different techniques of chaos control (Calvo, 2006)

Chaos is usually associated with randomness and intermittency (Paskota, 1998), because chaotic systems show random conducts with bursts of synchronization and almost periodic behavior. For instance, according to (Kapitaniak, 1996), chaos is defined as a superposition of (unstable) periodic motions. Another important observation of the chaotic dynamics is its sensitivity to small changes in the parameters or to the initial conditions (butterfly effect).

We can divide chaos controlling approaches into two broad categories (Castillo & Melin, 2006) :

- Firstly, a distinction of the techniques of chaos control could be made based on the use of feedback (Ott, Grebogi & Yorke, 1990). Closed-loop techniques monitor some variable in the phase plane and by perturbating temporarily a parameter of variable bring the dynamics to the desired orbit,

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- Secondly, there are non-feedback methods in which some other property or knowledge of the system is used to modify or explore of chaotic behavior.

One of the techniques based on the closed-loop approach consists of the following: while observing carefully the motion on the phase plane, we can detect the presence of unstable periodic orbits. These orbits are present in a infinite member and are embedded in compact area of the phase plane, marked as a chaotic attractor. A small perturbation in the parameters of the system can cause to jump from one orbit to another and system change dynamic behavior. This property was used for the control of chaos (Ott, Grebogi & Yorke, 1990; Barajas-Ramírez, 2006).

Open-loop system produced the same effect by changing slightly some parameter or property of system, permanently, without the feedback.

The examples of using feedback and non-feedback methods will by discussed in the next paragraphs.

3. Controlling chaos through feedback

Feedback methods do not change the controlled systems and stabilize unstable periodic orbits or strange chaotic attractor. For example, the Ott-Grebogi-Yorke method (Ott, Grebogi & Yorke, 1990), which is extremely general, relying only on the universal property of chaotic attractors, i.e., they have embedded within the infinitely many unstable periodic orbits. The method is based on observation of the trajectory and application of a feedback control system, which must by highly flexible. Such system in some experimental configurations may be large and expensive. It has the additional disadvantage that small perturbation on noise may cause large deviations from the desired operating trajectory.

A different approach to feedback control was proposed by Pyragas (1992). This method is based on the construction of special form of a time continuous perturbation, which does not change the form of the desired unstable periodic orbit, but under certain constraints can stabilize it. Have been proposed two feedback/controlling loops, shown in Fig. 1.

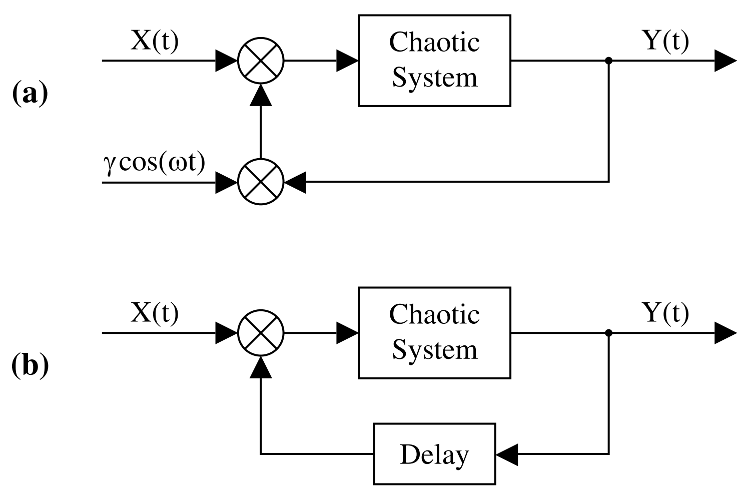

Figure 1. Feedback controlling loops, (a) control by periodic external perturbation and (b) control by time delay.

A combination of feedback and periodic external force $F(t)$ is used in the first access, see Fig. 1a. The second access, see Fig. 1b, does not require external source of energy and it is based on self/controlling delayed feedback. If the period of external force or time delay is equal to the period of one of unstable periodic orbit embedded in the chaotic attractor, it is possible to find such constant K, which allows stabilization of the unstable periodic orbit.

The first Pyragas's access can by considered as the special case of the direct application of classical controlling methods to the problem of controlling of chaos. Meditate of dynamic system, describe of first-order ordinary differential equations in time

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \mathbf{f}(\mathbf{x}) \qquad (1)$$

where $\mathbf{x} \in \mathbf{R}^n$ is controllable state of system, if exists a control function $\mathbf{u}(t)$, such, that

and

$$\begin{gathered} \frac{\mathrm{d}x}{\mathrm{d}t} = \mathbf{f}(\mathbf{x}) + \mathbf{u}(t) \\ \mathbf{y} = \mathbf{g}(\mathbf{x},\mathbf{u}) \ , \\ \mathbf{x}(t=0) = \mathbf{X}_0 \end{gathered} \qquad (2)$$

allows t_0 move trajectory $\mathbf{x}(t)$ from point $\mathbf{X}_0$ at time t_0 to the desired point $\mathbf{X}$ in finite time *t*. Out put from system representing value $\mathbf{y}$. Now, the concept on controllability can be applied to the chaos controlling problem.

For example, a generalized control system with inputs (i.e. including $\mathbf{u}(t)$ and disturbance input vector $\mathbf{v}$ is not necessary) is shown by the block diagram in Fig. 2.

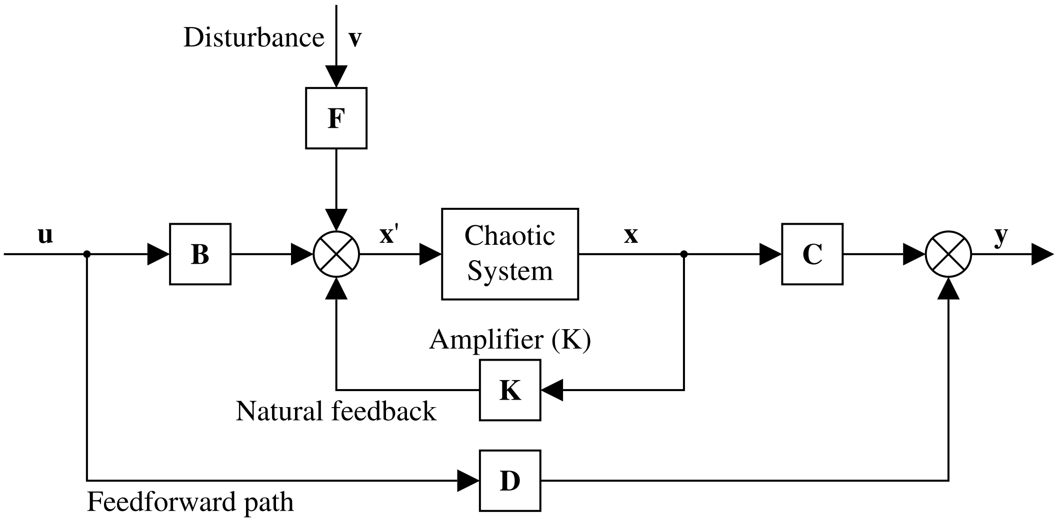

Figure 2. Multivariable control system – closed loop control configuration.

The state space representation of this system is

$$\mathbf{x}' = \mathbf{f}(\mathbf{x}, K) + \mathbf{B}\,\mathbf{u} + \mathbf{F}\,\mathbf{v} \ , \qquad (3)$$

$$\mathbf{y} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{u} \qquad (4)$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$ is state vector, $\mathbf{u} = [u_1, u_2, \ldots, u_r]^{\mathrm{T}}$ input vector, $\mathbf{y} = [y_1, y_2, \ldots, y_m]^{\mathrm{T}}$ output vector, $\mathbf{v}$ disturbance vector, $\mathbf{f}(\mathbf{x})$ vector function, $\mathbf{B}$ input distribution matrix

$(n \times r)$, $\mathbf{C}$ output (or measurement) gain matrix $(m \times n)$, $\mathbf{D}$ feed-forward gain matrix $(m \times r)$, $\mathbf{F}$ disturbance input distribution matrix and K is constant, which allow stabilization of unstable periodic orbits.

In equation (3) the disturbance term $\mathbf{F}\,\mathbf{v}$ may be dropped since it can be observed into the regular input $\mathbf{B}\,\mathbf{u}$.

Advantages of this method (Kapitaniak, 1996) :

- any solution of the original system can by controlled (fixed point, unstable periodic orbit, chaotic attractor, …),
- the controller has a very simple structure,
- accesses to system parameters are is not required,
- it is not affected by small parameter variations.

We will show, that chaotic system can be controlled by coupling it with another chaotic system (Kapitaniak, 1996). We call *A* and *B* two chaotic systems

$$\mathbf{x}' = \mathbf{f}(\mathbf{x}) \quad \text{respectively} \quad \mathbf{y}' = \mathbf{g}(\mathbf{y}) \tag{5}$$

where $\mathbf{x}, \mathbf{y} \in \mathbf{R}^n$, and we use the controlling strategy, which is illustrated in Fig. 3.

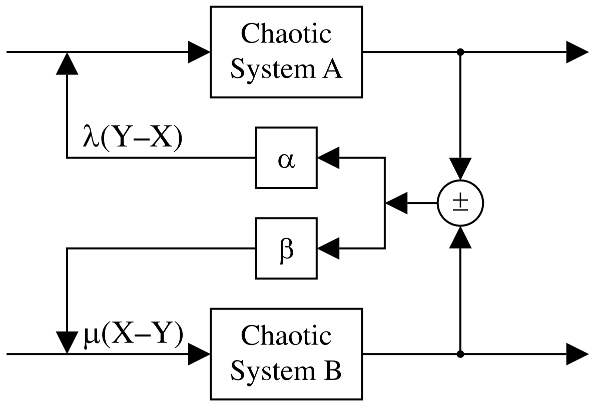

Figure 3. Controlling chaos-by-chaos scheme.

This two systems are coupled through the operators $\alpha, \beta > 0$, which have a very simple linear form. We will assume, that all state variables of both systems *A* and *B* can be measure, so we can measure signal $x(t)$ from system *A* and signal $y(t)$ from system *B*. The systems are coupled so, that the difference D_1 and D_2 between signals $x(t)$ and $y(t)$

$$\alpha\left[x(t) - y(t)\right] = \alpha\, D_1 = F_1(t) \ , \tag{6}$$

$$\beta\left[x(t) - y(t)\right] = \beta\, D_2 = F_2(t) \tag{7}$$

are used as control signal introduced, respectively, into each of the chaotic systems *A* and *B* as negative feedback.

Using the coupling according to Fig. 3, it has been state (Castillo & Melin, 2006), that one chaotic system coupled with order one can significantly change the behavior of one of them (unidirectional coupling, i.e. α or $\beta \neq 0$. In (Kocarev & Kapitaniak, 1995) are given rigorous conditions, under which chaotic attractors of systems A and B are equivalent, or the evolution of one of them is forced to take place on the attractor of the other one.

4. Controlling chaos without feedback

The non-feedback approach is much less flexible and requires more prior knowledge of equations of motion. On the other hand, to apply this method, we do not have know the trajectory of the system. The control procedures can be applied at any time and we can switch from one periodic orbit to another without returning to the chaotic behavior. This method can by very useful in mechanical systems, where feedback controllers are often very large. For example, a dynamical absorber having a mass of the of 1–2 % of mass of the basic systems, can evoke to convert chaotic behavior to periodic one over a substantial region of parameter space.

In this paragraph we describe a method for controlling chaos in which the chaos effect is achieved by coupling the chaotic main system to a simple autonomous system (controller), as shown in Fig. 4.

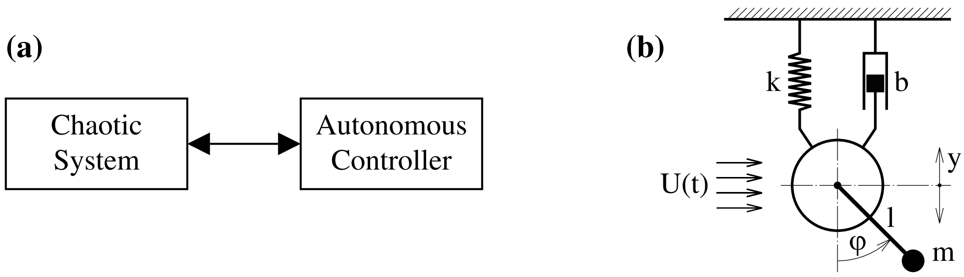

Figure 4. Coupling scheme (a) and dynamical damper as chaos controller (b).

This method is developed pro chaotic systems in which is difficult, if not impossible, to change any parameter of the basic system.

The complete dynamical system in Fig. 4b have two degree of freedom with parameters M, k, b and dynamical damper have one degree of freedom with parameters m and l. The equations for the extended system in dimensionless form are (Tondl, Kotek & Kratochvíl, 1998)

$$x''+\kappa x'+\Omega^2 x+\beta V^2\left(1-\gamma x^2\right)+\mu\left(\varphi''\sin\varphi+\varphi'^2\cos\varphi\right)=0\ , \tag{8}$$

$$\varphi''+\kappa_0\,\varphi'+\left(l+x''\right)\sin\varphi=0 \tag{9}$$

where $x=y/l$, $\mu=m/(m+M)$, $\Omega=\omega_0/\omega_1$, $\omega_0=(k/M)^{1/2}$, $\omega_1=(g/l)^{1/2}$, $V=u(t)/u_C$, $u_C=(\kappa/\beta)^{1/2}$ is critical velocity and β, κ and κ_0 are coefficient of linear viscous damping.

Initial conditions for numerical solution are :

$$x(0)=0.15\ ,\quad \varphi(0)=0.02\ ,\quad x'(0)=\varphi'(0)=0$$

and changes of parameters of system are :

$\beta=(0.02\div 0.08)$, $\kappa=0.02$, $\kappa_0=0.05$, $\gamma=16$, $\Omega=1.9$ and $V=(0.5\div 2.5)$.

Since the behavior of complete system can be unstable a chaotic, the role of controller is to change the chaotic behavior to some desired periodic, possible change some parameters of system. Parameters, which can be change are μ or β.

Some results of numerical experiments are shown on Figs. 5 and 6. On the Fig. 5 we can see a typical chaotic attractor (for $\beta = 0.02$ and $V = 1$) which controller changes from chaotic to periodic, when value μ sink from 0.2 to 0.075 and 0.5 . On the Fig. 6 we can see that periodic motion remain stable though the value V changes from $V = 1$ to 2.5 .

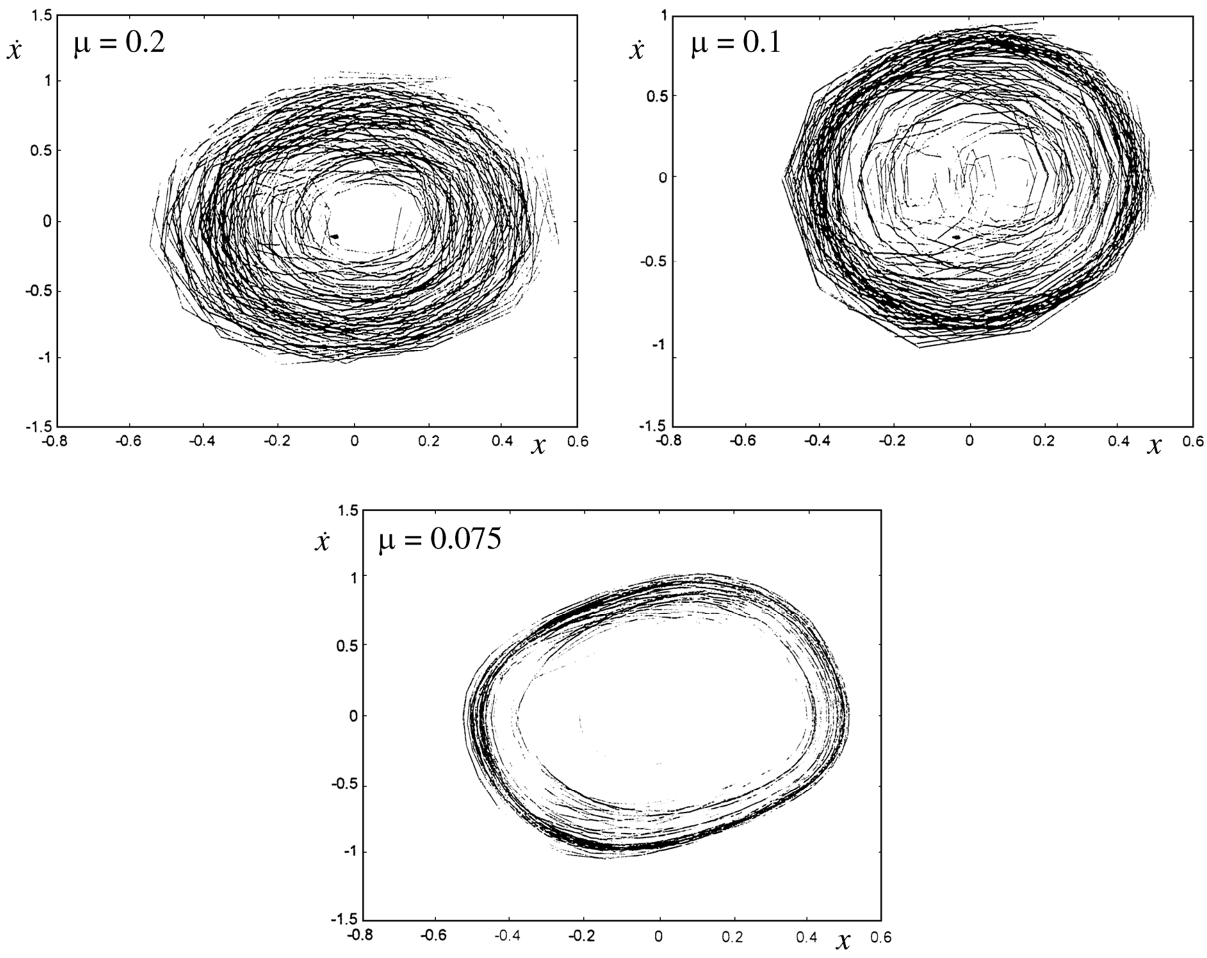

Figure 5. The change attractor from chaotic form to periodic form with the change value μ from 0,2 to 0.075 .

Although such a dynamical absorber (controller) can change the dynamical behavior of main system substantially, is usually small in comparison with the main system and does not require an increase of excitation force. It can be easily added to the existing system without major changes of design or construction. This contrasts with device based on feedback control, where it can be large and costly.

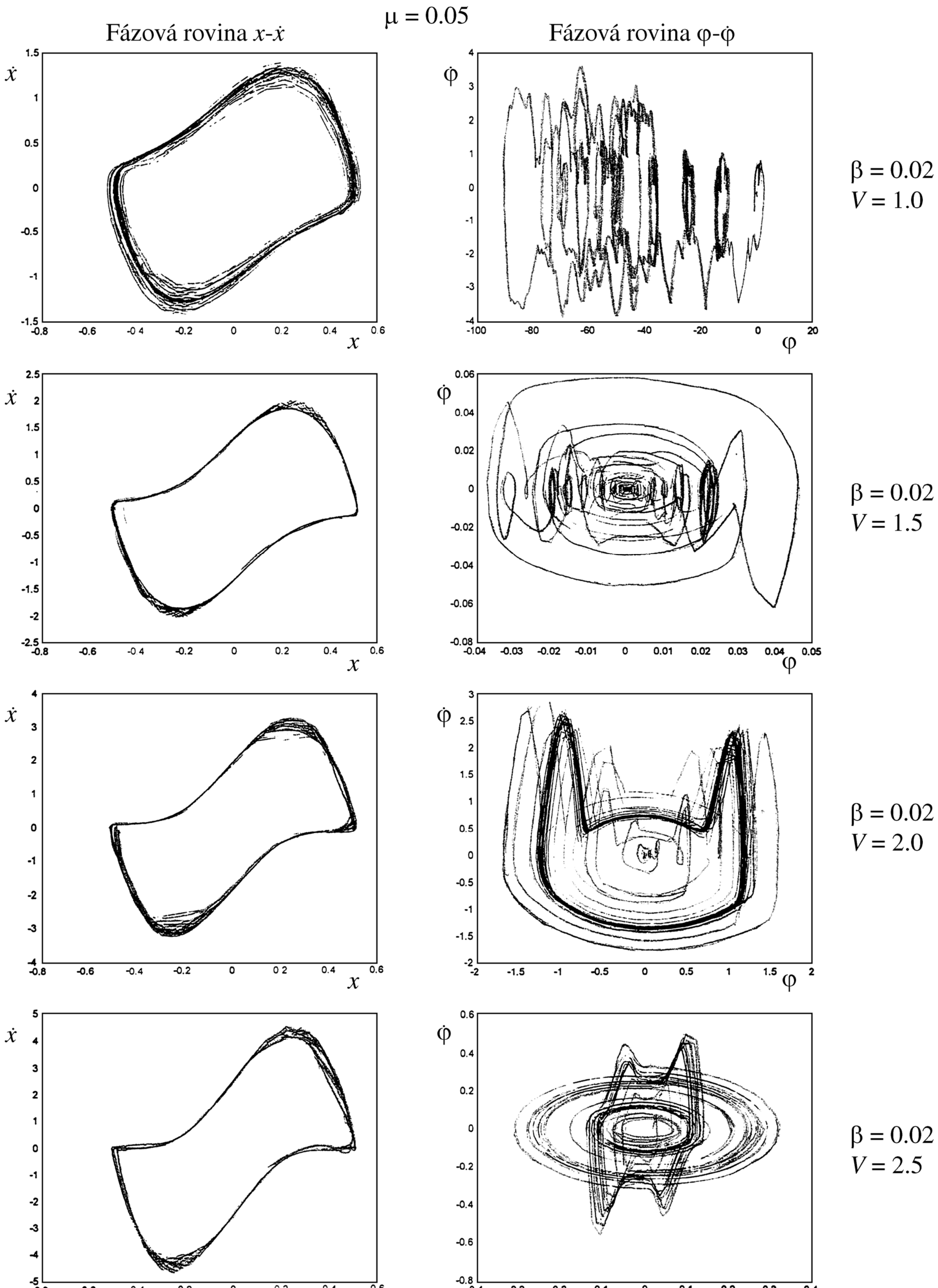

Figure 6. Behavior of the main system and controller with the change of parameter V from $V = 1$ to 2.5 .

5. Conclusion

In this article we have presented a some method of control of chaos for nonlinear dynamical systems. We can apply this methods for behavior identification. We also presented the results of numerical solution of chaos control of real system without feedback. As this method is designed mainly for experimental application, we stall now briefly suggest some guidelines for applying it (Castillo & Melin, 2006) :

- The coupled system has to be as simple as possible.
- This coupling should be chosen as small as possible.
- If it is possible one should couple the controller in such a way that the locations of the fixed points of the original system are not changed.

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