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STABILITY OF Q-Y CHARACTERISTIC OF CENTRIFUGAL PUMP

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Summary: *At the pumps of low specific speed it is possible, that near the point of zero discharge the Q-Y characteristic is unstable* ($\frac{\partial Y}{\partial Q} \rangle 0$)*. This phenomenon of unstable Q-Y characteristic causes big pressures and discharge pulsations, which are characterized by self- excited oscillations of the systems. It is not recommended to run the pump between points of unstable characteristic. For this pump it is impossible to work in exacting operations (atomic power stations, heating plant stations, chemical industry). The stability conditions (Y, Q) are analyzed on the basis of dissipation function. It is assumed, that power input of the pump is linear function of the discharge. Governing equations are Navier-Stokes equations and continuity equation for incompressible fluid in modification in dependence on swirl velocity* $\mathbf{\Omega}$.

1. Introduction

The characteristic of impeller pump is a dependence of specific energy Y on discharge Q.

At the pumps with low specific speed it is possible, that near the point of zero discharge is $\frac{\partial Y}{\partial Q} \rangle 0$. It induces, that dependence (Y,Q) has the maximum outside the point $Q = 0$ (fig. 1).

This effect causes two discharges Q_1, Q_2 for one value of specific energy.

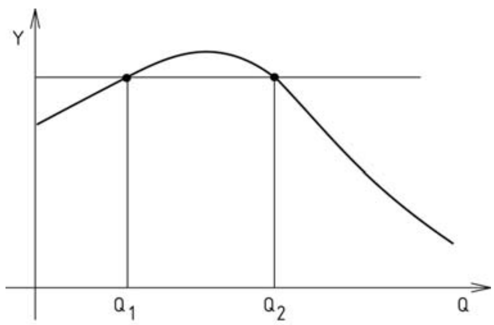

Fig.1

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This condition is physically impossible, so the characteristic, which is demonstrated on figure 1, has no real foundation; it is obtained from time – average values of pressure and discharge. In fact this phenomenon causes big pressure and discharge pulsations, which are characterized by self – excited oscillations of the systems.

It is not recommended run the pump between $Q \in \langle Q_1, Q_2 \rangle$. For this pump it is impossible to work in exacting operations (atomic power station, heating plant stations and chemical industry).

In the research report (1) the stability conditions of (*Y,Q*) characteristic are analyzed on the basis of dissipation function. It is assumed that power input of the pump is linear function of the discharge.

In this part the governing equations are Navier - Stokes equations and continuity equation for incompressible liquid, in modification in dependence on swirl velocity $\mathbf{\Omega}$, for which it is valid:

$$\mathbf{\Omega} = rot\mathbf{c} \ , \ \mathbf{c} \text{ – velocity vector of liquid.}$$

2. Specific energy

On the fig.2 the space out of the runner is introduced.

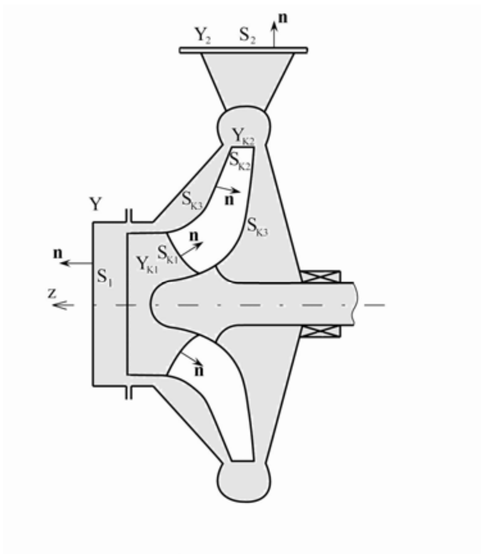

Fig. 2

The relation for specific energy (3) is possible to obtain from Navier-Stokes equation (1) and from continuity equation (2) for incompressible fluid.

$$\frac{\partial \mathbf{c}}{\partial t} + gradY_L - \mathbf{c}x\mathbf{\Omega} + \upsilon rot\mathbf{\Omega} = \mathbf{0} \qquad (1)$$

$$\text{div}\ \mathbf{c} \ = \ 0 \qquad (2)$$

$$Y_L = \frac{1}{2}|\mathbf{c}|^2 + \frac{p}{\rho} - \mathbf{g} \cdot \mathbf{x} \qquad (3)$$

where υ - kinematic viscosity , **c**- velocity vector; Y_L – local specific energy, p - pressure, ρ - density, **g** - gravity acceleration vector, **x** – position vector.

Since in next part only stationary flow will be considered, $\frac{\partial \mathbf{c}}{\partial t} = 0$, we obtain

$$gradY_L - \mathbf{c}x\mathbf{\Omega} + \upsilon rot\mathbf{\Omega} = \mathbf{0}. \tag{4}$$

2. The influence of dissipation function on Q-Y stability of the pump impeller

Assume, that specific energy on the inlet of the runner is Y_{k1}, on the outlet it is Y_{k2}.

The specific energy on the inlet of the pump is Y_1, on the outlet of the pump is Y_2. Final specific energy of the pump is

$$Y = Y_2 - Y_1. \tag{5}$$

It is possible to derive the input power of the pump $P_{\tilde{c}}$.

$$P_{\tilde{c}} = \rho YQ + 2D_s = \rho(Y_{k2} - Y_{k1})(Q + \Delta Q) + \rho\frac{\upsilon}{2}\int_{\tilde{S}} grad|\mathbf{c}|^2 \cdot \mathbf{n}dS. \tag{6}$$

In the relation (6) is meanwhile unknown difference of specific energies $Y_{k2} - Y_{k1}$. This difference can be determined based on the Navier-Stokes equations analysis in rotating coordinate system (y_i) joined with the runner.

We consider that the runner rotates around the axe z = y_i with constant angular velocity ω. Navier-Stokes equation in the rotating coordinate system is:

$$\rho\frac{\partial w_i}{\partial t} + \rho\frac{\partial w_i}{\partial y_j}w_j + \rho\omega^2\varepsilon_{i1n}\varepsilon_{n1m}y_m + 2\rho\omega\varepsilon_{i1n}w_n - \frac{\partial \Pi_{ij}}{\partial y_j} + \frac{\partial p}{\partial y_i} = \rho g_i. \tag{7}$$

ε_{ijn} - Levi-Civit antisymmetrical tensor, $i, j, n = 1,2,3$, w_i - velocity component .

In the next part the summing convention is not used.

If the equation (7) is multiplied in the region V_k of the runner (scalar multiplication) by velocity **w** and if we consider definition of specific energy of the runner Y_k

$$Y_k = \frac{|\mathbf{c}|^2}{2} + \frac{p}{\rho} - \mathbf{g}\cdot\mathbf{y}. \tag{8}$$

it is possible to write for difference of specific energies $Y_{k2} - Y_{k1}$ the following relation:

$$(Y_{k2} - Y_{k1})Q_k = +(u_2c_{u2} - u_1c_{u1})Q_k - \frac{\upsilon}{2}\int_{S_{k1}US_{k2}} grad|\mathbf{w}|^2 \cdot ndS_k - \frac{2D_k}{\rho}, \tag{9}$$

$$2D_k = \eta \int_{V_k} \frac{\partial w_i}{\partial y_j}\frac{\partial w_i}{\partial y_j} dV \quad . \tag{10}$$

In the condition of $\Delta Q \langle\langle Q$, it is possible to obtain the final equation for output power of the impeller pump:

$$P = \rho YQ = \rho(u_2 c_{u2} - u_1 c_{u1})Q + \frac{\eta}{2}\int_{S_{k3}} grad|\mathbf{c}|^2 \cdot \mathbf{n} dS - \\ - \frac{\eta}{2}\int_{S_{k1}US_{k2}} grad\left(|\mathbf{u}|^2 - 2uc_u\right)\mathbf{n} dS - (2D_s + 2D_{sk}). \tag{11}$$

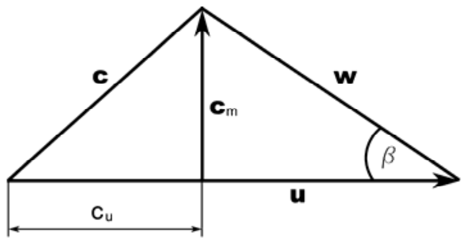

Fig. 3 Velocity triangle

After two times derivative of output power P (11) according to Q we obtain the relation for $\left.\frac{\partial Y}{\partial Q}\right/_{Q=0}$, which determines the condition of stability of (Y,Q) :

$$\left.\frac{\partial Y}{\partial Q}\right/_{Q=0} = -\frac{\partial^2}{\partial Q^2}(D_s + D_k)_{Q=0}. \tag{12}$$

From the last equation is possible to derive, that in the case of linear dependence of power input of the pump on discharge, the term in round parentheses is zero and the stability depends only on dissipation function $D = D_s + D_k$.

The impeller pump characteristic is stable if $\frac{\partial^2 D}{\partial Q^2} \rangle 0 \wedge \frac{\partial D}{\partial Q} \langle 0$ at the point $Q = 0$.

In the next step we can obtain results from CFD computations in the region of the runner, spiral case and suction part.

The distribution of dissipation function was obtained near the point of zero discharge. Diagrams are shown on figures 4 to 10. The stability is favourably influenced inside of the runner, on the contrary of flow inside of the spiral case. The cause of instability is consequently found out in the spiral case. Therefore the hydraulic design of this part of the pump is very important for Q-Y stability.

The next research has to be aimed first of all to spiral case design.

The distribution of dissipation function was computed near the point of zero discharge and at the point of best efficiency, too. It is possible to say, that local swirl in the runner has extensive influence for the dissipative energy (big values of dissipative energy). Big values of dissipative energy are at the output of the runner to spiral case, too. In the spiral case are big values of dissipative energy in the region of the nose. Results are motivating in the process of hydraulic design.

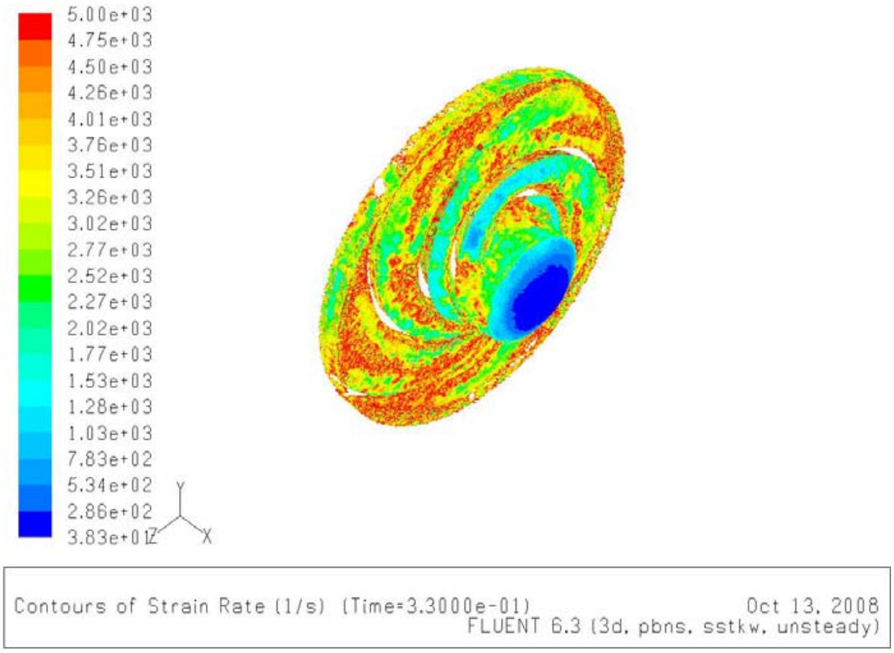

Fig. 4 Dissipative energy distribution, runner, Q = 1l/s

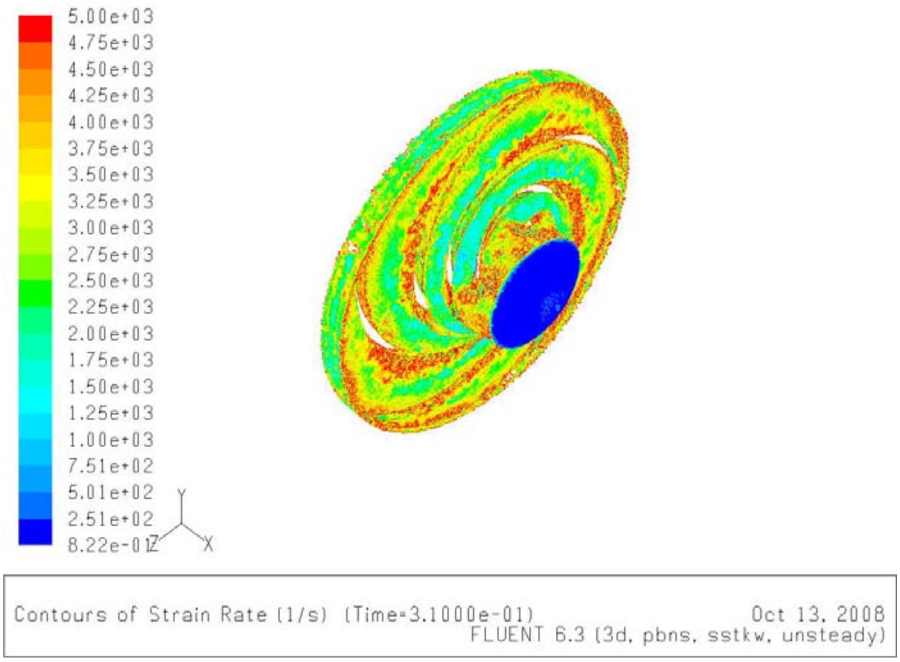

Fig. 5 Dissipative energy distribution, runner, Q = 7 l/s

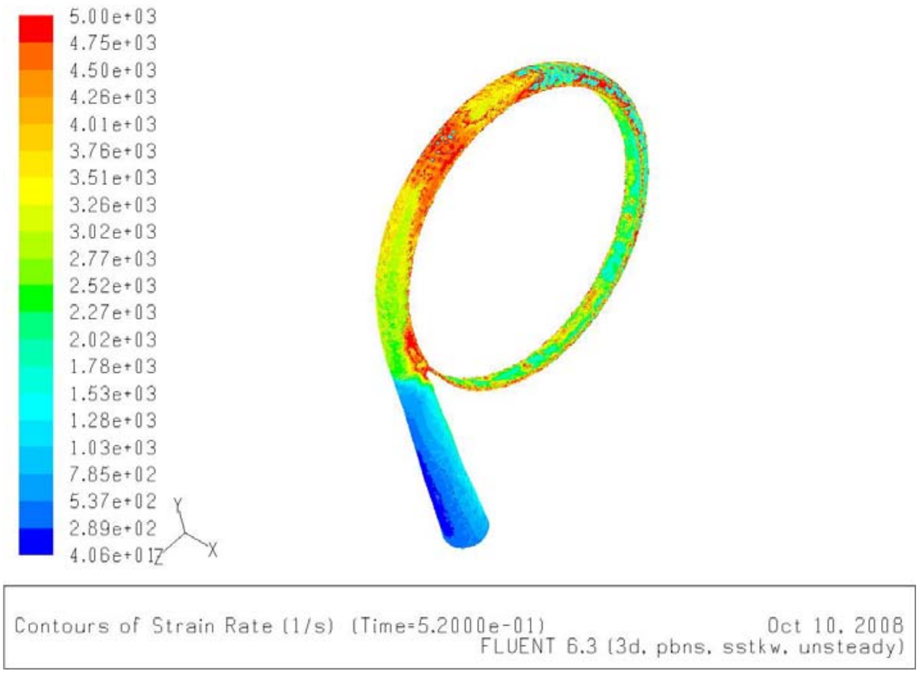

Fig. 6 Dissipative energy distribution, spiral case, Q = 1 l/s

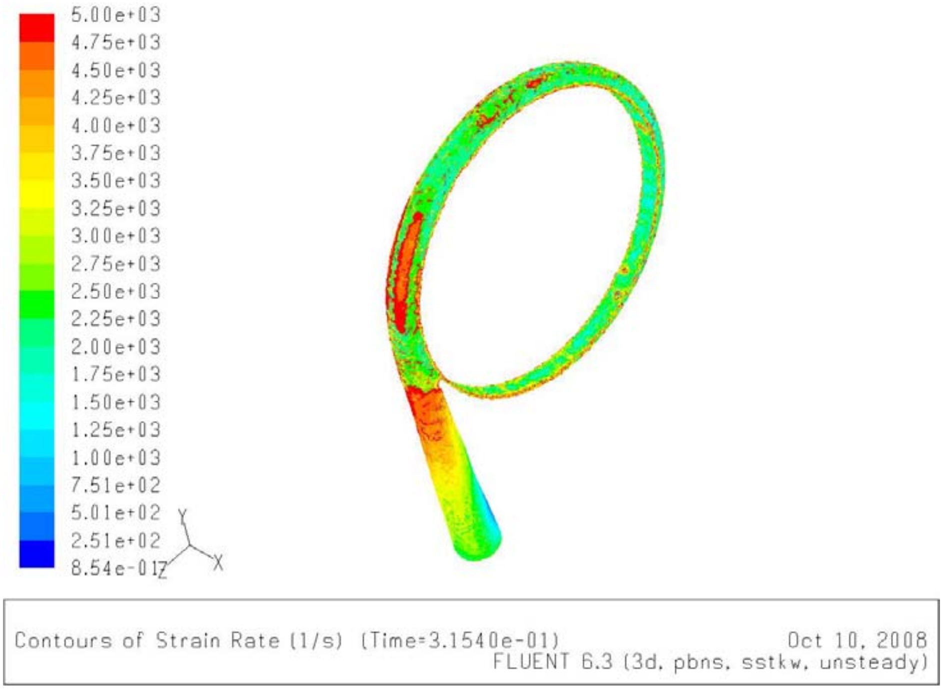

Fig. 7 Dissipative energy distribution, spiral case, Q = 7 l/s

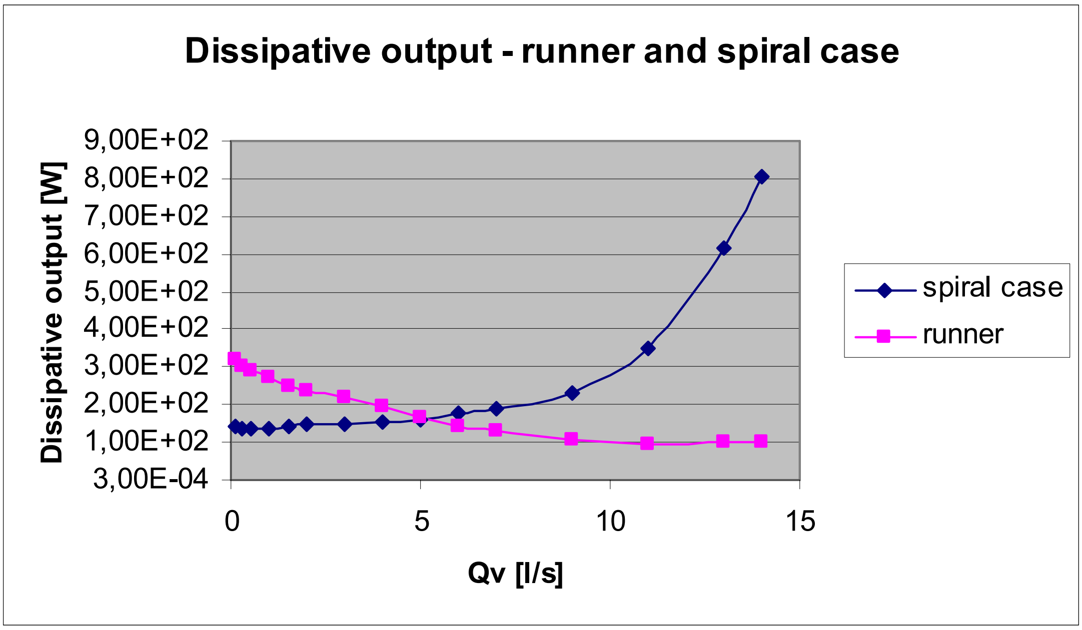

Fig. 8 Dissipative output for the runner and spiral case

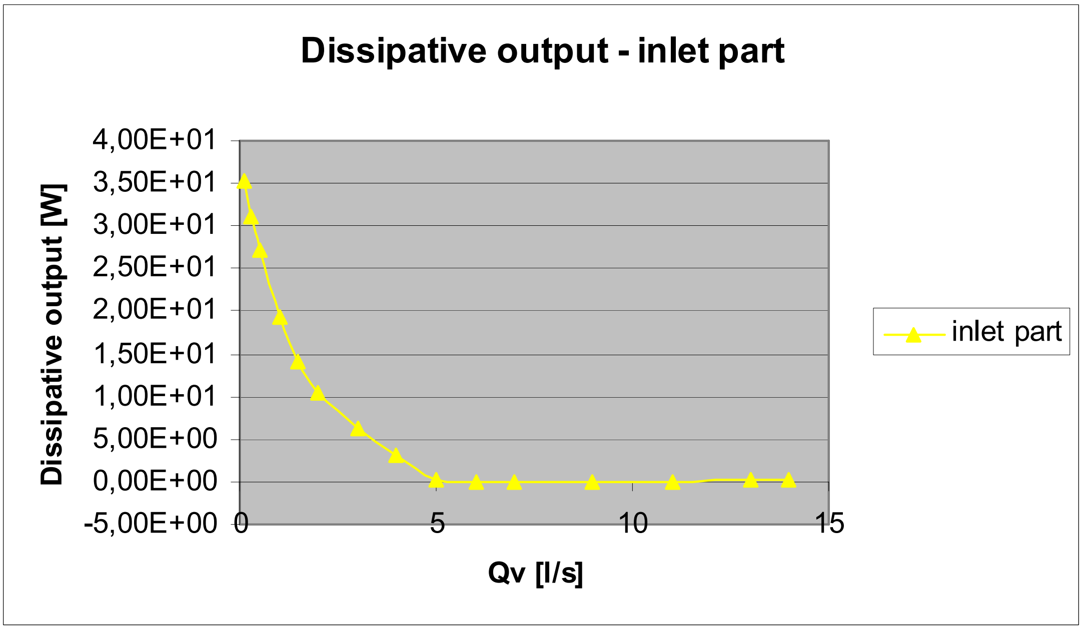

Fig. 9 Dissipative output in the input part of the runner

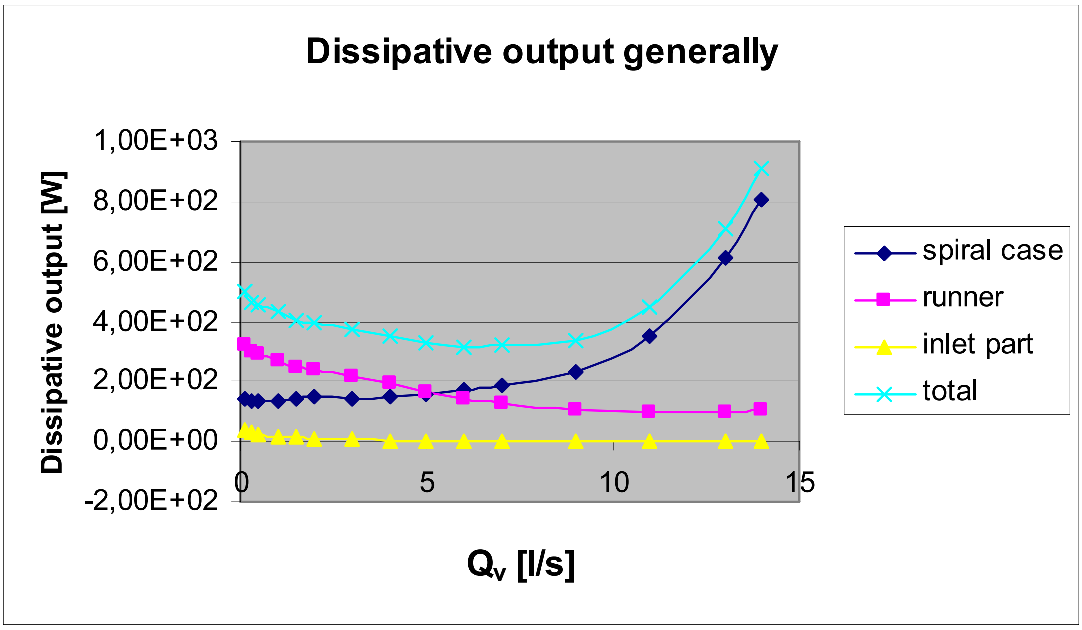

Fig. 10 Total dissipative output as the sum of input part, spiral case and runner dissipative outputs

3. Acknowledgement

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4. References

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