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PERIODICALY STIMULATED GROWTH AND REMODELLING OF THE 1D CONTINUUM-DYNAMICAL ANALYSIS

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Summary: *The contribution starts with the short summary of the growth and remodeling model. Then the multiple scale method is shortly introduced. The main part will be dedicated to the analysis of the dynamical system corresponding to the isometric behavior of the periodically stimulated 1D continuum. This kind of stimulation corresponds roughly with the excitation of the muscle fiber.*

1. Introduction

This contribution joins the previous papers of the authors – e.g. Rosenberg & Hynčík (2007) – dealing with the application of the growth and remodeling theory (GRT) according to DiCarlo & Quiligotti (2002) – on the muscle fiber modeling. This approach allows taking into account also the change of the stiffness of the muscle fiber during time. This effect was experimentally approved and modeled, see e.g. Herzog (2005). The same approach can be used to model the time evolution of the piezo-electric stack. In both cases the final formulation has the form of the dynamical system with two degrees of freedom. The numerical experiments have shown the interesting behavior of this system, e.g. the existence of some bifurcations. This contribution is devoted to the analysis of these properties using the multi-scale method (MSM), see e.g. Nayfeh (1973). This method is the kind of the perturbation method and in general allows decreasing the number of degrees of freedom. This will not be the case here but MSM will be used to model the behavior of this system close to the bifurcation point.

The contribution starts with the short summary of the model development. Then the MSM will be shortly introduced. The main part will be dedicated to the analysis of the dynamical system corresponding to the isometric behavior of the periodically stimulated muscle fiber. This kind of stimulation corresponds roughly with the excitation of the muscle fiber.

2. Problem setting

In GRT the starting point is the initial configuration B_0 that „growths“ and “remodels” , i.e. changes its volume (“growth”), anisotropy (“geometrical remodeling”) or material parameters (“material remodeling”). This process is expressed at first by the tensor **P** (further growth tensor) that relates the initial configuration to the relaxed one B_r with zero inner stress.

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To the real configuration B_t where the inner stress invoked by growth, geometrical remodeling and external loading can already exists, it is related by deformation tensor $\mathbf{F}_r$ (see Figure 1).

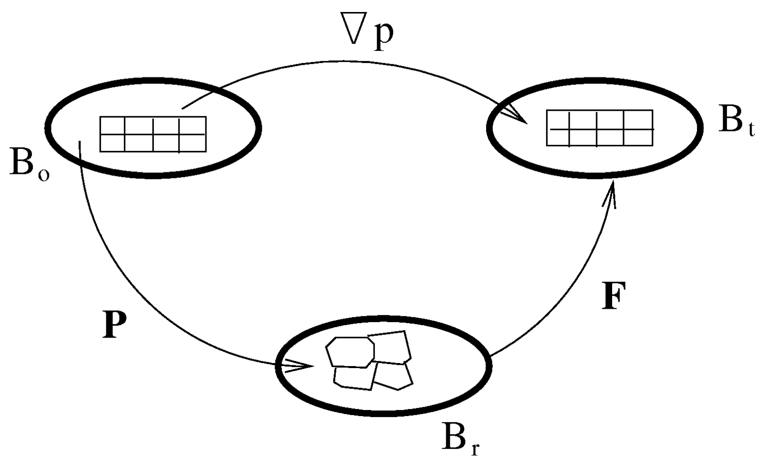


Figure 1 Initial, relaxed and current configurations

The deformation gradient between the configurations B_0 a B_t is

$$\nabla p = \mathbf{F}\ \mathrm{P} \tag{1}$$

According the work of DiCarlo was in DiCarlo & Quiligotti (2002) developed the following system of equations describing the behavior of the muscle fiber

$$\mu = \frac{\partial \psi}{\partial c};\ \boldsymbol{\tau}_{el} = \frac{\partial \psi}{\partial \mathbf{F}};\ \boldsymbol{\tau}_{dis} = \mathbf{H}\dot{\mathbf{F}};\ \mathbf{GV} = \mathbf{C} - \mathbf{E}$$
$$\mathbf{M}\dot{\mathbf{K}} = \mathbf{R} - \frac{\partial \psi}{\partial \mathbf{K}};\ \mathbf{m} = -\mathbf{K}_0\mathbf{Grad}\mu \tag{2}$$

where $\psi(\mathbf{F}, \mathbf{K}, c)$ is the free energy related to the relaxed volume. $\mathbf{K}$ represents the material parameters which can be changing during the material remodeling – $\dot{\mathbf{K}}$ is the corresponding velocity and $\mathbf{V} = \dot{\mathbf{P}}\,\mathbf{P}^{-1}$ is the velocity of growth. The stress $\boldsymbol{\tau}$ was decomposed into the elastic part $\boldsymbol{\tau}_{el}$ and the dissipative part $\boldsymbol{\tau}_{dis}$, $\mathbf{E}$ is the tensor of the Eshelby's type (further shortly Eshelby tensor), $\mathbf{M}, \mathbf{H}, \mathbf{K}_0$ and $\mathbf{G}$ are in the case of passive continua positively definite matrices. In Rosenberg & Hynčík (2007) was shown that in the application on muscle contraction this condition need not be fulfilled. The cause is the energy supply via ATP→ADP process. $\mathbf{C}$ is the generalized external remodeling force and μ and c are chemical potential and concentration of the relevant component respectively. Here we will not deal with the physical interpretation of all these parameters but we apply the equations (2) on 1D continuum. Let's the 1D continuum has the initial length l_0. Its actual length after growth, remodeling and loading will be l. The relaxed length (it means after growth and remodeling) is l_r. For the corresponding deformation gradients we can write

$$P = \frac{l_r}{l_0}, \quad F = \frac{l}{l_r}, \quad \nabla p = \frac{l}{l_0} \tag{3}$$

In the isometric case is $l = const$. For the free energy we will use at first the following form suggested by Fung:

$$\psi = \frac{k}{\mu}\left(e^{\frac{\mu}{2}(F-1)^2} - 1 \right) \tag{4}$$

For $\mu \to 0$ we obtain the common form for linear elastic continuum

$$\psi = \frac{1}{2}\, k(F-1)^2 \tag{5}$$

Introducing from (2) and (3) into (1) we obtain the system of equations for the evolution of relaxed length (after growth), stiffness and stress (eventually force)

$$\dot{k} = \frac{1}{m}\left[r - \frac{1}{\mu}\left(e^{\frac{\mu}{2}(F-1)^2} - 1 \right) \right] \tag{6}$$

$$\dot{l}_r = l_r^{\,3}\, \frac{C + \frac{k}{\mu} e^{\frac{\mu}{2}(F-1)^2} \left[\mu \frac{l}{l_r}\left(\frac{l}{l_r} - 1 \right) - 1 \right] + \frac{k}{\mu}}{g l_r^{\,2} + h l^2} \tag{7}$$

$$\tau = k\left(\frac{l}{l_r} - 1 \right) e^{\frac{\mu}{2}(F-1)^2} - h\, l\, l_r\, \frac{C + \frac{k}{\mu} e^{\frac{\mu}{2}(F-1)^2} \left[\mu \frac{l}{l_r}\left(\frac{l}{l_r} - 1 \right) - 1 \right] + \frac{k}{\mu}}{g l_r^{\,2} + h l^2} \tag{8}$$

Here m, h and g correspond with the matrices $\mathbf{M}, \mathbf{H}$ and $\mathbf{G}$ in 1D case. To be able to analyze the properties of the dynamical system we will rewrite the equations (6), (7) and (8) into the dimensionless form. The values in these equations have the following dimensions:

$$\begin{aligned}
&\tau, k, C \ldots\ldots\ldots [N] \\
&l_r, l \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [m] \\
&m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [N^{-1}s] \\
&g, h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [Ns] \\
&t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [s] \\
&r, \mu \ldots\ldots\ldots [1]
\end{aligned}$$

We will introduce the following dimensionless variables

$$k' = k\sqrt{\frac{|m|}{g}}$$
$$l_r' = {l_r}/{l} \qquad (9)$$
$$t' = {t}/{\sqrt{g|m|}}$$

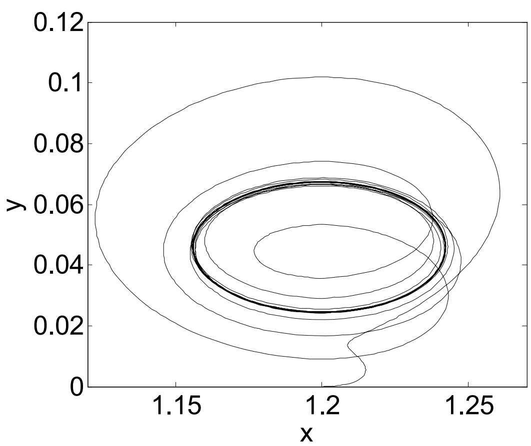
The dimensionless form of this nonautonoumos system is

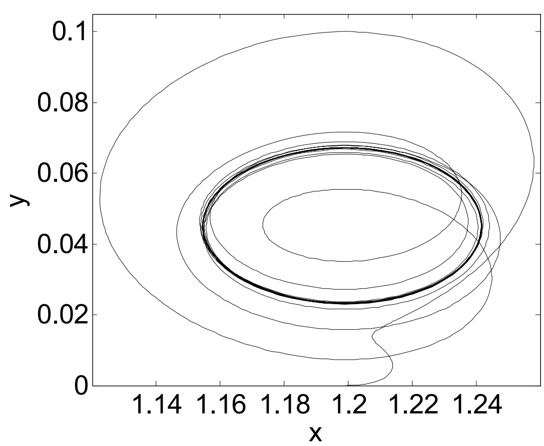
$$\dot{x} = -x\left\{C + D\sin\omega t + \frac{y}{\mu}e^{\frac{\mu}{2}(x-1)^2}\left[\mu x(x-1)-1\right] + \frac{y}{\mu}\right\} \;\; ; \;\; \dot{y} = \operatorname{sgn} m\left[r - \frac{1}{\mu}\left(e^{\frac{\mu}{2}(x-1)^2} - 1\right)\right] \qquad (10)$$

where $x = {1}/{l_r'}$; $y = k'$; $l_r' = {l_r}/{l}$ and l, l_r is the length of the continuum in actual and relaxed configuration and k' is the dimensionless stiffness . C, D, r, μ, m are the parameters. If we use the more simple form for the free energy (5), we obtain

$$\dot{x} = -x\left[C + D\sin\omega t + \frac{y}{2}\left(x^2 - 1\right)\right]; \qquad \dot{y} = \operatorname{sgn} m\left[r - \frac{1}{2}(x-1)^2\right] \qquad (11)$$

According to some numerical experiments we can see (Figures 2a and 2b) that both models have qualitatively same properties. Therefore further we will focus our attention on the simpler one (11).





a b

Figure 2 Response on the periodical stimulation for $\varepsilon C_1 = -0.001, r = 0.02, \varepsilon\nu = 0.001, \varepsilon^2 D_2 = 0.00001, a)\ \mu \to 0, b)\ \mu = 1$

3. Dynamical analysis for D = 0

This analysis was published in Rosenberg & Hynčík (2008). It was proved the existence of the degenerated Hopf's bifurcation for $C = 0$ and sgn $m = -1$. The situation is shown on Figure 3 in the right corner. Depending on the sign of C there exists one stable and one unstable equilibrium point and for $C = 0$ stable limit cycle around these points.

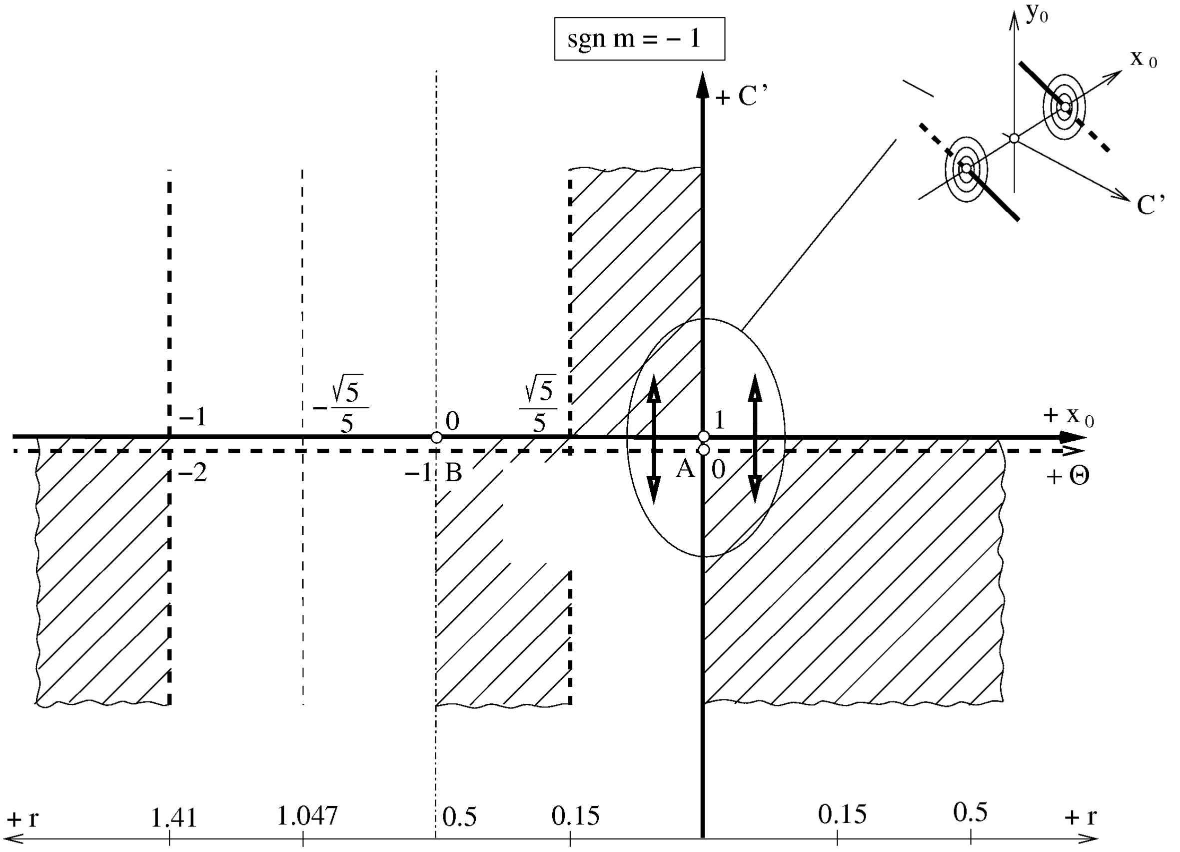


Figure 3 Stability domains in the parameter space

4. Dynamical analysis – soft resonant excitation

We try to analyze the behavior of this system with the periodical stimulation near the bifurcation point $C = 0$ for sgn $m = -1$, see Rosenberg & Hynčík (2008), using multiple-scale method, see e.g. Nayfeh & Balachandran (1995). We assume the following form of the solution

$$x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \varepsilon^3 x_3$$
$$y = y_0 + \varepsilon y_1 + \varepsilon^2 y_2 + \varepsilon^3 y_3 \tag{12}$$

where $0 < \varepsilon << 1$ is the scalar parameter. All variables in (12) are function of T_0, T_1 where $T_0 = t, T_1 = \varepsilon t$. Further we assume

$$C = \varepsilon C_1, \quad D \sin \omega t = \varepsilon^2 D_2 \sin(\Omega T_0 + \varepsilon \nu_1 T_0) \tag{13}$$

It means that we analyze the case when the stimulation is weak and the periodical term is smaller.
According $T_1 = \varepsilon T_0$ we can write

$$D \sin \omega t = \varepsilon^2 D_2 \sin(\Omega T_0 + \nu_1 T_1) \tag{14}$$

Putting from (12), (13), and (14) into (11) and comparing the terms with $\varepsilon^0, \varepsilon^1$ and ε^2 we obtain the following systems of equations:

For ε^0 :

$$\frac{\partial x_0}{\partial T_0} = -\frac{x_0 y_0}{2}\left(x_0^2 - 1\right); \quad \frac{\partial y_0}{\partial T_0} = -r + \left(x_0^2 - 2x_0 + 1\right) \tag{15}$$

The equilibrium point of this dynamical system has the coordinates

$$\begin{aligned} x_0 &= 1 \pm \Theta; \quad \Theta = \sqrt{2r} \\ y_0 &= 0 \end{aligned} \tag{16}$$

If we analyze the stability of this point using equation in variations, we obtain for the eigenvalues the relation

$$\lambda_{1,2} = \pm\sqrt{-\Omega^2}; \quad \Omega = \sqrt{\frac{1}{2} x_0 (x_0 - 1)(x_0^2 - 1)} \tag{17}$$

and therefore the points are in this stage of approximation stable.

For ε^1 :

For $y_0 = 0;\, x_0 = const.$ the corresponding system has the form

$$\begin{aligned} \frac{\partial x_1}{\partial T_0} &= -x_0 C_1 - \frac{1}{2} y_1 \left(x_0^2 - 1\right) x_0 \\ \frac{\partial y_1}{\partial T_0} &= x_1 (x_0 - 1) \end{aligned} \tag{18}$$

Corresponding equation only for x_1 is

$$\frac{\partial^2 x_1}{\partial T_0^2} + \Omega^2 x_1 = 0 \tag{19}$$

The solution has than the form

$$\begin{aligned} y_1 &= \frac{2}{x_0\left(x_0^2 - 1\right)}\left(K_1 \Omega \sin \Omega T_0 + K_2 \Omega \cos \Omega T_0 - x_0 C_1\right) \\ x_1 &= K_1 \cos \Omega T_0 - K_2 \sin \Omega T_0; \quad K_1(T_1), K_2(T_1) \end{aligned} \tag{20}$$

For ε^2 :

$$\frac{\partial x_2}{\partial T_0}+\frac{1}{2}x_0\left(x_0^2-1\right)y_2=-\frac{\partial x_1}{\partial T_1}-x_0D_2\sin\left(\Omega T_0+\nu_1T_1\right)-\frac{3}{2}x_1y_1x_0^2+\frac{1}{2}x_1y_1-x_1C_1$$
$$\frac{\partial y_2}{\partial T_0}=-\frac{\partial y_1}{\partial T_1}+\frac{1}{2}x_1^2+x_2\left(x_0-1\right) \tag{21a,b}$$

After putting from (20) we obtain the equation for x_2:

$$\frac{\partial^2 x_2}{\partial T_0^2}+\Omega^2x_2=2\left(K_1{}'\Omega\sin\Omega T_0+K_2{}'\Omega\cos\Omega T_0\right)-\frac{1}{4}x_0\left(x_0^2-1\right)\left(K_1\cos\Omega T_0-K_2\sin\Omega T_0\right)^2-$$
$$-x_0D_2\Omega\cos\Omega T_0\cos\nu_1T_1+x_0D_2\Omega\sin\Omega T_0\sin\nu_1T_1-$$
$$-\frac{3x_0^2-1}{x_0\left(x_0^2-1\right)}\left(K_1\cos\Omega T_0-K_2\sin\Omega T_0\right)\left(K_1\Omega^2\cos\Omega T_0-K_2\Omega^2\sin\Omega T_0\right)+$$
$$+\frac{3x_0^2-1}{x_0\left(x_0^2-1\right)}\left(K_1\Omega\sin\Omega T_0+K_2\Omega\cos\Omega T_0\right)\left(K_1\Omega\sin\Omega T_0+K_2\Omega\cos\Omega T_0-x_0C_1\right)+$$
$$+C_1\Omega\left(K_1\sin\Omega T_0+K_2\cos\Omega T_0\right) \tag{22}$$

From the setting the secular terms equal zero we obtain the equations for K_1, K_2 as

$$K_1{}'-\frac{C_1x_0^2}{x_0^2-1}K_1+\frac{1}{2}x_0D_2\sin\nu_1T_1=0$$
$$K_2{}'-\frac{C_1x_0^2}{x_0^2-1}K_2-\frac{1}{2}x_0D_2\cos\nu_1T_1=0 \tag{23}$$

Solution is

$$K_1=k_1e^{aT_1}+\frac{b}{\nu_1^2+a^2}\left(\nu_1\cos\nu_1T_1+a\sin\nu_1T_1\right)$$
$$K_2=k_2e^{aT_1}+\frac{b}{\nu_1^2+a^2}\left(\nu_1\sin\nu_1T_1-a\cos\nu_1T_1\right) \tag{24}$$

where

$$a=\frac{C_1x_0^2}{x_0^2-1};\quad b=\frac{x_0D_2}{2} \tag{25}$$

and k_1, k_2 are constants.

The first terms on the right sides of (24) tend to zero if $a<0$ and therefore for

$$C_1<0 \text{ and } x_0>1 \quad \text{that is } x_0=1+\Theta$$
or $$C_1>0 \text{ and } x_0<1 \quad \text{that is } x_0=1-\Theta\,.$$

Let $K_1=c\sin\varphi,\quad K_2=c\cos\varphi$ where c,φ are new parameters generally depending on T_1.

The solution is than

$$x = x_0 + \varepsilon c \cos(\Omega T_0 + \varphi); \quad y = y_0 + \varepsilon(x_0 - 1)c \sin(\Omega T_0 + \varphi) - \frac{2C_1}{x_0^2 - 1} \tag{26}$$

To analyze the stability of this solution we introduce the new parameters c, φ into (23). After substitution

$$\nu_1 T_1 - \varphi = \psi \tag{27}$$

we obtain the following system of equations

$$\begin{aligned} \psi' &= \nu_1 - \frac{b}{c}\cos\psi \\ c' &= ac - b\sin\psi \end{aligned} \tag{28}$$

Stationary point has the coordinates

$$tg\tilde{\psi} = \frac{a}{\nu_1}; \quad \tilde{c} = \frac{b}{\sqrt{\nu_1^2 + a^2}} = \frac{x_0 D_2 (x_0^2 - 1)}{2\sqrt{\nu_1^2 (x_0^2 - 1)^2 + C_1^2 x_0^4}} \tag{29}$$

Equations of variations for (28) are

$$\begin{aligned} \eta_\psi{}' &= \frac{b}{c}\sin\psi\, \eta_\psi + \frac{b}{c^2}\cos\psi\, \eta_c \\ \eta_c{}' &= -b\cos\psi\, \eta_\psi + a\, \eta_c \end{aligned} \tag{30}$$

For the eigenvalues we obtain the equation

$$\lambda^2 - \lambda\Sigma + \Delta = 0; \quad \Sigma = \frac{2C_1 x_0^2}{x_0^2 - 1}; \quad \Delta = a^2 + \nu_1^2 \tag{31}$$

and then

$$\lambda_{1,2} = a \pm i\,\nu_1 \tag{32}$$

Asymptotical stability occurs if

$$C_1 < 0 \ and \ x_0 > 1 \quad that \ is \ x_0 = 1 + \Theta$$

or $$C_1 > 0 \ and \ x_0 < 1 \quad that \ is \ x_0 = 1 - \Theta$$

Inserting from (27) into (26) we obtain the final form of solution

$$x = x_0 + \varepsilon\tilde{c}\cos((\Omega + \varepsilon\nu_1)T_0 - \tilde{\psi}); \quad y = y_0 + \varepsilon\left[(x_0 - 1)\tilde{c}\sin((\Omega + \varepsilon\nu_1)T_0 + \tilde{\varphi}) - \frac{2C_1}{x_0^2 - 1}\right] \tag{33}$$

We can conclude, that in the nearness of the eigenfrequency Ω („resonant excitation“) the resulting steady motion has approximately the same frequency ($\Omega + \varepsilon\nu_1$) – „the free oscillation component is entrained by the forced component“ – Nayfeh & Balachandran (1995). For the numerical example see Figure 4.

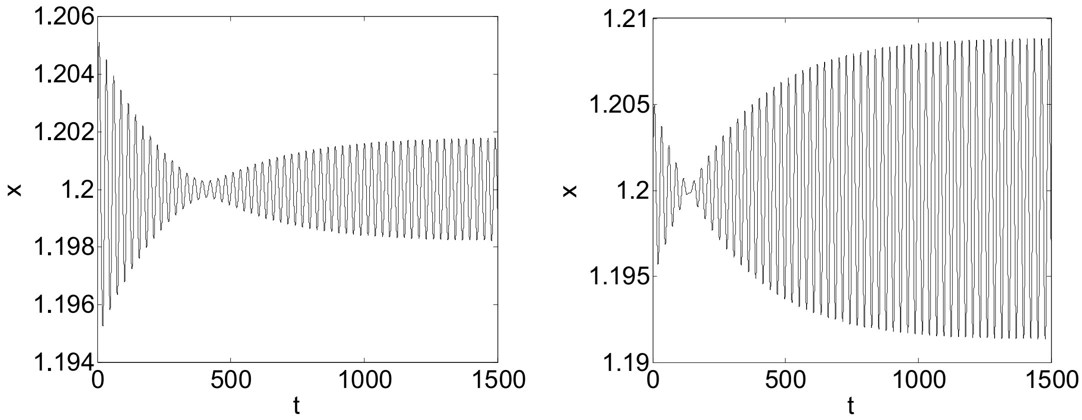


Figure 4 Response on the periodical stimulation for
$\varepsilon C_1 = -0.001,\ r = 0.02,\ \varepsilon\nu = 0.001,\ \varepsilon^2 D_2 = 0.00001,\ a)\ \varepsilon^2 D_2 = 0.00001,\ b)\ \varepsilon^2 D_2 = 0.00005.$

5. Dynamical analysis – soft nonresonant stimulation

Let

$$C = \varepsilon C_1, \quad D\sin\omega t = \varepsilon^2 D_2 \sin(\omega T_0) \tag{34}$$

The equations obtained for ε^0 and ε^1 and their solution will be the same as in the previous section. The difference will occur for ε^2:

$$\begin{aligned}\frac{\partial^2 x_2}{\partial T_0^2} + \Omega^2 x_2 &= 2\left(K_1'\Omega\sin\Omega T_0 + K_2'\Omega\cos\Omega T_0\right) - \frac{1}{4}x_0\left(x_0^2-1\right)\left(K_1\cos\Omega T_0 - K_2\sin\Omega T_0\right)^2 - \\ &- x_0 D_2 \Omega\cos\omega T_0 - \\ &- \frac{3x_0^2-1}{x_0\left(x_0^2-1\right)}\left(K_1\cos\Omega T_0 - K_2\sin\Omega T_0\right)\left(K_1\Omega^2\cos\Omega T_0 - K_2\Omega^2\sin\Omega T_0\right) + \\ &+ \frac{3x_0^2-1}{x_0\left(x_0^2-1\right)}\left(K_1\Omega\sin\Omega T_0 + K_2\Omega\cos\Omega T_0\right)\left(K_1\Omega\sin\Omega T_0 + K_2\Omega\cos\Omega T_0 - x_0 C_1\right) + \\ &+ C_1\Omega\left(K_1\sin\Omega T_0 + K_2\cos\Omega T_0\right)\end{aligned} \tag{35}$$

For the secular terms we can write

$$2K_i'\Omega - \frac{3x_0^2-1}{x_0\left(x_0^2-1\right)}x_0 C_1 K_i \Omega + C_1 \Omega K_i = 0; \quad i = 1,2 \tag{36}$$

and after short simplification

$$K_i' - aK_i = 0 \Rightarrow \quad K_i = ke^{aT_1} \tag{37}$$

For $a < 0$ the amplitudes K_i converge to zero, for a = 0 are constant and for a > 0 diverge to infinite. Let $a < 0$, that is

$$C_1 < 0 \ \textit{and}\ x_0 > 1 \quad \textit{that is}\ x_0 = 1 + \Theta$$

$$\text{or}\ C_1 > 0 \ \textit{and}\ x_0 < 1 \quad \textit{that is}\ x_0 = 1 - \Theta$$

Then the equation (35) will have very simple form

$$\frac{\partial^2 x_2}{\partial T_0^2} + \Omega^2 x_2 = -x_0 D_2 \Omega \cos \omega T_0 \tag{38}$$

The solution is

$$x_2 = A \sin \Omega T_0 + B \cos \Omega T_0 - \frac{x_0 D_2 \Omega}{\Omega^2 - \omega^2} \cos \omega T_0 \tag{39}$$

and from (21b), which has the same form as in the case of soft resonant stimulation, we obtain

$$y_2 = \frac{2}{x_0 \left(x_0^2 - 1\right)} \left(-A\Omega \cos \Omega T_0 + B\Omega \sin \Omega T_0 + \frac{x_0 D_2 \Omega}{\Omega^2 - \omega^2} \omega \sin \omega T \right) \tag{40}$$

If we change the constant parameters

$$A = c \sin \varphi;\ B = c \cos \varphi \tag{41}$$

the final solution has the form

$$x = x_0 + \varepsilon^2 \left[c \cos(\Omega T_0 - \varphi) - \frac{x_0 D_2 \Omega}{\Omega^2 - \omega^2} \cos \omega T_0 \right] \tag{42}$$

$$y = y_0 + \varepsilon^2 \frac{2\Omega}{x_0 \left(x_0^2 - 1\right)} \left[c \sin(\Omega T_0 - \varphi) - \frac{x_0 D_2 \omega}{\Omega^2 - \omega^2} \sin \omega T_0 \right]$$

Second term on the RHS of (42) is the rest of y_1, compare with (20). For incommensurate frequencies we represent these equations the quasiperiodic motion. From the initial conditions $T_0 = t = 0 : x = x_0, y = y_0$

$$c = \frac{x_0 D_2 \Omega}{\Omega^2 - \omega^2}; \quad \varphi = 0 \tag{43}$$

Shifting the origin into the point with coordinates (x_0, y_0) we obtain for the orbit in the state space

$$\begin{aligned} \xi &= c(\cos \Omega t - \cos \omega t) \\ \eta &= \frac{2\Omega c}{x_0 \left(x_0^2 - 1\right)} (\sin \Omega t - \sin \omega t) \end{aligned} \tag{44}$$

To construct the Poincare mapping we will sample the time with period $2\pi n / \Omega$. Then

$$\begin{aligned} \xi_n &= c\left(1 - \cos 2\pi \frac{\omega}{\Omega} n\right) \\ \eta_n &= \frac{-2\Omega c}{x_0 \left(x_0^2 - 1\right)} \sin 2\pi \frac{\omega}{\Omega} n \end{aligned} \tag{45}$$

Eliminating parameter n we obtain the equation of ellipse. On Figure 5 we can see the corresponding numerical example – the attractor in Poincare mapping. Figure 6 shows its destruction and transition to chaos for further decreasing *C*. The analysis of this process needs the change of (13) and the approximation of higher order.

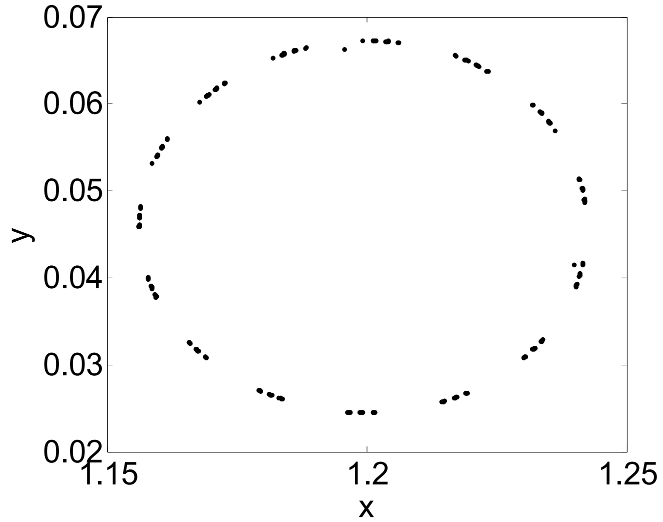


Figure 5 Poincare mapping for $\varepsilon C_1 = -0.01, r = 0.02, \omega = 0.4, \Omega = 0.2298, \varepsilon^2 D_2 = 0.01.$

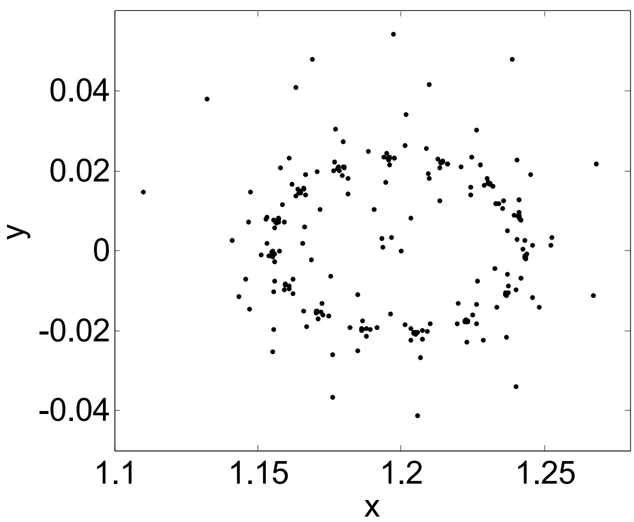


Figure 6 Destruction of the attractor for
$\varepsilon C_1 = -0.0003, r = 0.02, \omega = 0.4, \Omega = 0.2298, \varepsilon^2 D_2 = 0.01.$

6. Conclusion

Shortly summarized results of the analysis in case $D = 0$ (in detail see in Rosenberg& Hynčík (2008)) when the control parameter is *C* show the existence of the degenerated Hopf's bifurcation. For C = 0 the limit cycle with the frequency Ω can be observed.

Main part of the contribution is devoted to the analysis of the soft (small *D*) resonant ($\omega \to \Omega$) and non-resonant stimulation using MSM.

The results of this analysis are compared with the numerical experiments. It is analyzed also the influence of the parameter μ. This parameter doesn't change the qualitative properties of the system.

7. Acknowledgement

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