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KINEMATIC ANALYSIS OF DOUBLE-BODY MANIPULATORS

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Summary: *The work aims at making a kinematical analysis of basic manipula¬tors structures, with the use of the computer program developed by the authors. In the paper there have been presented twelve structures of double-body manipulators divided into four groups in respect to used kinematical pairs, and their occurrence order (place of situation) in the mechanism. Each group contains three mechanisms in which axes of particular bodies are placed perpendicularly, parallelly and obliquely.*

1. Basis of the mechanism kinematic analysis

The analytic method of kinematic analysis involves:

- assigning systems of coordinates to particular elements
- defining constant and variable parameters which account for dependencies between particular links of the kinematical chain (Denavit and Hartenberg notation has been chosen).
- determination of transformation matrixes (indispensable for description of the tool or grip system points in the structure of the base).
- determination of motion parameters (position, velocity, and acceleration).
- determination of the mechanism work space.
- determination of the working element (and other elements) motion within the work space.

Accomplishment of the above mentioned tasks requires matrix transformations. The paper presents a computer aided kinematic analysis of possible configurations of the manipulators.

2. Position of reference systems on elements of series manipulators

Relations of particular systems of reference with the successive links of the mechanism are such as follows: all z axes are consistent with geometric axes of the nodes. It is assumed that axis x_i lies on the normal to the axis of rotational pairs of links i and $i+1$, whereas axis z_i lies on the axis of the rotational pair connecting links $i-1$ with i, axis y is a completion of

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the right handed system of perpendicular coordinates. Besides, for each element, and for each motion connection, there can be provided values of four parameters describing their position and orientation. These parameters are called Deniavit-Hartenberg's parameters and are defined as:

- a_i – length of the i^{th} element measured along axis x_i,
- α_i – angle of the i^{th} element torsion,
- d_i – shift of the i^{th} element along axis z_i,
- θ_i – angle of configuration of the i^{th} motion connection.

Position of the coordinate system was matched in such a way that, according to Denavit and Hatengerg notation, the systems transformations were reduced to two rotations (one around axis z, and one around x) and two translations (along axis z and x), and thanks to it the transformation matrix between two systems has been reduced to the following form:

$$ {}^{\mathrm{i}}_{\mathrm{i+1}}\mathrm{T} = \left(R_{Xi}\left(\alpha_i\right)\cdot D_{Xi}\left(a_i\right)\right)\cdot\left(R_{Zi}\left(\theta_i\right)\cdot D_{Zi}\left(d_i\right)\right) \tag{1} $$

Particular matrixes of translation and rotation will have the following forms:

- Matrix of rotation by angle α_i around axis x_i:

$$ R_{Xi}\left(\alpha_i\right)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

- Matrix of translation by value a_i along axis x_i:

$$ D_{Xi}\left(a_i\right)=\begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3} $$

- Matrix of rotation by angle θ_i around axis z_{i+1}:

$$ R_{Zi+1}\left(\theta_i\right)=\begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4} $$

- Matrix of translation by value d_i along axis z_{i+1}:

$$ D_{Zi+1}\left(d_i\right)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5} $$

According to notation (2.1):

$$ {}^{i}_{i+1}\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

After multiplication of matrixes we receive:

$$ {}^{i}_{i+1}\mathbf{T} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_i \\ \sin\alpha_i \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i & -d_i \sin\alpha_i \\ \sin\alpha_i \sin\theta_i & \sin\alpha_i \cos\theta_i & \cos\alpha_i & d_i \cos\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6} $$

where:

- $\mathbf{R}_{Xi}(\alpha_i)$ – rotation matrix of system $\{i+1\}$ in relation to the system $\{i\}$, and around axis x_i by angle α_i,
- $\mathbf{D}_{Xi}(a_i)$ – translation matrix of system $\{i+1\}$ center in relation to system $\{i\}$, along axis x_i, by value a_i,
- $\mathbf{R}_{Zi}(\theta_i)$ – rotation matrix of system $\{i+1\}$ in relation to system $\{i\}$, along axis z_{i+1} by angle θ_i,
- $\mathbf{D}_{Zi}(d_i)$ – translation matrix of system $\{i+1\}$ center in relation in system $\{i\}$ along axis z_{i+1}, by value d_i,
- α_i – rotation angle around axis x_i measured from axis z_i to z_{i+1},
- a_i – value of translation along axis x_i, measured from axis z_i to axis z_{i+1}, is positive if the translation vector sense is consistent with the sense of axis x_i,
- θ_i – rotation angle around axis z_i measured from axis x_i to axis x_{i+1},
- d_i – value of translation along axis z_i measured from axis x_i to x_{i+1} is positive when the translation vector sense is consistent with the sense of axis z_i.

Thanks to such a notation each transformation is described by means of four parameters α, a, θ, d. In pairs of V class, three of these parameters are constant (depending on geometric dimensions of the mechanism) and one variable (exploitation). E.g.: α, a, d are constant whereas θ is variable. In pairs of class IV there are two constant (α, a) and two variable ones (θ, d).

Determination of general transformation matrix ${}^{0}_{P}\mathbf{T}$, describing transformation of the coordinate system in grip P to base system 0, is reduced to multiplying particular matrixes of transformations of successive reference systems connected with particular kinematic pairs, according to formula (7).

$$ {}^{0}_{P}\mathbf{T} = {}^{0}_{1}\mathbf{T} \cdot {}^{1}_{2}\mathbf{T} \cdot \ldots \cdot {}^{i\text{-}1}_{i}\mathbf{T} \cdot \ldots \cdot {}^{P\text{-}1}_{P}\mathbf{T} \tag{7} $$

Position of a given point connected with the grip system in a global coordinate system is:

$$ {}^{0}r = {}^{0}_{P}\mathbf{T} \cdot {}^{P}r \tag{8} $$

where :

^{0}r – vector of position of point P in system $\{0\}$

^{P}r – vector of position of point P in system $\{i\}$

${}^{0}_{P}\mathbf{T}$ – transformation matrix of system $\{P\}$ into $\{0\}$.

In rotation pairs, angle θ is a variable parameter (exploitation), whereas the others α, a, d are constant ones resulting from the chain geometry. Thus, in this case, angle θ is a configuration parameter, variable in time. In translational pairs, shift d is variable (in time), whereas α, a, θ are constant.

An advantage of the applied Denavit-Hatenberg notation is that transformations of any kinds of pairs are denoted by four, always the same, configuration parameters, thereby in each case rotation and shift matrixes also have the same form. Thanks to it, is easy to transfer the whole notation to computer algorithm and, using computers, accelerate the whole process of manipulator analysis.

3. Algorithm of analysis of series two-bodies mechanisms

The aim of the program, developed by the authors for analysis and determination of parameters for spatial mechanisms motion, is to enhance the process of matrix transformations (multiplying, transposition) occurring during determining motion parameters, i.e. location, velocity, and acceleration of the mechanism given point. This point is usually the beginning of the coordinate system of the object mounted in the jaws of the manipulator grip.

Block scheme of the order of calculations performed by the program is demonstrated in fig. 1. To begin with, system $\{0\}$ connected with the manipulator base is defined. Then, after having defined the quantity and type of kinematic pairs which make up the manipulator, the description of particular coordinate systems connected with the successive element, is made. Particular transformations of the systems are described by providing the axis in relation to which there occurs the translation or rotation, and this transformation value (distance or angle). Algorithm of the program in this stage does not require providing a concrete value of the translation or rotation, only a symbol representing this value.

After having entered the program data, the first calculations are performed and in effect, matrix ${}^{0}_{P}\mathbf{T}$ of transformation of system $\{P\}$ to system $\{0\}$ or, expression of system $\{P\}$ in coordinates of system $\{0\}$ are obtained. The successive calculation stage is derivation of formulas for position, velocity and acceleration of point $\{P\}$ in the system of the base. Now, is possible to enter the numerical values of variable parameters in order to obtain numerical values r, v and p.

The main advantage of the program is a possibility of determining formulas (not only values) for parameters of special mechanism motion and the transformation matrix. Additionally, the program defines work space for a manipulator with given parameters. The program has been used for an analysis of two-bodies mechanisms with pairs: rotational R and translational T, in four combinations TT, RR, RT, and TR. The program, completed with additional modules, can be used for analysis of mechanisms with a bigger number of bodies.

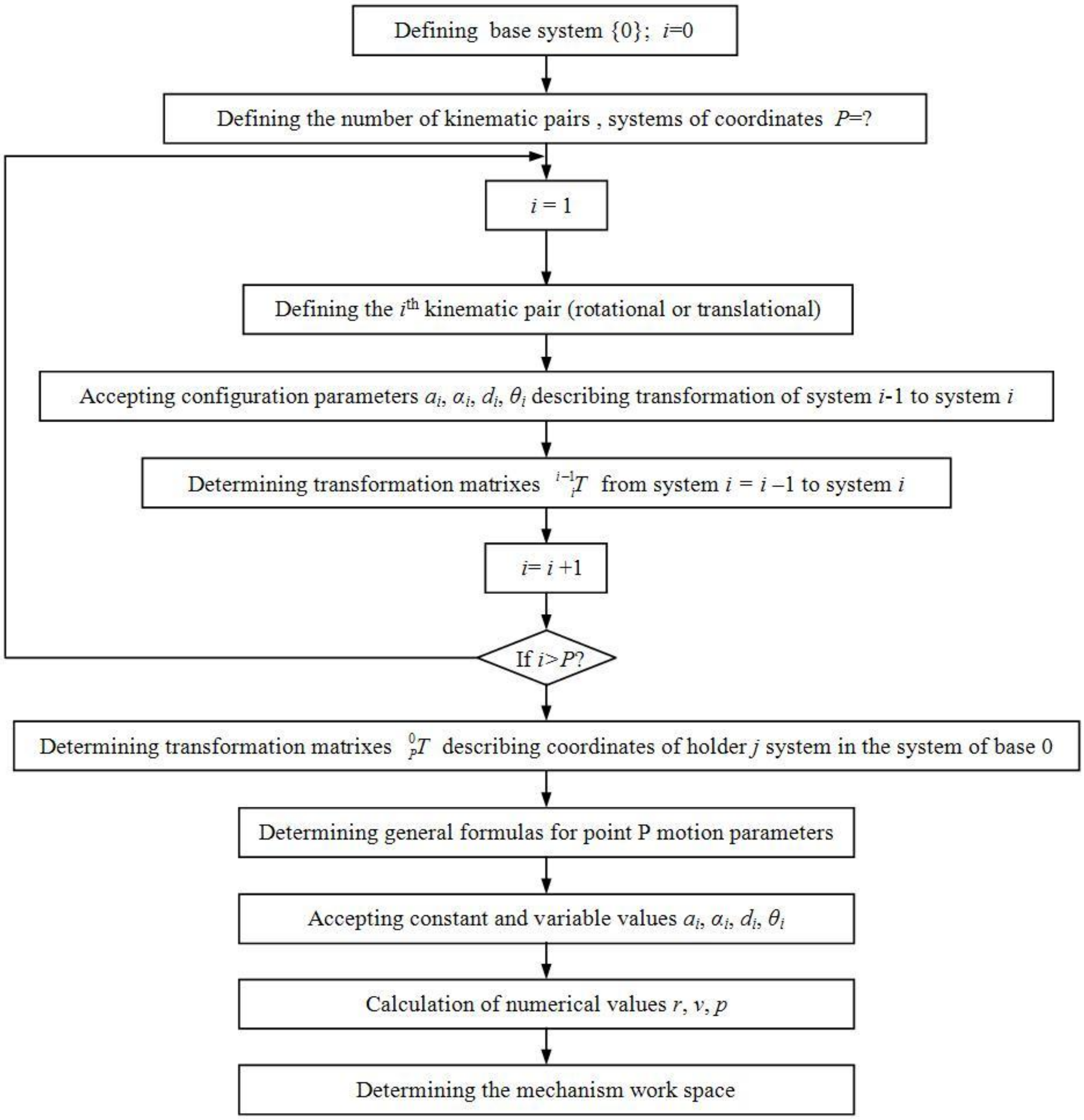

Fig. 1 Algorithm of spatial mechanism analysis

4. Software 'Mech3D' for kinematical analysis of two-bodies mechanisms

Figure 2 shows the program main window. Working with the program starts with choosing the kind of structure of the mechanism then, there follows choice of one of twelve structures of two-element mechanisms (fig. 2) [Siemieniako F., Kuźmierowski T. (2001)].

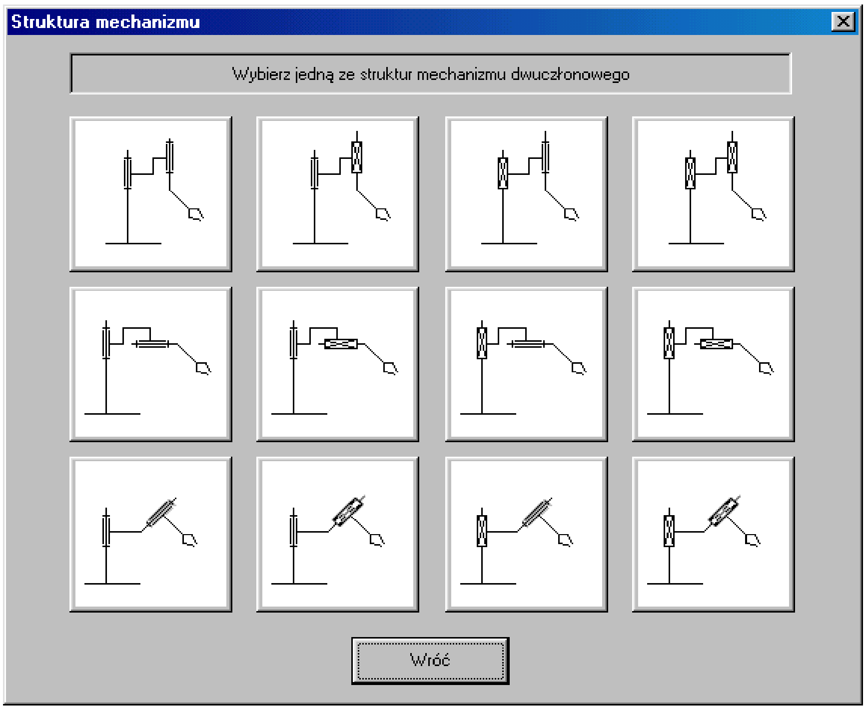

Fig. 2 Choosing the structure mechanism

After having chosen one of them, there opens a window by which constant geometric dimensions of the mechanism will be defined (fig. 3), according to denotations presented in the scheme. These are constant Denavit-Hatenberg parameters defining mutual positions of coordinate systems connected with successive elements of kinematic pairs.

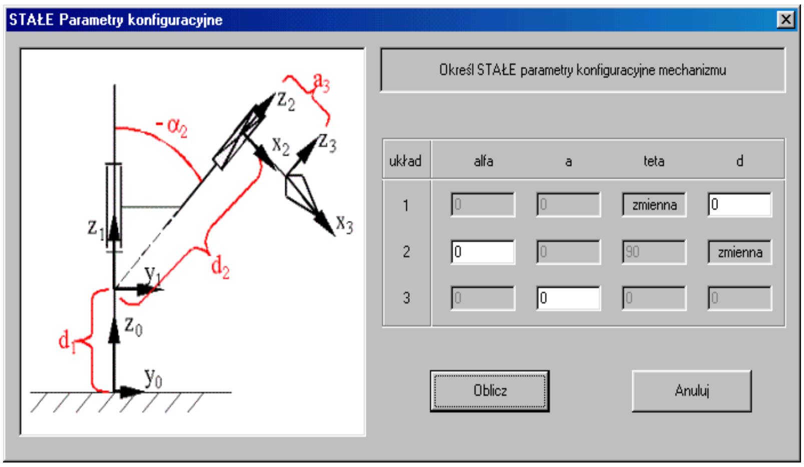

Fig. 3 Defining the mechanism constant parameters

Next, after having performed the first calculations the program presents transformation matrix. In this matrix, particular positions are expressed by formulas. In the next window (fig. 4) it is possible to see formulas of determined parameters of the chosen mechanism motion. To make the recording shorter, the following denotations have been introduced: $\mathrm{st} - \sin\theta$, $\mathrm{ct} - \cos\theta$.

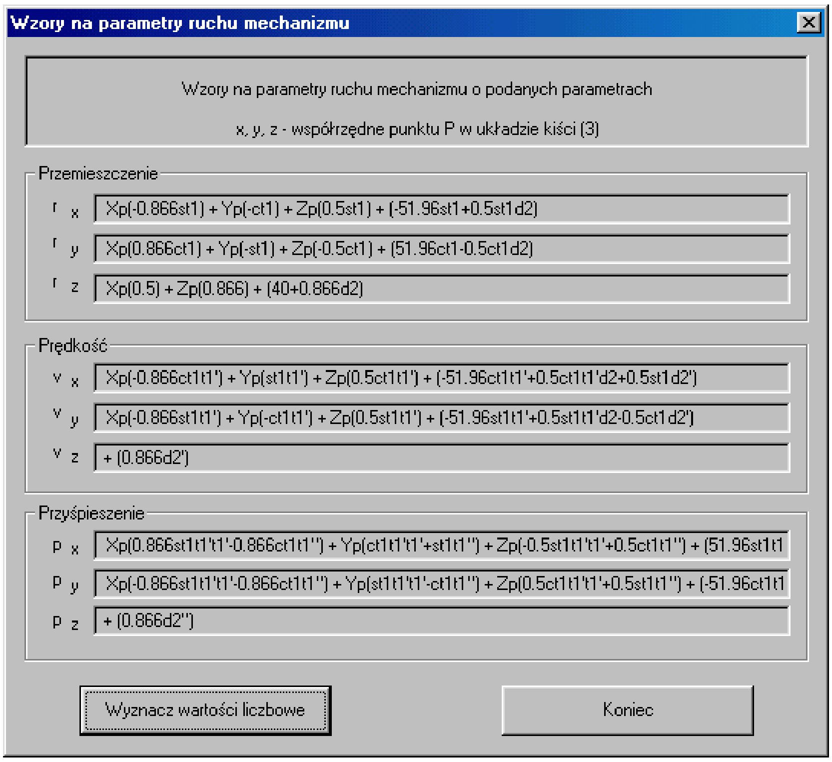

Fig. 4 Formulas for motion parameters

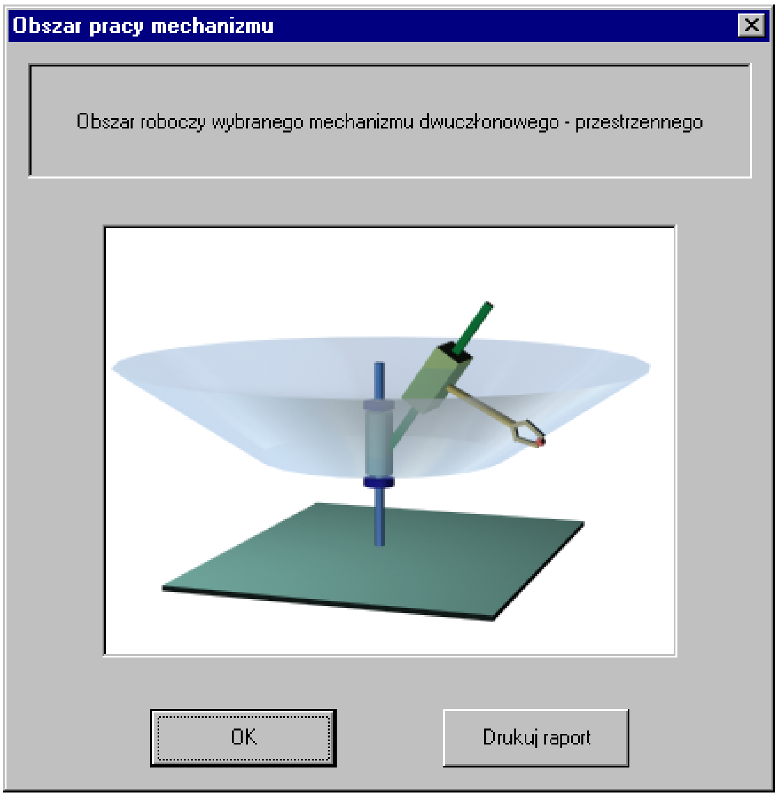

Fig. 5 The mechanism work space

On the basis of the calculated transformation matrix and geometric parameters values, the program determines and outlines the mechanism work space (fig. 5).

5. Summary

In the paper there have been presented twelve structures of two-bodies mechanisms (fig. 2) divided into four groups in respect to used kinematical pairs, and their occurrence order (place of situation) in the mechanism. Each group contains three mechanisms in which axes of particular members are placed perpendicularly, parallelly and obliquely.

The proposition contains all possible combinations of two-bodies mechanisms structures which are applied in manipulators. The work aims at making a kinematical analysis of these structures, with the use of the computer program developed by the authors.

On the example of the presented manipulators it is possible to see the complexity of the kinematic task involving determination of motion parameters of chosen points of the mechanism. Making the constructor's job easier and accelerating calculations is one of the most important advantages of the developed computer program 'Mech 3D'. The program changes the notation of the manipulator structure from the form of given configuration parameters of elements into matrix notation, and then, it determines formulas for transformation matrix of reference systems coordinates, and parameters of the working element motion. The authors hope that the presented program as well as the results of analytical study on two-bodies mechanisms structures, will facilitate engineers work in the field of kinematic analysis of manipulators.

6. References

Craig J. J. (1995) *Wprowadzenie do robotyki. Mechanika i sterowanie.* (In English: Introduction to Robotics. Mechanics and Control) WNT. Warszawa.

Miller S. (1989) *Teoria maszyn i mechanizmów. Analiza układów mechanicznych* (In English: Machine and Mechanism Theory. Mechanical Systems Analyses). Wydawnictwa Politechniki Wrocławskiej. Wrocław.

Siemieniako F., Kuźmierowski T. (2001) *Wspomagane komputerowo wyznaczanie parametrów ruchu efektorów manipulatorów* (In English: Computer Aided Determination of Manipulators Effectors Motion Parameters). Wydawnictwa Politechniki Białostockiej. Białystok.

Wawrzecki J. (1995) *Teoria manipulatorów* (In English: Manipulators Theory). Wyd. Politechniki Łódzkiej. Łódź.