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ABOUT A POSSIBILITY OF THE SIMPLIFICATION OF THE DYNAMICAL SYSTEM WITH GYROSCOPIC STABILISER

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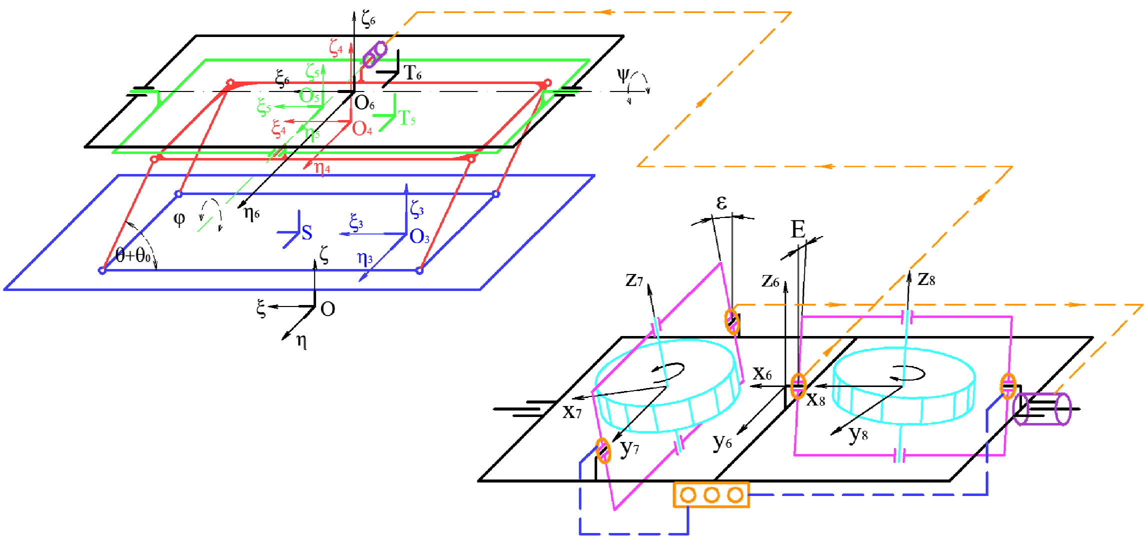
Summary: *The problem of a simplification of the vibroisolation system with gyroscopic stabilizer is solved. According to the presumption, that the impuslmoments of gyroscopes is large and according to a weak coupling between the both subsystems it is possible to analyze the both relevant characteristic equations separately.*



1. Introduction

In the paper (Šklíba, J., 2009) there was analyzed the problem of the simplifications of the equations of a vibro-isolation system with a gyroscopic stabilizer. The main result of this analysis has been: If we neglect passive resistances in axes of precession frames and its inertia forces, the characteristic polynomial can bee expressed as a product of two polynomials of second and fourth degree. The first annulled polynomial gives the characteristic equation for parallelogram (the first subsystem) and the second annulled polynomial there is the characteristic equation of simplified system of gyroscopic stabilizer of Cardan suspension (the second subsystem). The results of numerical experiments confirm this separation in more general cases and therefore we attempt to prove this consequence of weak coupling between the both subsystems generally.

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2. Preliminary considerations

The starting system (disturbed system for equilibrium state of the nonlinear system with parametric and external excitation, deduced in (Šklíba, J., 2007) is

$$\mathbf{A}\ddot{\vec{q}} + (\mathbf{B}+\mathbf{G})\dot{\vec{q}} + (\mathbf{C}+\mathbf{K})\vec{q} = \vec{0}\,. \tag{1}$$

Its characteristic determinant is

$$\begin{vmatrix} A_{11}\lambda^2 + B_{11}\lambda + C_{11} & A_{12}\lambda^2 & A_{12}\lambda^2 & 0 & 0 \\ A_{21}\lambda^2 & A_{22}\lambda^2 + B_{22}\lambda + C_{22} & A_{23}\lambda^2 & A_{24}\lambda^2 & G_{25}\lambda + K_{25} \\ A_{31}\lambda^2 & A_{32}\lambda^2 & A_{33}\lambda^2 + B_{33}\lambda + C_{33} & G_{34}\lambda + K_{34} & A_{35}\lambda^2 \\ 0 & A_{42}\lambda^2 & G_{43}\lambda + K_{43} & A_{44}\lambda^2 + B_{44}\lambda + C_{44} & 0 \\ 0 & G_{52}\lambda + K_{52} & A_{53}\lambda^2 & 0 & A_{44}\lambda^2 + B_{44}\lambda + C_{44} \end{vmatrix}, \tag{2}$$

where the mass matrix **A**, damping matrix **B** and stiffness matrix **C** are symmetrical, matrix of gyroscopic forces (**G**) and matrix of correction and relief (**K**) are antisymmetrical; $\vec{q} = \vec{q}(\vartheta, \varphi, \psi, \varepsilon, \mathrm{E})$ is a vector of angle coordinates.

3. Transformation of disturbance system

Using this transformation of variables we let the first variable ϑ in original position and then we apply the linear transformation of the other variables in such a way to that the mass and damping matrices are diagonal**.** At the same time we introduce two small parameters δ (for damping) and μ (for ratio of non-diagonal and diagonal members in original mass-matrix), The large parameter (for impulse moment of gyroscopes) H and the formal parameter κ are also introduced. The transformed disturbed system is:

$$A_{11}\ddot{\vartheta} + B_{11}\dot{\vartheta} + C_{11} + \mu\sum_{j=2}^{5} a_{1j}\ddot{x}_j = 0\,, \tag{3a}$$

$$\mu A_{k1}\ddot{\vartheta} + (\ddot{x}_k + \delta b_{kk}\dot{x}_k) + \sum_{j=2}^{5}\left(Hg_{kj}\dot{x}_j + \kappa^2\gamma_{kj}x_j\right) = 0\,, \qquad k = 2{,}3\,, \tag{3b}$$

$$(\ddot{x}_k + \delta b_{kk}\dot{x}_k) + \sum_{j=2}^{5}\left(Hg_{kj}\dot{x}_j + \kappa^2\gamma_{kj}x_j\right) = 0\,, \qquad k = 4{,}5\,.$$

The characteristic determinant relevant to the system (3) is

$$\begin{vmatrix} A_{11}\lambda^2 + B_{11}\lambda + C_{11} & \mu\cdot a_{12}\lambda^2 & \mu\cdot a_{13}\lambda^2 & \mu\cdot a_{14}\lambda^2 & \mu\cdot a_{15}\lambda^2 \\ \mu\cdot A_{21}\lambda^2 & \lambda^2 + \delta b_{22}\lambda + \kappa^2\gamma_{22} & Hg_{23}\lambda + \kappa^2\gamma_{23} & Hg_{24}\lambda + \kappa^2\gamma_{24} & Hg_{25}\lambda + \kappa^2\gamma_{25} \\ \mu\cdot A_{31}\lambda^2 & Hg_{32}\lambda + \kappa^2\gamma_{32} & \lambda^2 + \delta b_{33}\lambda + \kappa^2\gamma_{33} & Hg_{34}\lambda + \kappa^2\gamma_{34} & Hg_{35}\lambda + \kappa^2\gamma_{35} \\ 0 & Hg_{42}\lambda + \kappa^2\gamma_{42} & Hg_{43}\lambda + \kappa^2\gamma_{43} & \lambda^2 + \delta b_{44}\lambda + \kappa^2\gamma_{44} & Hg_{45}\lambda + \kappa^2\gamma_{45} \\ 0 & Hg_{52}\lambda + \kappa^2\gamma_{52} & Hg_{53}\lambda + \kappa^2\gamma_{53} & Hg_{54}\lambda + \kappa^2\gamma_{54} & \lambda^2 + \delta b_{55}\lambda + \kappa^2\gamma_{55} \end{vmatrix}. \tag{4}$$

We develop this characteristic determinant for the first column

$$\Delta = \left(A_{11}\lambda^2 + B_{11}\lambda + C_{11}\right)D_{11}\left(H^4, \mu^0\right) + \mu A_{21}\lambda^2 D_{21}\left(H^3, \mu^1\right) + \mu A_{31}\lambda^2 D_{31}\left(H^3, \mu^1\right). \tag{5}$$

where D_{11}, D_{21} and D_{31} are denoted the minors, relevant to three members of the first column. The dependency of powers μ and H is designated. From this equation we can explain the above mentioned weak coupling of the both subsystems. If we neglect the second and third members on the right side (5), we can write the characteristic equation as a product of the both characteristic equations relevant to the both subsystems. The second subsystem (Cardan suspension with gyroscopes) has a characteristic equation of 8.degree:

$$D_{11} = \left|\delta_{kj}\left(\lambda^2 + \delta \cdot b_k \lambda\right) + Hg_{kj}\lambda + \kappa^2\gamma_{kj}\right| = \sum_{i=0}^{8} a_i'\lambda^{8-i} = 0,\ a_0' = 1. \tag{6}$$

The coefficients of this polynomial are the entire rational functions of parameters H, δ and κ. By identification of their structure we use the fact, that every member of the equation (6) is a homogeneous function of parameters λ, H, δ, κ and therefore the resultant power is 8, then we have:

$$\begin{aligned}
&a_1' = a_1\delta, && a_5' = a_5H^3\delta^2 + a_5^{(1)}\delta H^2 + a_5^{(2)}\delta^2 H + ...,\\
&a_2' = a_2H^2 + a_2^{(1)}\delta^2, && a_6' = a_6H^2(\kappa^2)^2 + a_6^{(1)}H^2\delta^2\kappa^2 + ...,\\
&a_3' = a_3H^3 + a_3^{(1)}\delta H^2 + ..., && a_7' = a_7H(\kappa^2)^3 + a_7^{(1)}\delta(\kappa^2)^3 + ...,\\
&a_4' = a_4H^4 + a_4^{(1)}\delta H^3 + a_4^{(2)}\delta^2H^2 + ..., && a_8' = a_8(\kappa^2)^4.
\end{aligned} \tag{7}$$

The dependency of the coefficients on the large parameter H, is such that its power initially increases, the largest is by a_4' and then decreases. It is evident that

$$a_1 = \sum_{i=2}^{5} b_i, \qquad a_4 = \det\left\|g_{ij}\right\|, \tag{8}$$

and we suppose that $a_4 \neq 0$.

In the same time we will analyze two equations

$$\Delta^{(n)} = \left|\delta_{kj} \cdot \left(\lambda^2 + \delta \cdot b_k\right) \cdot \lambda + H \cdot g_{kj}\right| = 0, \tag{9}$$

$$\Delta^{(p)} = \left|\delta_{kj} \cdot \delta \cdot b_k \cdot \lambda + H \cdot g_{kj} + \kappa^2\gamma_{kj}\right| = 0. \tag{10}$$

According to the fact, that this system has favourable even number of coordinates we can use the theorems (in Merkin, 1956) and by relevant assumptions substitute (6) by two equations (9) and (10) for precession and nutation frequencies.

The equation (9) is the characteristic equation of the system in which do not actuate the conservative and radial correction forces.

$$\Delta^{(n)} = \lambda^4 + a_1''\lambda^3 + a_2''\lambda^2 + a_3''\lambda + a_4'' = 0. \tag{11}$$

The equation (10) is the characteristic equation of the simplified system (in which do not actuate inertia forces).

$$\Delta^{(p)} = a_4''\lambda^4 + a_5''\lambda^3 + a_6''\lambda^2 + a_7''\lambda + a_8'' = 0. \tag{12}$$

For the coefficients of these equations we have relations:

$$\begin{aligned}
&a_1'' = a_1\delta, && a_5'' = a_5H^3\delta^2 + \overline{a}_5^{(1)}\delta H^2 + \overline{a}_5^{(2)}\delta^2 H + ..., \\
&a_2'' = a_2H^2 + \overline{a}_2^{(1)}\delta^2, && a_6'' = a_6H^2(\kappa^2)^2 + \overline{a}_6^{(1)}H^2\delta^2\kappa^2 + ..., \\
&a_3'' = a_3H^3 + \overline{a}_3^{(1)}\delta H^2 + ..., && a_7'' = a_7H(\kappa^2)^3 + \overline{a}_7^{(1)}\delta(\kappa^2)^3 + ..., \\
&a_4'' = a_4H^4 + \overline{a}_4^{(1)}\delta H^3 + \overline{a}_4^{(2)}\delta^2H^2 + ..., && a_8'' = a_8(\kappa^2)^4.
\end{aligned} \tag{13}$$

Now it is possible to demonstrate that the first members (by highest power of *H*) in relations (13) are the same as in relations (7). The middle coefficient a_4 is common for the both characteristic equations. The roots of equation (11) are relevant to nutation frequencies (very high) and the roots of equation (12) are relevant to precession frequencies (very low or null).

When Hurwitz conditions are fulfilled for the both equations (11) and (12) the simplification of system (3b) is possible and legal (see Merkin).

4. Conclusion

It is possible to substitute the stability solution of our system (and also the computation of roots of characteristic equation) by the solution:

1. Characteristic equation of parallelogram, of nutation frequencies and of simplified system (for precession frequencies).
2. At the same time the successive assumptions must be fulfilled: the impulse moment of gyroscopes is large, the coupling of both subsystems is weak, Hurwitz conditions of equation (9) and (10) are fulfilled and determinant of gyroscopic members is not equal to zero.

5. Acknowledgement

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