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APPLICATION OF VISCOELASTIC HOMOGENIZATION FOR CONCRETE CREEP

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Summary: *Multiscale viscoelastic homogenization is employed for the assessment of concrete creep. Recent nanoindentation results testified creep origin in calcium silicate hydrates (*C-S-H*) on the scale of nanometers. First, the viscous properties of* C-S-H *are identified by means of numerical inverse analysis from the level of cement paste. Second, the upscaling to the scale of concrete is carried out via correspondence principle. Predicted creep of concrete corresponds reasonably well to experimental data.*

1. Introduction

The renewed interest in concrete creep is driven by several considerations. To mention a few of them, slender concrete members calls for a sophisticated prediction model for the assessment of their service life, exceeding easily 100 years. Prestressing loses due to concrete creep need to be accurately determined. On the material side, pure Portland cement becomes expensive material and secondary cementitious materials (slag, fly ash) are incorporated. To assure acceptable creep performace, creep experiments, lasting several months, need extrapolation to tens of years. Such extrapolation may induce significant error (Bažant, 1999).

It has been recognized that concrete creep is attributed dominantly to cement paste. Fine and coarse aggregates behave as an isotropic elastic material. The effect of aggregates for creep attenuation was recognized experimentally, e.g. Brooks and Neville (1975) mentioned the formula relating creep between cement paste and concrete under the same curing conditions

$$\varepsilon^{creep}_{concrete} = \varepsilon^{creep}_{paste}(1 - f_a)^{\alpha} \quad (1)$$

where f_a is the volume fraction of aggregates and α is the exponent which has to be calibrated. Continuing further to the submicrometer scale, it became evident that the main hydration product, calcium silicate hydrate (C-S-H), exhibit significant creep during nanoindentation (Jennings et al., 2005).

Instead of relying solely on empirical data, multiscale approach offers a tentative way. Proposed methodology is based on the assumption that C-S-H is the only viscous component in

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concrete. However, direct measurement on C-S-H samples is impossible and viscous C-S-H properties must be obtained from top-bottom inverse analysis. Here, the analysis is restrained to the so-called basic creep, which is the state of no moisture transport in or out the sample.

2. Multiscale nature of concrete

Concrete is doubtless a multiscale material. Typical levels may be found at separable and remarkable length scales

C-S-H level typically spans the characteristic length between 10 nm – 1 μm (Bernard et al., 2003). Two morphologies of C-S-H were found,

cement paste level is found on the scale of 1 μm – 100 μm. Clinker minerals, gypsum, CH, homogenized C-S-H and some capillary porosity are present (Bernard et al., 2003). Entrained and entrapped air is accommodated here,

mortar level is considered on the scale between 1 mm and 1 cm. It contains homogenized cement paste, fine aggregates such as sand and associated ITZ,

concrete level spans in typical concrete the characteristic length of 1 cm – 1 dm. Mortar, coarse aggregates such as gravel and associated ITZ are typically found.

Multiscale nature of concrete helps to understand unlocalized phenomena on various scales; concrete elasticity is a typical subject of homogenization (Bernard et al., 2003). Elastic homogenization relies on identification of intrinsic elastic properties and, using homogenization methods, bridging the scales. The same idea will be pursuited here for the assessment of creep.

The extension from elasticity to viscoelasticity has been proposed and partially validated for all scales (Pichler and Lackner, 2008). Although such approach might be predictive, proper validation and coupling among levels must be carried out.

3. Constitutive Law for C-S-H

C-S-H is assumed to exhibit viscoelastic behavior of the B3 model, often used for the scale of concrete (Bažant, 2001). For the case of no moisture or temperature change, the flow term in model B3 can be simplified to a logarithmic law, which will be adopted here. If drying creep is excluded, the B3 compliance function for uniaxial stress has the form

$$J(t,t') = q_1 + C_v(t,t') + q_4 \ln\left(\frac{t}{t'}\right) \tag{2}$$

$$C_v(t,t') = \int_{t'}^{t} v^{-1}(t)\dot{C}_g(\tau - t')\,\mathrm{d}\tau = \int_{t'}^{t} v^{-1}(t)\frac{n(\tau-t')^{n-1}}{\lambda_0^n + (\tau-t')^n}\,\mathrm{d}\tau \tag{3}$$

$$v^{-1}(t) = \left[q_2\left(\frac{\lambda_0}{t}\right)^m + q_3\right] \tag{4}$$

Several parameters were found to be constant for concrete, independent of its type and curing conditions: $\lambda_0 \approx 1$ day, m = 0.5 and n = 0.1 (Bažant and Prasannan, 1989).

When uniaxial stress changes, the response is hypothesized to obey linearity with respect to stress

$$\varepsilon(t) = \int_0^t J(t,t')\mathrm{d}\sigma(t') + \varepsilon_0 \tag{5}$$

where ε_0 is the initial strain, which includes shrinkage, thermal strain and cracking strain.

The numerical solution of Eq. (5) is carried out with exponential algorithm for Kelvin chain (Jirásek and Bažant, 2002) and continuous retardation spectrum (Bažant and Xi, 1995). The exponential algorithm leads to

$$\Delta y_\mu = \Delta t/\tau_\mu \tag{6}$$

$$\lambda_\mu = \frac{1-\exp(-\Delta y_\mu)}{\Delta y_\mu} \tag{7}$$

$$\frac{1}{v_{n+1/2}} = q_2\left(\frac{\lambda_0}{t_{n+1/2}}\right)^m + q_3 \tag{8}$$

$$E'' = \left[q_1 + \frac{1}{v_{n+1/2}}\left(\sum_{\mu=min}^{0}\frac{1}{E_\mu} + \sum_{\mu=1}^{max}\frac{1-\lambda_\mu}{E_\mu}\right)\right]^{-1} \tag{9}$$

where subscript $n+1/2$ refers to time $t_{n+1/2}$ at mid-step in log-time scale, i.e. the geometrical mean between t_n and t_{n+1}, τ_μ are properly chosen retardation times of the μ-th Kelvin unit ($\mu = 1, 2 \ldots N$), $E_\mu(t)$ are the corresponding moduli, which in general are age-dependent.

Note that when the time step Δt is constant, Eq. (9) provides the same tangent incremental modulus E'' for non-aging material ($q_2 = 0$). Poisson's ratio is assumed to remain constant during creep, and so the incremental tangent stiffness tensor in three dimensions, $\mathbb{L}''$, may be assembled from E'' directly. The three-dimensional version of exponential algorithm is completed as follows

$$\Delta\boldsymbol{\varepsilon}'' = \frac{1}{v_{n+1/2}}\sum_{\mu=1}^{max}[1-\exp(-\Delta y_\mu)]\boldsymbol{\gamma}_\mu^n + \frac{q_4}{t_{n+1/2}}\Delta t\boldsymbol{\sigma} \tag{10}$$

$$\Delta\boldsymbol{\sigma} = \mathbb{L}'' : (\Delta\boldsymbol{\varepsilon} - \Delta\boldsymbol{\varepsilon}'') \tag{11}$$

$$\boldsymbol{\gamma}_\mu^{n+1} = \boldsymbol{\gamma}_\mu^n \exp(-\Delta y_\mu) + \frac{\lambda_\mu E''}{E_\mu}(\Delta\boldsymbol{\varepsilon} - \Delta\boldsymbol{\varepsilon}'') \tag{12}$$

where strain tensor $\Delta\boldsymbol{\varepsilon}''$ represents an inelastic strain increment tensor corresponding to the history of loading, and $\boldsymbol{\gamma}_\mu$ are the internal tensor variables (or partial strains) of the Kelvin units which must be updated after each time step.

4. Inverse analysis based on FFT

The inverse analysis aims at identification of C-S-H viscous properties from the level of cement paste. Numerical homogenization technique will be adopted, although the analytical correspondence principle would give the same result in this particular case. The numerical homogenization techniques rely on replacing the real microstructure by a representative volume element (RVE) of the material.

The RVE of cement paste is here approximated by Bentz's discrete hydration model CEMHYD3D from the National Institute of Standards and Technology (NIST), which has the resolution of 1 μm (Bentz, 2005).

The macroscopic viscoelastic response of RVE is obtained through the solution of the local problem, which consists of the equilibrium and constitutive equations complemented by boundary conditions. According to the exponential algorithm, two equations must be satisfied in each time step

$$\text{div}\,\Delta\boldsymbol{\sigma}(\mathbf{x}) = 0 \tag{13}$$

$$\Delta\boldsymbol{\sigma}(\mathbf{x}) = \mathbb{L}''(\mathbf{x}) : (\Delta\boldsymbol{\varepsilon}(\mathbf{x}) - \Delta\boldsymbol{\varepsilon}''(\mathbf{x})) = \mathbb{L}''(\mathbf{x}) : \Delta\boldsymbol{\varepsilon}(\mathbf{x}) - \Delta\boldsymbol{\lambda}(\mathbf{x}) \tag{14}$$

where inelastic strain increment $\Delta\boldsymbol{\varepsilon}''(\mathbf{x})$ is known from previous time step and can be replaced after multiplication with incremental stiffness tensor by eigenstress tensor $\Delta\boldsymbol{\lambda}(\mathbf{x})$. Assumed periodic boundary conditions are expressed here as decomposition of strain increments into average $\Delta\mathbf{E}$ and fluctuating part $\Delta\boldsymbol{\varepsilon}^*(\mathbf{x})$ over the RVE

$$\Delta\boldsymbol{\varepsilon}(\mathbf{x}) = \Delta\mathbf{E} + \Delta\boldsymbol{\varepsilon}^*(\mathbf{x}) \tag{15}$$

The usual way to solve Eqs. (13)-(15) is with FEM. Much faster solution is obtained by a recursive iterative procedure based on FFT (Moulinec and Suquet, 1994)

$$\Delta\boldsymbol{\varepsilon}^{k+1}(\boldsymbol{x}) = \Delta\boldsymbol{\varepsilon}^{k}(\boldsymbol{x}) - \boldsymbol{\Gamma}^0(\boldsymbol{x}) * \Delta\boldsymbol{\sigma}^k(\boldsymbol{x}) \tag{16}$$

where $\boldsymbol{\Gamma}^0(\boldsymbol{x})$ is the Green operator for arbitrary reference medium, which is explicit for linear elastic material. The convolution in Eq. (16) corresponds to multiplication in Fourier space.

4.1. Inverse creep analysis for two-year old cement paste

The creep of C-S-H will be obtained from the creep experiment of two year old hardened cement paste, with the water-to-cement ratio w/c = 0.5 (Beaudoin and Tamtsia, 2004). Two RVEs $10\times10\times10$ and $50\times50\times50$ μm of cement paste were reconstructed using CEMHYD3D model (Bentz, 2005).

Parameters $q_1, q_3, q_4, \lambda_0, n$ for the C-S-H creeping phase in model B3 are unknown. Parameter q_4 quantifies the irrecoverable flow strain which can be determined from unloading, independently of the other parameters of mature paste. Parameters q_1 and q_3 are in fact related through nano-indentation data, but are subjected to a certain range of uncertainty, limited by the creep attained.

Fig. 1 shows assigned viscoelastic behavior to C-S-H, in addition without flow term ($q_4 = 0$) to illustrate the effect of irrecoverable creep part. An arrow points to the time when indentation modulus is typically evaluated. The asymptotic compliance $q_1 = 0.0381$ GPa^{-1} is about 93 % of compliance from nano-indentation data $J(t + 0.00003\ \mathrm{day}, \mathrm{t}) = 0.0411$ GPa^{-1}. The elastic strain immediately after loading was not reported (Beaudoin and Tamtsia, 2004) and was assumed to be $800 \cdot 10^{-6}$ for static load 10.35 MPa at 0.01 day, Fig. 1. The choice has negligible effect on data after unloading when elastic strain is immediately recovered. The RVE size $50\times50\times50$ μm is obviously sufficient and the response does not differ from $10\times10\times10$ μm, Fig. 1. The concurrence is attributed to a weak heterogeneity due to well hydrated cement paste with the absence of large unhydrated cement grains.

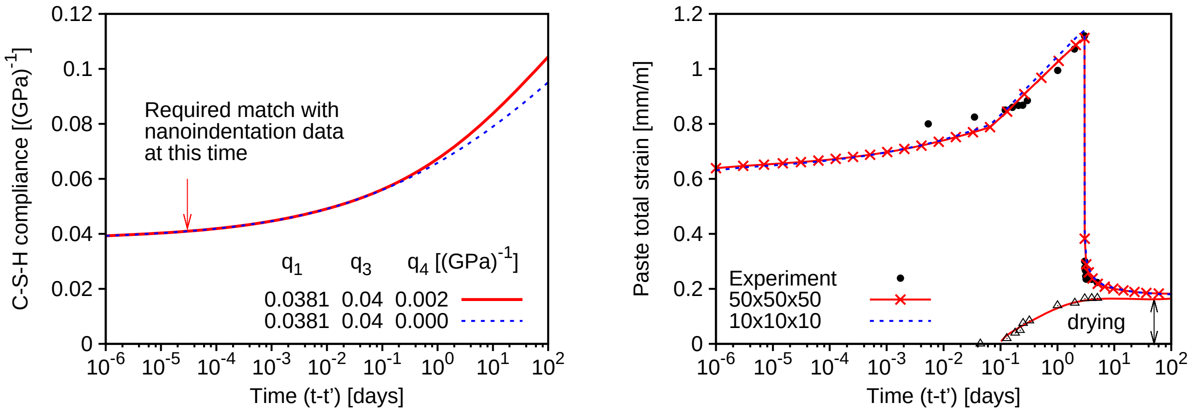

Figure 1: Compliance assigned to C-S-H (left), for comparison without the flow term ($q_4 = 0$). Measured total strain of two-year old cement paste (right) with simulated strain for two RVE sizes.

5. Correspondence principle

Numerical homogenization is generally a time-consuming process as opposed to analytical counterpart. The most simple viscoelastic analytical homogenization relies on correspondence principle proposed by Lee-Mandel via integral transform to Laplace or Laplace-Carson domain. The latter integral transformation is defined as

$$F(s) = s \int_0^\infty f(t) e^{-st} \mathrm{d}t \tag{17}$$

$$f(t) = \frac{1}{2\pi i} \int \frac{F(s)}{s} e^{st} \mathrm{d}s \tag{18}$$

where s is a variable in the frequency domain. Once a local constitutive law per phase r is formulated as a convolution, the integral transformation yields the same formalism as in linear elasticity (Matzenmiller and Gerlach, 2001)

$$\varepsilon_r(t) = \int_{t'=0}^{t'=t} \mathbb{J}_r(t-t') : \frac{\partial \sigma_r(t')}{\partial t'} \mathrm{d}t' = \mathbb{J}_r * \sigma_r \tag{19}$$

The transformation relies on compliance tensor in the form $\mathbb{J}_r(t-t')$ which is valid for non-aging, non-solidifying materials. Certain extension to aging materials is possible through time-shifting procedure, when compliance tensor has the form $\mathbb{J}_r(\xi(t) - \xi(t'))$. This approach was used for creep analysis of cement paste with partial success but the creep response at different loading times t' was unsuccessful (Grasley and Lange, 2007).

The major challenge is the inverse transform from Laplace-Carson domain to the real time domain, Eq. (18). The collocation method implemented in Pierard (2006) is generally unstable, strongly depending on the selection of retardation times and collocation points. Stehfest (1970) proposed almost flawless numerical algorithm, based on the expectation of $f(t)$ in the form of probability density.

The comparison between Laplace-Carson and FFT-based homogenization approach was carried out on two year old cement paste (Beaudoin and Tamtsia, 2004). Input data relied on volume fractions of chemical phases and viscous parameters of previously calibrated B3 model for C-S-H. Only C-S-H was allowed to creep, other chemical phases behaved elastically. The results are plotted in Fig. 3 for $q_1 = 0.0381$ 1/GPa, $q_2 = q_4 = 0$, $q_3 = 0.040$ 1/GPa, $n = 0.25$, $m = 0.5$. The difference among numerical and analytical approaches is not significant, although the paste porosity is 0.202.

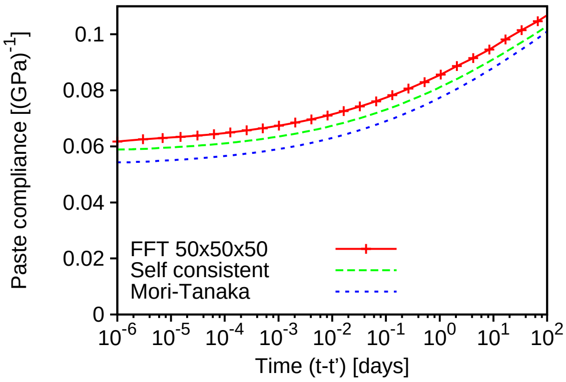

Figure 2: Comparison of FFT-based and analytical methods for two year old cement paste.

6. Identification of C-S-H creep

In the preceding sections, analytical homogenization methods were found to provide similar results as the numerical solution. Now, the challenge lies in transferring viscoelasticity to the scale of concrete, distinguishing the role of aggregates. From an engineering point of view, creep is quite significant for long-time durations. The combination of analytical homogenization in Laplace-Carson domain and calibrated C-S-H creep law should yield approximate solution.

Brooks (2005) measured basic creep and autogeneous shrinkage of concrete with different w/cs or aggregate types for the duration of 30 years. Compressive uniaxial stress, with the magnitude 0.3 of 14 day compressive strength, was imposed on 14-day old concrete. Autogeneous shrinkage was subtracted from total strain. Only basic creep will be simulated in order to circumvent differential shrinkage caused by drying.

The aim was to maintain as many parameters as possible from the previous simulation, assuming again $q_1 = 0.0381$ 1/GPa, $q_2 = 0$, $q_3 = 0.040$ 1/GPa, $n = 0.25$, $m = 0.5$. Apparently, irreversible creep caused by q_4 has the most influencing effect on long-term properties. This was confirmed again by Brooks (2005) who found reversible part under 30-year load duration as low as 5 - 14 %, determined at six months after unloading.

Fig. 3 shows the results from uniaxial creep at four scales. In order to match concrete creep, the fit yielded $q_4 = 0.018$ 1/GPa. Young's modulus of coarse and fine aggregates was assumed as 60 GPa, Poisson's ratio as 0.2. Air introduced in concrete was considered to act at the level of cement paste. Mori-Tanaka method was used for the transition among all three scales.

It must be emphasized that the identification of creep parameters is not unique and proposed approach has several limitations. The most neglected fact is the aging of C-S-H which needs to be disregarded in the present analytical approach. If not so, compliance expressed in the general form $\mathbb{J}_r(t, t')$ can not be used directly in the convolution in Eq. (19). Certain remedy is the time-shifting approach, which would yield compliance mapping in the form $\mathbb{J}_r(\xi(t) - \xi(t'))$. Therefore, irreversible creep is treated only approximately. The onset of loading, originally

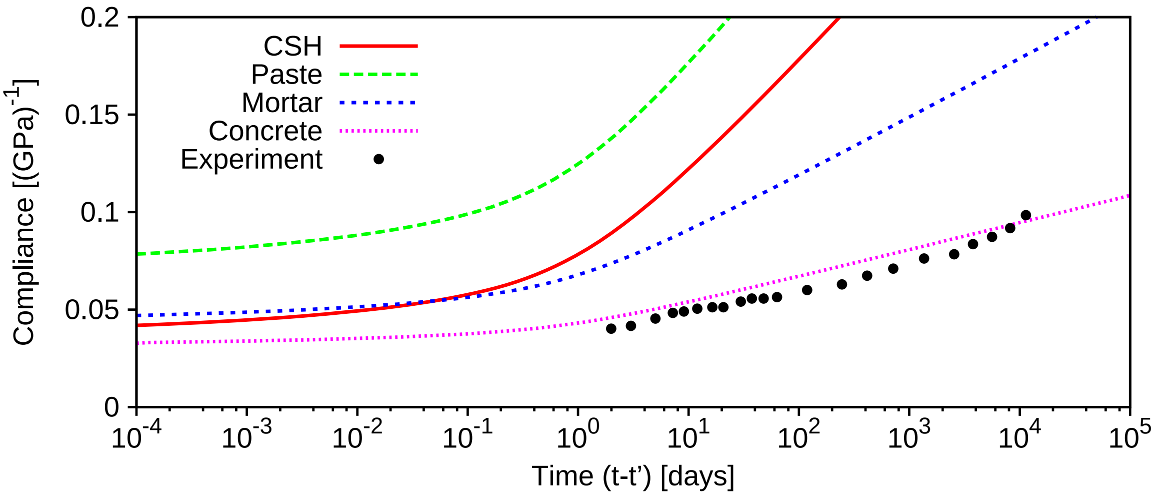

Figure 3: Creep at four scales as predicted by analytical homogenization methods.

proposed in B3 model, is set to one day

$$q_4 \ln\left(\frac{t}{t'}\right) = q_4 \ln\left(1 + \frac{t-t'}{t'}\right) \approx q_4 \ln\left(1 + \frac{t-t'}{1}\right) \tag{20}$$

From a material point of view, fine and coarse aggregates are surrounded by iterfacial transition zone. In this zone, higher w/c exist due to increased amount of porosity. Neglecting the transition zone localizes all deformation into cement paste. Viscous properties of cement paste represent the upper bound.

7. Conclusion

Presented multiscale viscoelastic approach shed the light on the deformation during basic creep at three levels; cement paste, mortar and concrete. Basic assumption is creep localization in C-S-H phase found on the scale of nanometers. Proposed methodology led to the identification of C-S-H constitutive law. Although the extension to the level of concrete seems applicable, several phenomena are not treated correctly and need refinement. Validation shows correct identification of elastic properties and homogenization methods; short-term creep corresponds to elastic solution.

8. Acknowledgment

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