ENGINEERING MECHANICS 2009
National Conference with International Participation
Svratka, Czech Republic, May 11 – 14, 2009

pp. 1267–1272
Paper **#254**

OPTIMALIZATION OF THE ELBOW

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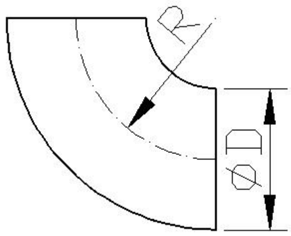
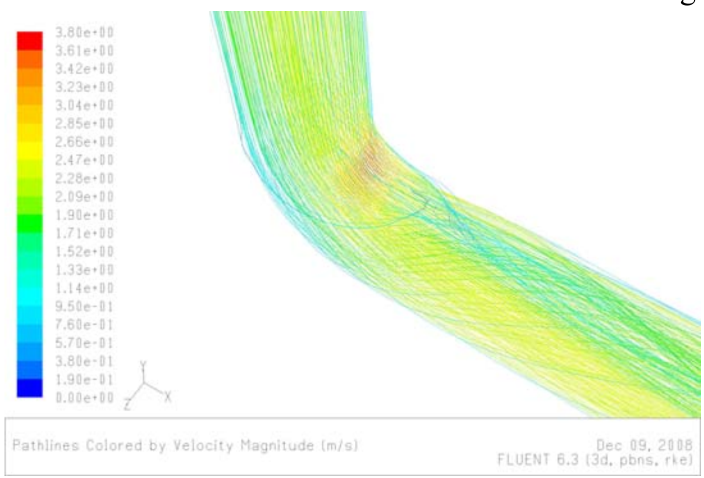
Summary: *This paper is concerning with the shape optimization of the elbow in order to minimize energy losses caused by the flowing of fluid. The flow field is solved by CFD. The Nelder-Mead algorithm is used for optimization.*

1. Introduction

Curved pipeline is a significant part of any pipeline network. The minimal radius of curvature when flowing fluid fluently follows the curvature of pipe is equal to two diameters, it means that R/D = 2.



With every change of the direction of the flow is friction losses increased by the bend losses. The fluid flowing through the bend is influenced by the centrifugal force which is compensated by the radial pressure gradient. The particles of fluid with higher momentum that are situated close to the axis of symmetry from where they are transferred from inner to outer side of the curvature. In order to preserve mass flow law, the particles with the lower momentum are transferred from boundary layer in opposite direction. This transfer of particles from boundary layer to axis of symmetry causes the cross section flow with two counter-moving eddies. The cross section flow is responsible for energy exchange between boundary layer and core of the stream, which further increases the energy losses of the elbow.



Streamlines of curved pipes are variously deformed spirals.

Energy losses caused by the elbow are not realized only in the elbow itself, but mostly in direct pipe behind the elbow (more than 50 diameter behind the elbow), where the spiral movement is subsequently suppressed.

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Mark	Title	Unit
c_s	Mean velocity	[m/s]
p	Pressure	[Pa]
ρ	Density	[kg/m3]
α	Corriolis number	[-]
Re	Reynolds Number	[-]
ξ	Loss coefficient	[-]
Q(r,s)	Point of Bézier surface for parameters r and s	[-]

2. Optimization of the Elbow

Our goal is to minimize energy losses caused by the flowing fluid by means of suitable shape change. The shape optimization is consisted of several steps, which are repeated until preset condition is fulfilled.

First step is to define boundary conditions and criteria function. The second step is parametric description of geometry. Then we must create computational net and solve the flow field. After that we must evaluate the criteria function and decide if we found minimum of criteria function or not. If solution is not suitable, then we obtain parameters from optimization algorithm for the new shape and begin over again. All these steps are made automatically until the minimum of criteria function is found.

3. Boundary conditions

The inflow velocity to domain was set to 2 m/s. Diameter of pipe is 50 mm. The temperature of water is 20 °C. The Reynolds-number corresponding to these conditions is 97 433 and the R/D is 0.8.

4. Criteria function

As criteria function is used loss coefficient for the elbow derived from Bernoulli equations for real fluid.

$$\xi = \frac{2}{c_{(2S)}^2}\left(\left(\frac{\alpha_{(1)} \cdot c_{(1S)}^2 - \alpha_{(2)} \cdot c_{(2S)}^2}{2}\right) + \left(\frac{p_{(1)} - p_{(2)}}{\rho}\right)\right)$$

Points 1 and 2 should be in the places that are not influenced by the elbow. This condition will not be fully fulfilled because of reduced length of inflow and outflow pipe for overall reduction of computational time. Mean velocity and pressure are average values on cross-section at each point.

5. Optimization Algorithm

As optimization algorithm is used Nelder-Mead method. This method uses the concept of simplex (in R^2 is simplex triangle in R^3 is simplex tetrahedron and so on) which has N+1 vertices in N dimension. The idea of this method is replacing the vertices with the worst value of criteria function with new one.

Disadvantage of Nelder-Mead method is that if searched function contains more then one local extreme, the method can get stuck and global extreme cannot be found at all.

In general the big problem of the optimization is that number of calculations needed for finding the solution is adequate dimension of optimization as shown at fig.1 And because obtaining one value of criteria function is equal to finding of converged solution of flow field, which is very time-consuming task, we try to keep number of changed parameters as low as possible. In our case each control point of surface has generally three changeable parameters, that means that we have three axes in which control point can generally move.

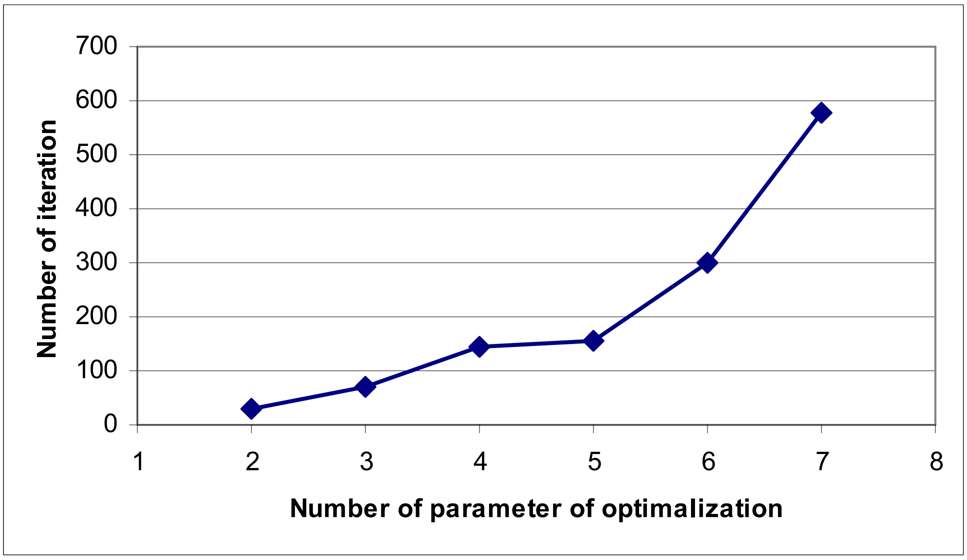
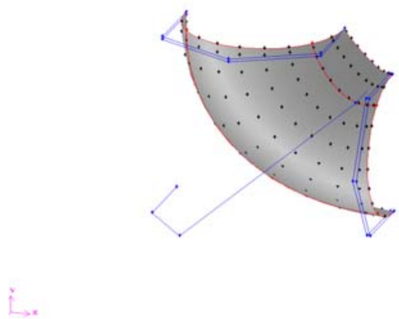


Fig. 1 Dependence of parameters of optimization and Number of iteration needed for finding the solution. The graph is built for Nelder Mead algorithm (parameter of reflection = 1.2, parameter of expansion = 2 and parameter contraction = 0.5) for the random begging simplex. Solved function is $f(x) = \Sigma x_i^2$

6. Parametric description of geometry



In order to obtain the whole shape of elbow based on the few changed parameters we use parametric description the geometry. The parametric description is made by means of Bézier surfaces. Parameters, which are changed by the optimization algorithm, are control points of surface. Part of the computational net described by the Bézier surface refers only the elbow itself. Supply and outlet pipe do not change. Bezier surfaces are described by equation

$$Q(r,s) = \sum_{i=0}^{m} \sum_{j=0}^{n} P_{ij} B_i^m(r) B_j^n(s)$$

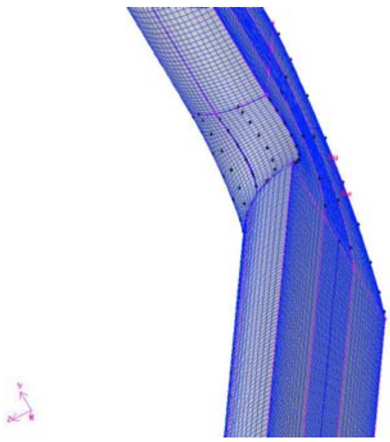
$$B_i^n(t) = \binom{n}{i} \cdot t^i (1-t)^{n-i}$$

where P_{ij} is the matrix of control points. B_i is Bernstain polynomials of "n" and "m" degree. "r" and "s" are parameters of surface and they can changing in interval <0,1>. Bezier curves have several characteristics that are used in creation of computational net. Mainly, the Bézier surface passes through beginning and ending control points. Furthermore the Bézier surface transforms in the same way as its control points under all linear transformations and translations. The parametric description of the elbow is shown at the picture. Where blue lines are control polygons (control points match together to create logical group). Duplicating of control polygon on the entrance and exit is due to smooth passage between Bézier surface and rest of computational domain.

In this case of parameterization is governing only middle polygon. Each point of the polygon has three directions in which he can move (three degrees of freedom). Because we are solving only the half of computational net and also we desire to obtain smooth cross-section, so that the pair of control points of governing polygon, which are situated on the symmetry plane, have to have same "z" coordinates. In order to further lowering of parameters of optimization, the "z" coordinates of the center point of polygon and as well as the rest of them are functionally dependent. When we apply all above-mentioned reduction of degrees of freedom, we obtain 5 parameters for which we find the solution.

7. Creation of computational net

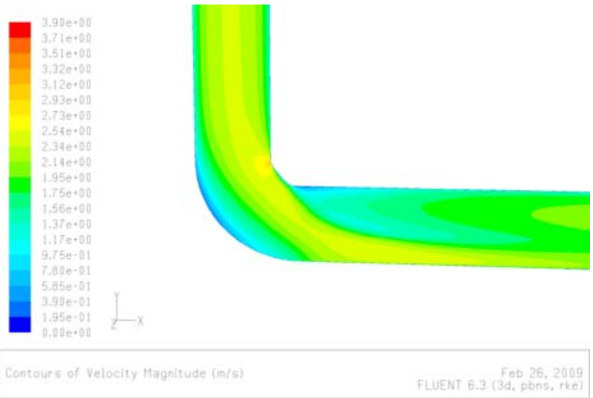
When we have parametrically described the geometry we can create the computational net for CFD solver. The Computational net is created automatically by means of journal files. Journal files are the lists of commands for preprocessor (Gambit) which are realized step by step. The Journal file itself is created in two steps. Firstly there is written part that is unique for each geometry and after that there is written common part. In order to lower the computational time needed for solving the flow field, several simplification are applied. The biggest simplification is that we apply symmetry hence we solve the flow field only in the half of the computational net. The second simplification is reducing of the length of inflow and outflow pipe. Altogether have computational net 303000 of cells, which is completely mapped.



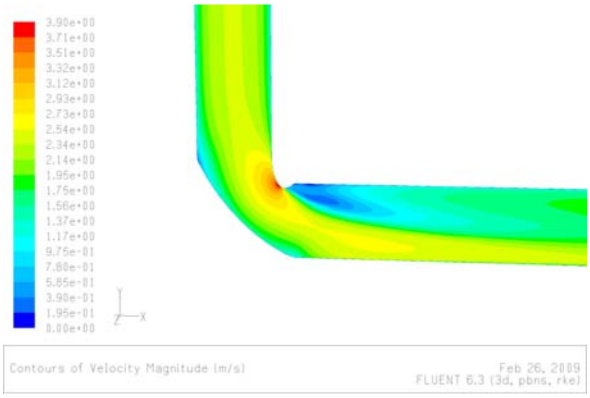
8. Conclusion

Overall improvement of criteria function was 4%. Most probable reason is bad parameterization of the geometry. On the next four pictures there are shown flow fields of four variations of shape, including the best solution.

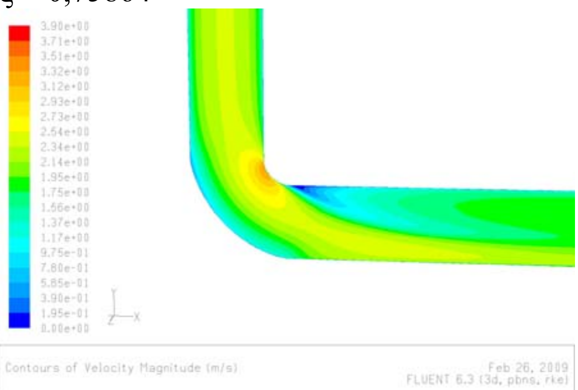
If we look at the all four pictures we can see that on the place behind the bend there is flow separation that cannot be fixed with current parameterization of geometry. Therefore a new parameterization of geometry is needed for further lowering of energy loses. The current parameterization is concerning only the elbow itself. But as we can see, the parametric part of geometry should interfere further to outflow area in order to suppress the flow separation behind the bend.



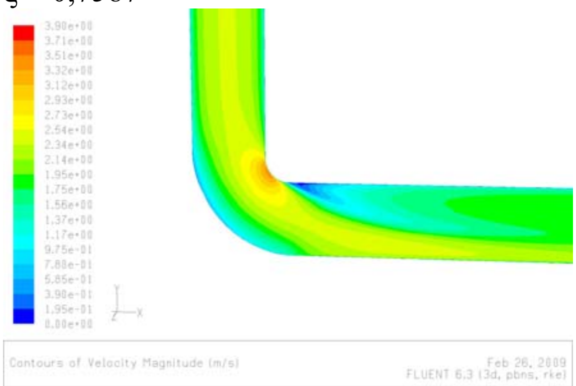
ξ = 0,75864



ξ = 0,7587



ξ = 0,65411



ξ = 0,6609

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