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GLOBAL FACTORS FOR RELIABILITY VERIFICATION OF REINFORCED CONCRETE STRUCTURES

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Summary: *The concept of global resistance and load factors provides a rational approach to reliability verification of reinforced concrete structures that require non-linear or dynamic analysis. In such cases the mean values of basic variables need to be used in the analysis to determine adequate structural response. Reliability of these structures can be then conveniently verified using results of the analysis and the global resistance and load factors. The submitted proposal for the specification of global factors is based on probabilistic analyses of selected structural members (slab, beam and column). It appears that the global resistance factors are dependent on the type of structural members and on reinforcement ratio. The global load factors are strongly dependent on load ratio of variable and total actions and type of action. The total reliability factors can be further defined as products of the global resistance and load factors.*

1. Introduction

Available European documents for the design of structures (Eurocodes) EN 1990 (2002) and EN 1992-1-1 (2004) are primarily focused on structures exposed to static loads and the linear dependence between load effect and deformations. It is then possible to analyse and verify structural reliability using the design values of basic variables. For structures with non-linear behaviour the Eurocodes provide simplified rules and recommendations only. One of the general rules introduced in the basic document EN 1990 (2002) suggests an alternative procedure for determining design resistance using the characteristic value (determined from characteristic values of basic variables) and an appropriate partial factor called here the global resistance factor. However, this approach may not be satisfactory in case of dynamic actions or significantly non-linear behaviour of structures when the mean values of basic variables need to be considered as indicated by Gulvanessian et al. (2002) and Cervenka (2007).

The submitted study, which is an extension of the previous study by Holicky (2007), attempts to derive the global resistance factors taking into account the general rules of EN 1990 (2002) and relevant findings of the recent studies by Holicky and Retief (2005) and Gulvanessian and Holicky (2005). Similarly as in the previous investigations, the submitted study is based on probabilistic analysis of selected reinforced concrete members (slab, beam and column). The probabilistic approach guarantees a compliance of derived global factors with reliability requirements of the Eurocodes EN 1990 (2002) and EN 1992-1-1 (2004).

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The global resistance factors are derived for two fundamental cases when the design resistance is derived from:

- the characteristic value of resistance,
- the mean value of resistance.

The first approach may be useful for verification of ultimate limit states based on previous analysis of serviceability limit states while the second approach may be applied when non-linear or dynamic analysis is required.

2. Design value of resistance

EN 1990 (2002) gives a recommendation for determining the design value r_{d} of a resistance R. Based on the FORM reliability procedure, the following relationship is provided:

$$\mathrm{Prob}(R \le r_{\mathrm{d}}) = \Phi(-\alpha_R\beta) \quad (1)$$

Here Φ denotes the cumulative distribution function of standardised normal distribution; α_R resistance sensitivity factor for which EN 1990 (2002) allows an approximation $\alpha_R = 0{,}8$; and β is the reliability index that is in common cases of structures with the design lifetime 50 years considered as $\beta = 3{,}8$. For $\alpha_R = 0{,}8$ and $\beta = 3{,}8$, the design resistance r_{d} is a fractile of R corresponding to the probability:

$$\Phi(-\alpha_R\beta) \approx \Phi(-3{,}04) = 0{,}00118 \quad (2)$$

It is generally expected, cf. Gulvanessian et al. (2002), that the resistance of reinforced concrete members may be described by two-parameter lognormal distribution with the lower bound at the origin. The design value r_{d} can be then approximated as:

$$r_{\mathrm{d}} = \mu_R \exp(-\alpha_R\beta\, V_R) \quad (3)$$

where μ_R denotes the mean and $V_R = \sigma_R/\mu_R$ is the coefficient of variation of resistance. The characteristic value r_{k} is the fractile of resistance corresponding to the probability 0,05:

$$r_{\mathrm{k}} = \mu_R \exp(-1{,}65\ V_R) \quad (4)$$

Here the factor -1,65 is the fractile of standardised normal distribution corresponding to the probability 0,05. Equations (3) and (4) provide a good approximation when the coefficient of variation V_R is small, say $V_R < 0{,}25$.

3. Global resistance factor

The design resistance r_{d} may be derived from the characteristic value r_{k} or from the mean μ_R using formulae:

$$r_{\mathrm{d}} = r_{\mathrm{k}} / \gamma_R \quad (5)$$

$$r_{\mathrm{d}} = \mu_R / \gamma_R^{*} \quad (6)$$

where the global resistance factors γ_R and γ_R^{*} follow from (3), (4), (5) and (6):

$$\gamma_R = \exp(-1{,}645 V_R) / \exp(-\alpha_R\beta\, V_R)$$
$$= \exp[(\alpha_R\beta - 1{,}65)\ V_R] \approx \exp(1{,}39\ V_R) \quad (7)$$

$$\gamma_R^{*} = 1 / \exp(-\alpha_R\beta\, V_R) = \exp(\alpha_R\beta\, V_R) \approx \exp(3{,}04 V_R) \quad (8)$$

The global factors are obviously dependent on the coefficient of variation V_R that should be somehow estimated. However, the coefficient V_R depends on the type of member and its reinforcement ratio. In common cases (slab, beam and column) the coefficient may be expected within the range from 0,1 to 0,2. In addition the characteristic and the mean value r_k and μ_R should be assessed. Approximately they may be estimated from the characteristic and mean values of the basic variables $\mathbf{X}$ describing the resistance $R = \mathrm{R}(\mathbf{X})$, thus $r_k \approx \mathrm{R}(\mathbf{X}_k)$ and $\mu_R \approx \mathrm{R}(\mu_{\mathbf{X}})$ (see an example in the following section). Figure 1 shows variation of the global resistance factors γ_R a γ_R^* with the coefficient of variation V_R.

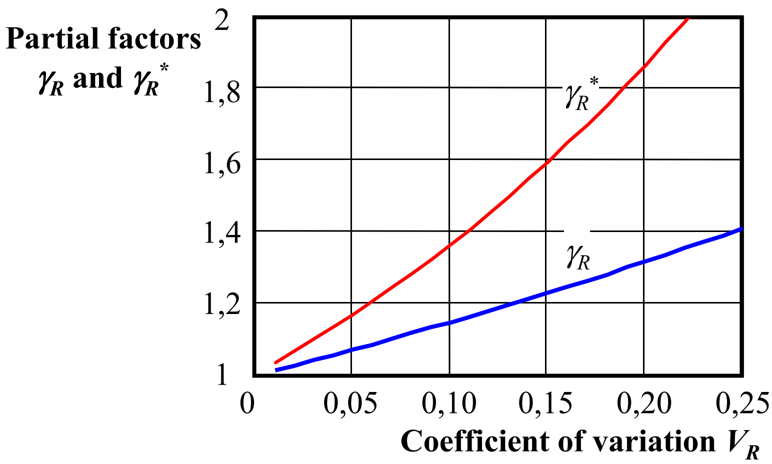

Figure 1: Variation of the global resistance factors γ_R and γ_R* with the coefficient V_R

It follows from Figure 1 that for V_R within the considered range the global factor γ_R varies from 1,15 to 1,32 and the factor γ_R^* from 1,36 to 1,86. As the first approximation the values γ_R = 1,25 and γ_R^* = 1,60 corresponding to the coefficient of variation V_R = 0,15 may be used.

Significance of the coefficient of variation is evident from Figure 2 where the lognormal distribution with the lower bound at the origin, the mean μ_R = 1 and the coefficient of variation V_R = 0,1 and 0,2 is assumed. Figure 2 shows the corresponding characteristic and design values r_k and r_d. It should be, however, noted that the assumed lognormal distribution may be just an approximation of the actual distribution.

Figures 1 and 2 clearly indicate that the coefficient of variation V_R may significantly affect estimation of the characteristic and design resistance. Indicative values of V_R for selected reinforced concrete members (slab, beam and column) are shown in Figure3.

Variation of the coefficient V_R with the reinforcement ratio ρ is derived using probabilistic methods for a slab of the thickness 0,25 m, a beam of the cross-section dimensions 0,30 × 0,60 m, and a centrically loaded short column of the cross-section dimensions 0,30 × 0,30 m, concrete C20/25 and steel S500. In addition model uncertainties (the coefficient of variation 0,1) are included to cover simplifications related to considered models for resistance and probabilistic modelling.

It follows from Figure 3 that with the increasing reinforcement ratio ρ the coefficient of variation V_R decreases for the centrically loaded short column and increases in the case of the slab or beam.

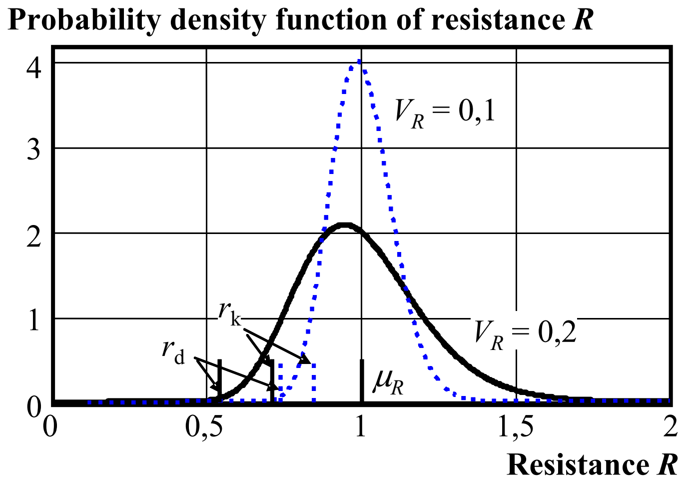

Figure 2: The characteristic and design values r_k and r_d assuming a lognormal distribution of resistance and the coefficient of variation V_R = 0,1 and 0,2

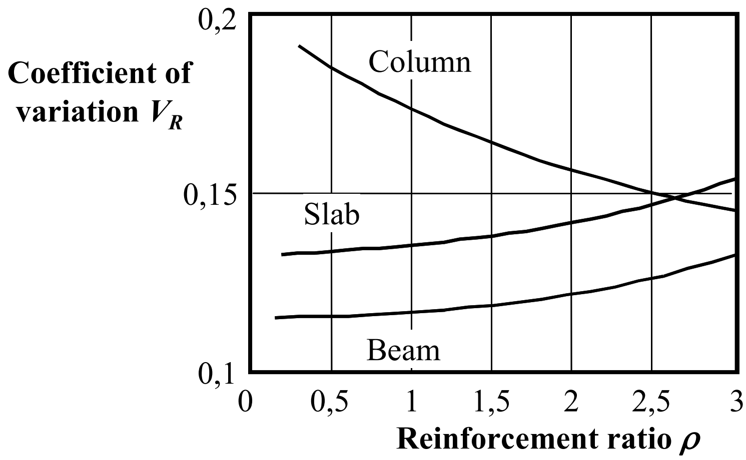

Figure 3: Variation of the coefficient V_R with the reinforcement ratio ρ

4. Numerical example - global resistance factors for a slab, beam and column

In a numerical example the global resistance factor for a reinforced concrete slab is initially estimated. It is assumed that the resistance may be expressed as follows:

$$R = \mathrm{R}(\mathbf{X}) = K_R\, A_s f_y[h - a - 0{,}5\, A_s f_y / (b\alpha_{cc} f_c)] \qquad (9)$$

Basic variables **X** include the coefficient of model uncertainty K_R, reinforcement area A_s (the mean corresponds to the reinforcement ratio 0,01), yield strength f_y, slab depth h, the distance of reinforcement from the surface a, slab width b (deterministic quantity 1 m), the coefficient of concrete strength α_{cc} (deterministic α_{cc} = 1) and concrete strength f_c. Probabilistic models for the basic variables listed in Table 1 are accepted from the working materials of JCSS (2006) and from previous studies by Holicky and Holicka (2004), Holicky and Markova (2003) and Holicky and Retief (2005).

Table 1: Probabilistic models of basic variables

Variable	Symbol	Basic variable	Distrib.	Unit	Mean	St. dev.	Char. v.	Des. v.
Material properties	A_s	Reinforcement area	LN	m^2	0,0022	0,00011	0,0022	0,0022
	f_c	Concrete strength	LN	MPa	30	5	20	13,3
	f_y	Yield strength	LN	MPa	560	30	500	435
Geometric data	h	Slab height	N	m	0,25	0,008	0,25	0,25
	a	Distance	GAM	m	0,03	0,006	0,03	0,03
Model unc.	K_R	Uncertainty	N	-	1,0	0,10	1,0	1,0

Partial factors for concrete and steel are considered by the values recommended in EN 1992-1-1 (2004), γ_c = 1,5 and γ_s = 1,15. It follows from Figure 3 that the coefficient of variation V_R is about 0,13. The mean, characteristic value and the design value may be estimated from equation (9) and appropriate data in Table 1 as follows:

$$\mu_R \approx 0{,}245 \text{ kNm},\ r_k \approx 0{,}212 \text{ kNm},\ r_d \approx 0{,}176 \text{ kNm} \qquad (10)$$

The global resistance factors γ_R and γ_R^* follow from (7) a (8) or from Figure 1 as:

$$\gamma_R = \exp(1{,}39V_R) \approx 1{,}20 \qquad (11)$$

$$\gamma_R^* = \exp(3{,}04V_R) \approx 1{,}48 \qquad (12)$$

Using the global resistance factors it follows from (5) and (6) that:

$$r_d = r_k / \gamma_R = 0{,}212 / 1{,}20 = 0{,}177 \text{ kNm} \qquad (13)$$

$$r_d = \mu_R / \gamma_R^* = 0{,}245 / 1{,}48 = 0{,}166 \text{ kNm} \qquad (14)$$

It appears that r_d determined from r_k using the global factor γ_R is very close to the value 0,176 kNm estimated from (9) while r_d determined from μ_R using the global factor γ_R^* is rather conservative. Nevertheless, the differences up to 6 % are small.

Using a deterministic approach, the global factors γ_R and γ_R^* may be estimated from (5) and (6):

$$\gamma_R = r_k / r_d \approx 1{,}20 \qquad (15)$$

$$\gamma_R^* = \mu_R / r_d \approx 1{,}39 \qquad (16)$$

These estimates are close to those based on the assumption of the two-parameter lognormal distribution of resistance, given in (11) and (12).

The two-parameter lognormal distribution assumed above may not always be an appropriate approximation for the resistance of reinforced concrete members. In such a case a more general three-parameter lognormal distribution, when the skewness is an independent parameter, may be a more suitable theoretical model and is assumed hereafter. The skewness of the distribution is obtained from the input data given in Table 1.

Based on the three-parameter lognormal distribution, the probability density function of resistance and estimated characteristic and design values r_k and r_d determined using equation (9) and data from Table 1 are indicated in Figure 4. It should be noted that the coefficient of model uncertainty K_R (see Table 1) is estimated only. Available information including the working materials of JCSS (2006) is incomplete and inconclusive.

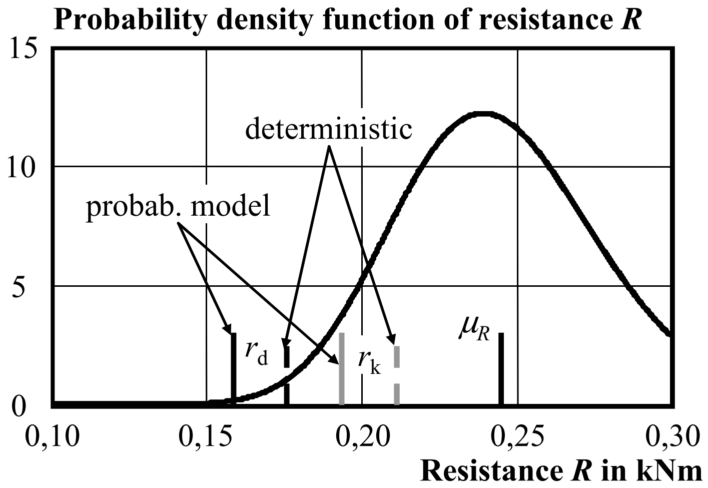

Figure 4: Probability density function of the resistance of the slab

It appears that the characteristic and design values derived from the probabilistic model are rather lower than those given in (10) (differences up to 10 %). The global factors $\gamma_R \approx 1,22$ and $\gamma_R^* \approx 1,52$ are slightly greater than those corresponding to the two-parameter lognormal distribution given in (11). This increase is due to a lower skewness of the derived three-parameter distribution than the skewness corresponding to the two-parameter lognormal distribution. For all the three alternatives - equations (11) and (15) and the probabilistic model, the global factors γ_R are similar while the global factors γ_R^* vary within the range from 1.38 up to 1.52.

Figure 5 shows the global resistance factors γ_R and γ_R^* derived for slab, beam and column, assuming the three-parameter lognormal distribution of the resistance. It confirms the previous findings published by Holicky and Holicka (2004) and Holicky (2007) that the global resistance factors γ_R and γ_R^* are dependent on both, the type of structural member and the reinforcement ratio ρ. It appears that a simple approximation valid in general for all members and reinforcement ratios may be incorrect. The factors $\gamma_R \approx 1,3$ and $\gamma_R^* \approx 1,6$ should be considered as informative estimates only. Note that the global factors derived by the deterministic approach (15) and (16) may be unconservative.

5. Global load factor

An anticipated extension of the concept of global resistance factor is the idea of global load factor and the total factor of safety (reliability). Similarly to the case of resistance, EN 1990 (2002) gives a recommendation for determining the design value e_d of a load effect E. The following probabilistic relationship is provided:

$$\text{Prob}(E > e_d) = \Phi\,(\alpha_E \beta) \tag{17}$$

Here α_E denotes the resistance sensitivity factor for which EN 1990 (2002) allows approximation $\alpha_E = -0,7$ and $\beta = 3,8$ is the reliability index. For $\alpha_E = -0,7$ and $\beta = 3,8$, the design resistance e_d is a fractile of E corresponding to the probability:

$$\Phi(-\alpha_E \beta) \approx \Phi\,(-2,77) = 0,00309 \tag{18}$$

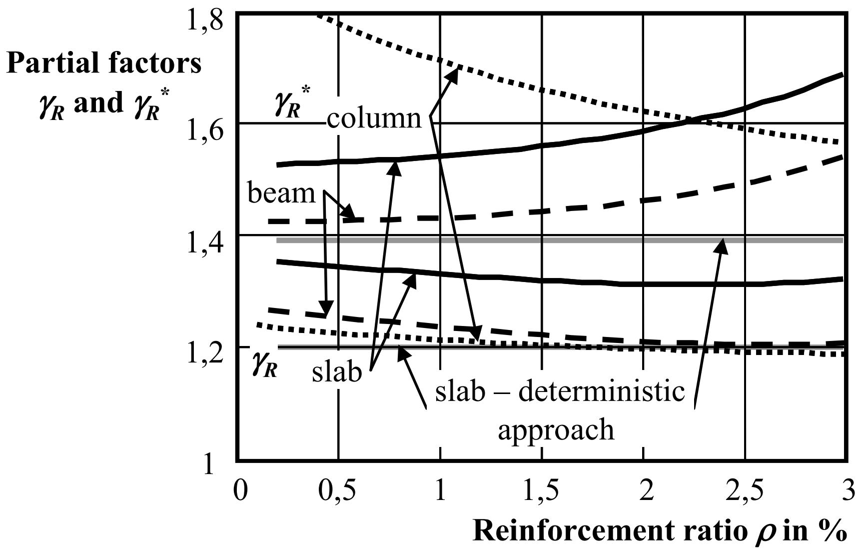

Figure 5. Variation of the global resistance factors γ_R and γ_R^* with the reinforcement ratio ρ

The permanent load is usually described by a normal distribution and life-time extremes of variable action often by Gumbel distribution as indicated e.g. by Gulvanessian et al. (2002) and JCSS (2006). The combination of actions (the total load effect) is considered here as a simple sum of a single permanent load G and a single variable load Q. The total load effect is approximately described by a three-parameter lognormal distribution. It appears that the resulting coefficient of variation and skewness of the load effect E are strongly dependent on the load ratio χ of characteristic variable and the total actions:

$$\chi = q_k /(q_k + g_k) \tag{19}$$

where g_k and q_k denote the characteristic values of G and Q. The ratio χ may vary within the interval from nearly 0 (underground structures, foundations) up to nearly 1 (local effects on bridges, crane girders). For common buildings, the realistic range may be from 0 up to 0.8.

As an example assume a permanent action G described by the normal distribution having the mean $\mu_G = g_k$ and the coefficient of variation $V_G = 0,1$, and 50-year maxima of a variable action described by Gumbel distribution. For the variable action, three alternatives (with different parameters of the Gumbel distribution) are further considered:

- the mean $\mu_Q = 0,6q_k$ and the coefficient of variation $V_Q = 0,35$: this model may well describe 50-year maxima of imposed loads,
- $\mu_W = 0,7w_k$ and $V_W = 0,35$: wind action,
- $\mu_S = s_k$ and $V_S = 0,22$: snow load,

The models for the variable actions are accepted from Gulvanessian and Holicky (2005) and Holicky et al. (2007). In addition, the coefficient of model uncertainty is considered by a normal distribution having the mean 1 and the coefficient of variation 0,05. These characteristics are listed in Table 2.

The global load factors γ_E and γ_E^* of the load effect E related to the characteristic value e_k and the mean μ_E are defined as:

$$\gamma_E = e_d / e_k \tag{20}$$

Table 2: Probabilistic models for action effects

Variable	Symbol	Basic variable	Distrib.	Mean	St. dev.	Char. v.	Des. v.
Actions	G	Permanent load	N	g_k	$0{,}1g_k$	g_k	$1{,}35g_k$
	Q	Imposed load	GUM	$0{,}6q_k$	$0{,}35q_k$	q_k	$1{,}5q_k$
	W	Wind action	GUM	$0{,}7w_k$	$0{,}35w_k$	w_k	$1{,}5w_k$
	S	Snow load	GUM	s_k	$0{,}22s_k$	s_k	$1{,}5s_k$
Model unc.	K_E	Uncertainty in load effect	N	1	0,05	1	1

$$\gamma_E^* = e_d / \mu_E \tag{21}$$

where the design value e_d of the load effect is defined in (17) and (18).

Figure 6 shows the variation of the global load effect factors γ_E and γ_E^* and variation of the coefficient of variation V_E with the load ratio χ (skewness, not indicated in Figure 6, varies within the range from 0 to 1,1). Obviously, both the global factors γ_E and γ_E^* are significantly dependent on the load ratio χ and type of load. Considering the practical range of the load ratio from 0 up to 0,8, it appears that:

- For the imposed load, the global factor γ_E varies within the interval from 1,2 to 1,35 (the value 1,3 might be an informative approximation) and the factor γ_E^* varies within the interval from 1,3 to 2 (for the load ratio χ around 0,5, the indicative value 1,5 may be considered),
- For the wind action, the factor γ_E varies within the interval from 1,2 to 1,5 (for χ around 0,5 $\gamma_E \approx 1{,}35$) and the factor γ_E^* within the interval from 1,3 to 2 (for χ around 0,5 $\gamma_E^* \approx 1{,}6$),
- For the snow load, the global factors γ_E and γ_E^* vary within the interval from 1,3 to 1,7 (for χ around 0,5 $\gamma_E = \gamma_E^* \approx 1{,}45$).

Note that for the snow load, the global factor γ_E^* equals to the factor γ_E since $\mu_S = s_k$.

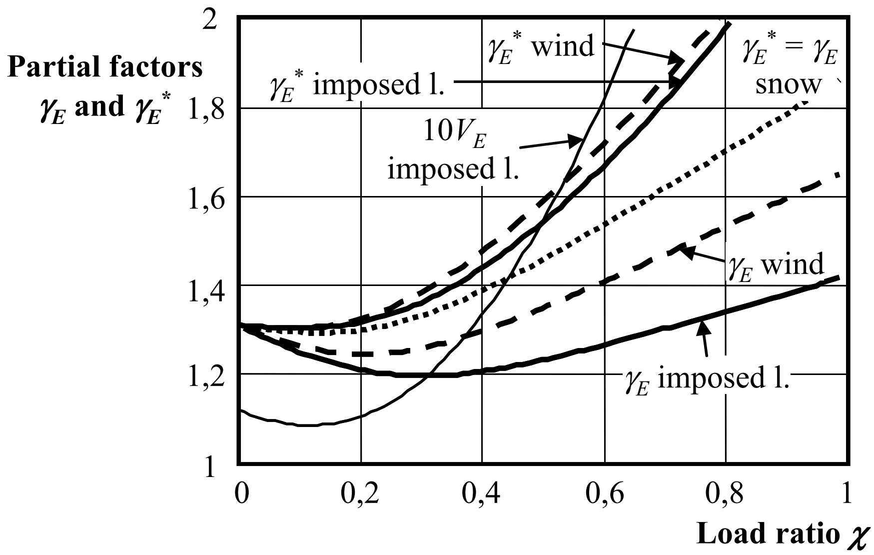

Figure 6: Variation of the global factors γ_E and γ^*_E and the coefficient of variation V_E with the load ratio χ for the different types of loads

6. Total factor of safety

The fundamental reliability condition $e_d < r_d$ may be expressed in terms of the characteristic values e_k and r_k using the factors γ_E and γ_R as follows:

$$r_k > \gamma_E \gamma_R e_k \tag{22}$$

Similarly in terms of the mean values μ_R and μ_E using the factors γ_E^* and γ_R^*, the fundamental inequality may be expressed as:

$$\mu_R > \gamma_E^* \gamma_R^* \mu_E \tag{23}$$

The products $\gamma_E\gamma_R$ and $\gamma_E^*\gamma_R^* < \gamma_E \gamma_R$, corresponding to the total factor of safety (reliability), may be assessed using both the graphs in Figures 5 and 6. However, it follows that the total factor of safety is dependent on the type of a structural member, reinforcement ratio ρ, load ratio χ and type of load. Simple approximations are difficult.

7. Concluding remarks

The resistance of reinforced concrete members can be well approximated by a two-parameter lognormal distribution having the lower bound at the origin. The coefficient of variation of the reinforced concrete members investigated here seems to be within an interval from 0,12 to 0,18. The coefficient of variation is, however, dependent on the reinforcement ratio; it increases in the case of a slab and beam (less than 0,15) and decreases in the case of a column (from 0,18 to 0,15).

Having the mean μ_R or the characteristic value r_k and the coefficient of variation V_R of the resistance R, the design value r_d may be well estimated using the global resistance factors. In general, the type of member and its reinforcement ratio should be taken into account. As the first (informative) approximation, the following global resistance factors may be considered: $\gamma_R \approx 1,30$ for the global factors related to the characteristic value r_k, and $\gamma_R^* = 1,60$ for the global factors related to the mean μ_R.

The global factors γ_E and γ_E^* of the load effects are strongly dependent on the load ratio χ of characteristic variable and total actions and a type of action. Considering the common range of the load ratio up to 0,8, the global factor γ_E for imposed load varies within the interval from 1,2 to 1,35 (the value 1,3 may be used as the first approximation), the factor γ_E^* varies within the interval from 1,3 to 2 (the value 1,6 may be used as the first approximation). For the load ratio χ around 0,5, the global factors of wind action may be approximated as $\gamma_E \approx 1,35$ and $\gamma_E^* \approx 1,6$ and for snow load $\gamma_E \approx \gamma_E^* \approx 1,45$.

The total factor of safety may be obtained from the products $\gamma_E \gamma_R$ or $\gamma_E^* \gamma_R^*$.

8. Acknowledgement

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