ENGINEERING MECHANICS 2009
National Conference with International Participation
Svratka, Czech Republic, May 11 – 14, 2009

pp. 969–979
Paper **#177**

EXPERIMENTAL STUDY OF PEDESTRIAN DYNAMICS IN VERTICAL AND HORIZONTAL DIRECTION

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Summary: *The authors have noticed the newest observations and few analysis of excitation mechanism. They suppose that the import knowledge of the forces frequencies of step or strides for different walking velocities is the most important for the further analysis and analysis of the mechanisms.*

The authors received the vertical force dependence on the walking velocity, on stride length and on the weight of pedestrian. The new research step is focused on lateral horizontal forces.

1. Introduction

ITAM investigated a large number of footbridges and asserted that these constructions were very sensitive to pedestrians' movements as their eigenfrequencies were close to the step frequencies of pedestrians due to the light weight of the footbridges investigated.

2. Dynamic load

The dynamic load has at least three components – one vertical, two horizontal and in the case of a curved pedestrian movement, one torsion component (Harper, 1962; Barker, 2002).

2.1. The vertical load

At first we deal with the most important one, i.e. the vertical. The dynamic load is expressed by the dynamic coefficient for a single person

$$\delta_p = \frac{\max F_{dyn} + F_{stat}}{F_{stat}} \tag{1}$$

and for a group of people

$$\delta_{pc} = \frac{\max \sum F_{dyn} + \sum F_{stat}}{\sum F_{stat}} \tag{2}$$

where:

F denotes strength.

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The sum $\sum$ runs over all the strengths at a given time t which is chosen in such a way that δ_{pc} is maximal.

We have also denoted by *stat*, respectively *dyn* the static, respectively dynamic parts of strengths.

2.1.1. A single pedestrian

2.1.1.1. A pedestrian on a horizontal plane

The vertical component of the strength reaches its maximum if the center of mass of the pedestrian is at its maximum over the horizontal plane. The so-called saddle point between two peaks (see Fig. 1) is the occasion when the center of mass is at its minimum, i.e. the pedestrian's two legs pass each other. If the pedestrian walks fast or runs, the saddle point does not occur and the two peaks merge.

In the fig. 1, time is on the horizontal axis and the dynamic coefficient δ_p is on the vertical axis. We denote the duration of stride by t_k and duration of step by t_s. The speed of walking is 1.1 ms^{-1}. The intersection of the pressure functions (of time) of the left and right leg is the moment when both legs are touching the plane.

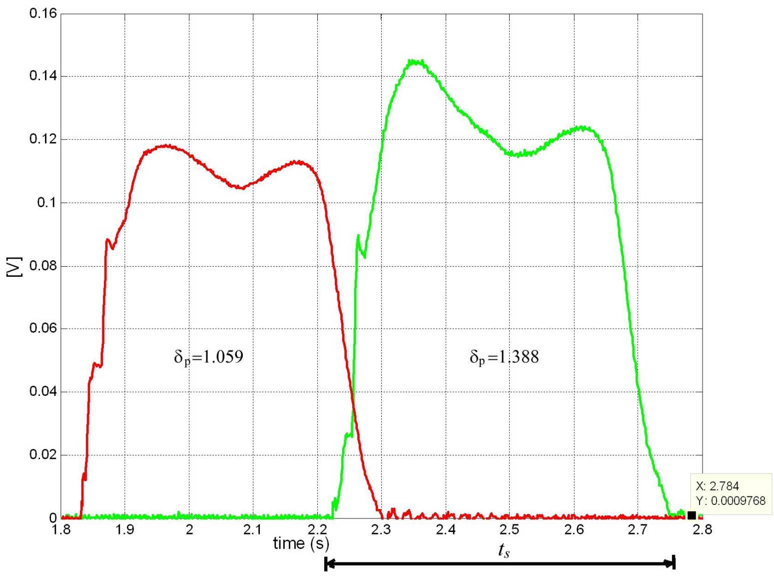

Figure 1. Time histories of the left and right leg.

According to our measurements (Pirner & Urushadze, 2007), the borderline between walking and running lies somewhere between 1.4 and 1.8 ms^{-1}. It appears in the series of subsequent strides in the way that the end of one stride and the beginning of the next one merge at one point (Bachmann & Ammann, 1987).

We have measured the walking of ten men and two women and can confirm that every individual has its own characteristic "handwriting" of walking.

In Figure 2 we see the dependence of the dynamic coefficient δ_p on the step frequency $f_s = 1/t_s$ and stride frequency $f_k = 1/t_k$. The datasets are interpolated by polynoms through their means (dashed line) and through their maxima (full line).

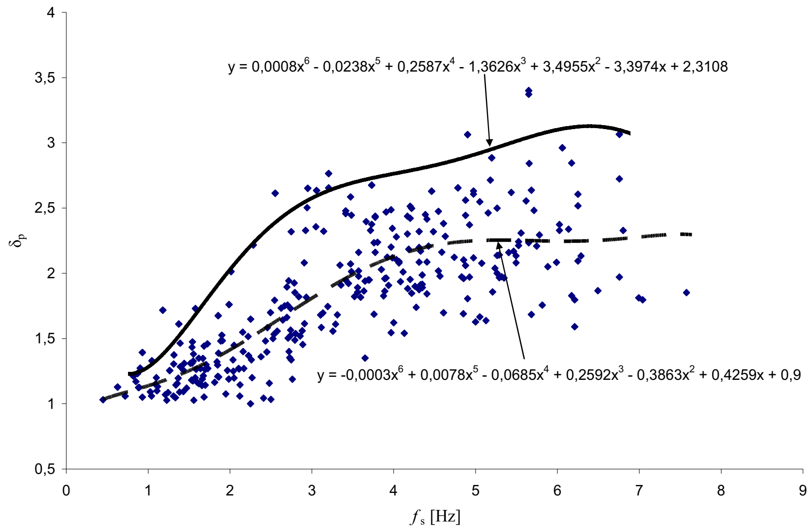

Figure 2. The dynamic coefficient δ_p versus step frequency f_s.

In Figure 3 we see the dependence of the step frequency f_s and the stride length l_k on the stride frequency f_k. From this dependence we can derive an approximate relation between f_s and f_k.

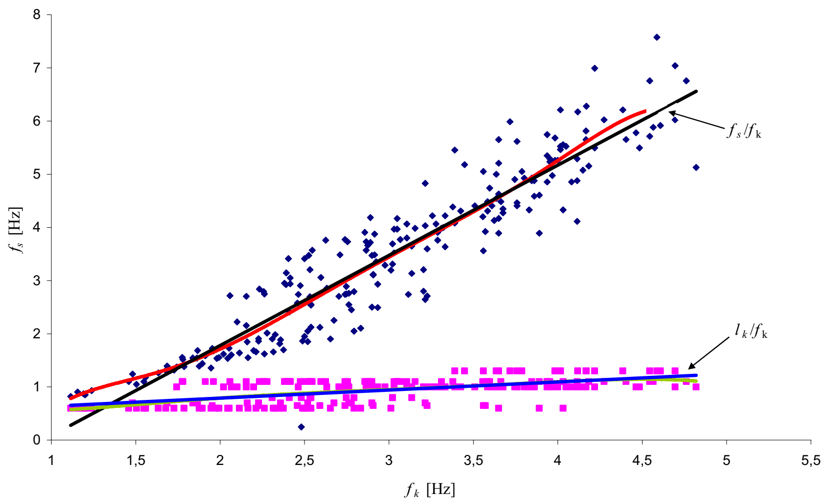

Figure 3. The dependence of step frequency f_s on stride frequency f_k and the stride length l_k on stride frequency f_k.

Furthermore, the relation between the stride length l_k and f_k can be derived from Fig 3.

In Figure 7 we see the dependence of the dynamic coefficient δ_p and the striding velocity on stride frequency. It is apparent from this figure, which contains all the records of the individuals tested, that the aforementioned dependencies have a large variance; in spite of that it was possible to establish an approximate relation between the striding velocity $\dot{v}$ [m/s] and the step and stride frequencies

$$\dot{v} \doteq l_k \cdot f_k \tag{3a}$$

$$\dot{v} \doteq 0.8 f_k \doteq 0,6 f_s \tag{3b}$$

The relations (2) and (3) do not capture any differences between men and women due to their approximate nature.

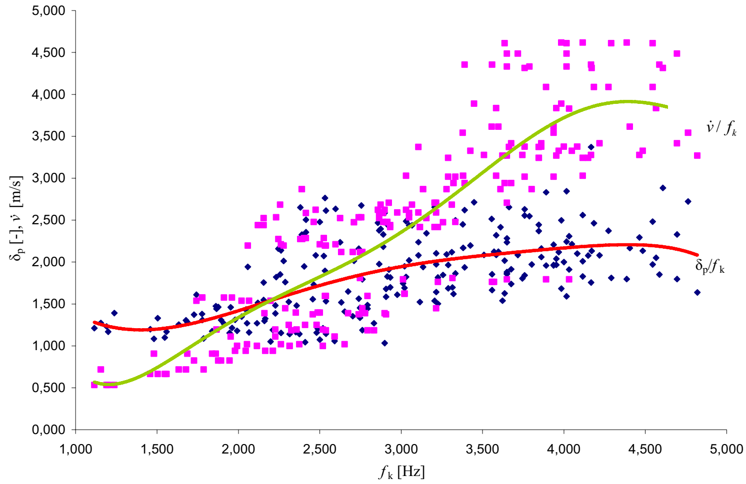

Figure 4. The dependence of dynamic coefficient δ_p on stride frequency f_k and striding velocity $\dot{v}$ on stride frequency f_k.

2.1.1.2. A pedestrian on an inclined plane

The next part of the experimental investigation was a measurement of the dynamic characteristics (δ_p, f_s,, f_k, l_k) for walking on an inclined plane. The selected slopes for experiments were (in percentage) 16%, 21.2%, and 33%.

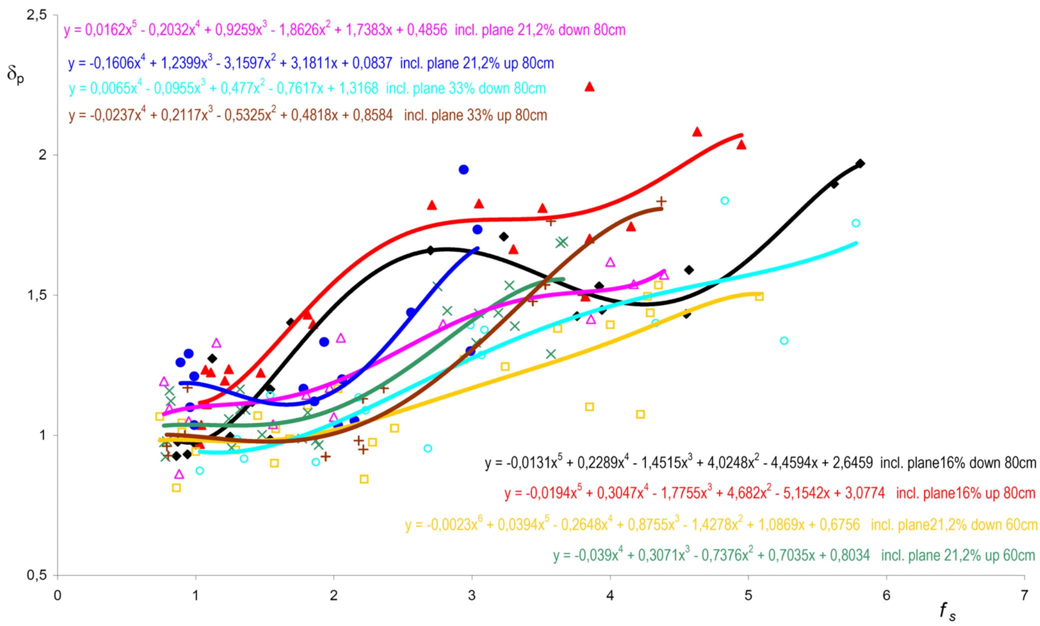

Figure 5. The dynamic coefficient δ_p versus frequency f_s – inclined plane

In Figure 5 we see the dependence of dynamic coefficient δ_p on the step frequency f_s when the length of one step is 80 cm; it is obvious that if the slope is more than 16% the pedestrian is more careful, i.e., her dynamic load is smaller. Polynomial curves represent probable dependencies of the quantities involved.

Some of the results obtained:

- The vertical component of the strength as a function of time is roughly similar to the corresponding function in the case of the horizontal plane, i.e., they have two peaks for striding velocities $\dot{v}$ = 0.8 ÷ 1.5 ms^{-1} (the first one is usually higher than the second one when walking down and other way round when walking up), if the pedestrian walks quickly or runs they merge into one peak.
- "Decrease" of the dependence δ_p / f_s at $f_s \cong 3.5$ Hz occurs only if the slope is equal to 16%.
- The rate of frequencies f_s and f_k, is more complicated – it differs from the formula (3b) which is valid for the horizontal plane only. The results of measurement are shown in table 1. It is apparent that the rate is higher when walking up. The slope and stride length have only minor effects.
- The rate of $\dot{v}$ and $\varnothing f_s$ is $\dot{v} / \varnothing f_s$ = 0.59 ÷ 0.86 [m];

  it was impossible to obtain more precise information from the measured values.

2.1.1.3. A pedestrian on a staircase

A staircase can be a part of a footbridge. This is why we have measured dynamic effects of a pedestrian on stairs when he or she moves up and down.

In Figure 6 we depicted the dependence of the dynamic coefficient δ_p on the step frequency f_s.

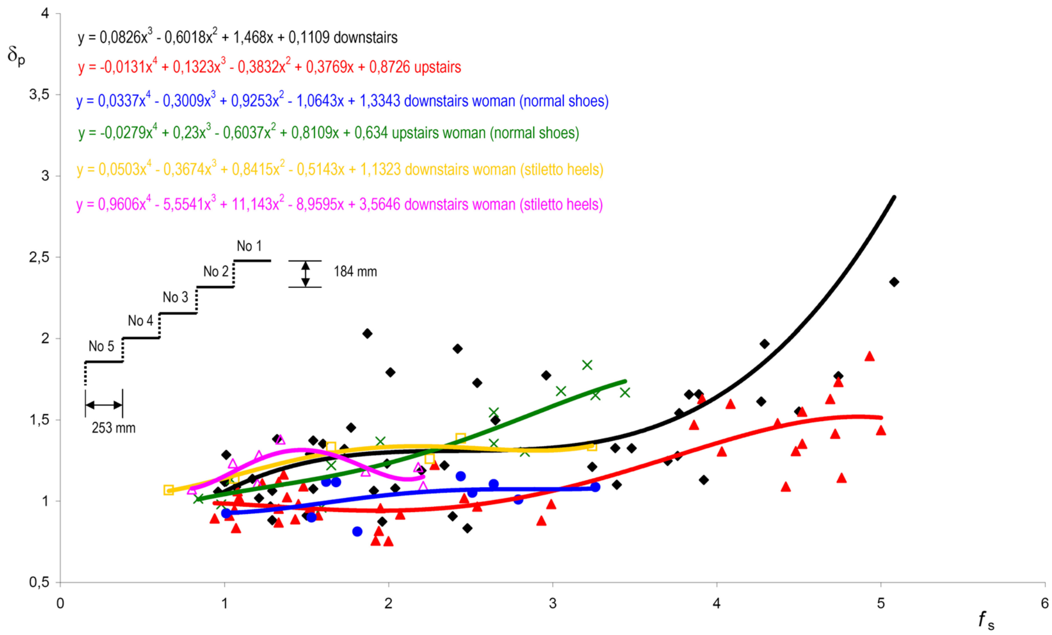

Figure 6. The dynamic coefficient δ_p versus frequency f_s.

From the measurement results it follows:

- men cause more dynamic effects when walking downstairs than upstairs
- women are more careful and when they walk downstairs they cause less dynamic effects than when they walk upstairs
- no effects of the height of heels (worn by women) were observed.

The bar chart for slow walking has a similar shape as seen in Figure 1 for footsteps on a horizontal plane. For fast walking, the two peaks in the bar chart merge.

2.1.2 A group of pedestrians on a horizontal plane

We have investigated the vertical components of the strength exerted by a group of pedestrians because of our search for a theoretical expression of load and response of a footbridge in both synchronous and asynchronous cases. We have used five pedestrians walking side by side and in a variant setting three pedestrian side by side and two behind them (Fig. 10b showing the horizontal projection of sensors).

In both cases the distance between two outer sensors was 2 m, corresponding to the width of the footbridge between guardrails of 3 m. The "density" of pedestrians was 0.9 [person/m^2].

The Figure 7 contains the record of vertical components of strengths exerted by five pedestrians when they walk with the velocity $\dot{v}$ =1.4 ms^{-1} (the latter variant with 3 pedestrians in front and 2 behind them; Fig. 10b).

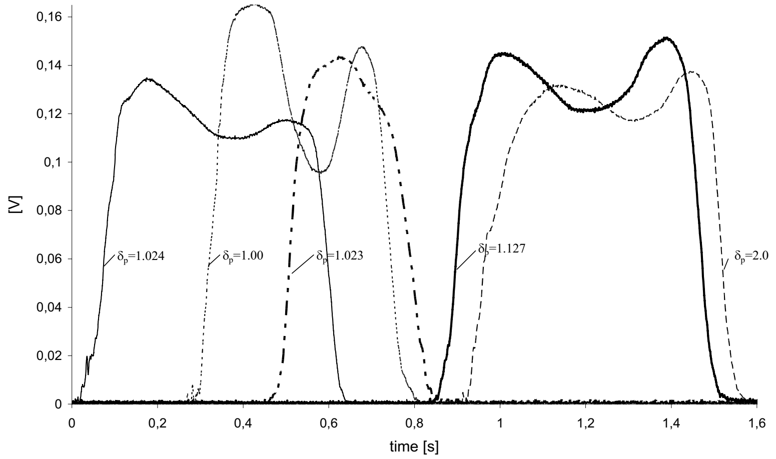

Figure 7. The vertical forces of five persons; $\dot{v} = 1.4$ m/s ; the sensors were located as in Fig. 10b.

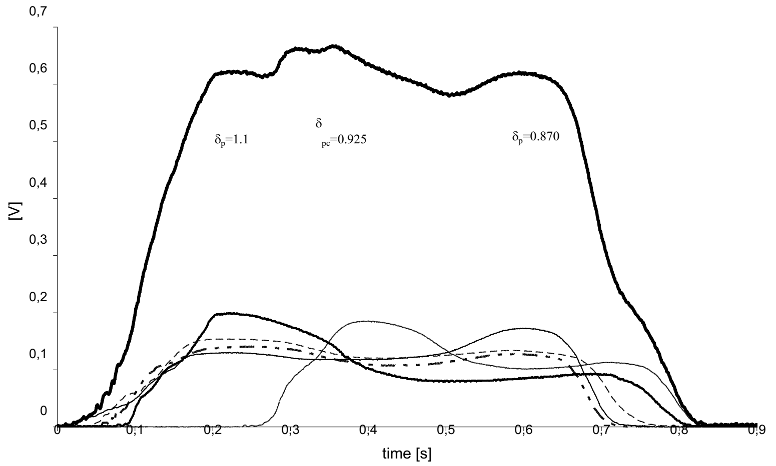

Figure 8. The vertical forces of five persons; $\dot{v} = 1.5$m/s ; the sensors were located as in Fig. 10c. The symbol δ_{pc} = dynamic coefficient of all persons.

The Figure 8 shows the record of vertical components of strengths exerted by five pedestrians when they walk with the velocity $\dot{v}$=1.5 ms^{-1}. The setting of sensors was in agreement with the situation depicted in the Figure 10c.

The dependencies of the dynamic coefficient δ_{pc} on the step frequency f_s for 2, 3, 4 and 5 people side by side, respectively is shown in the Figure 9.

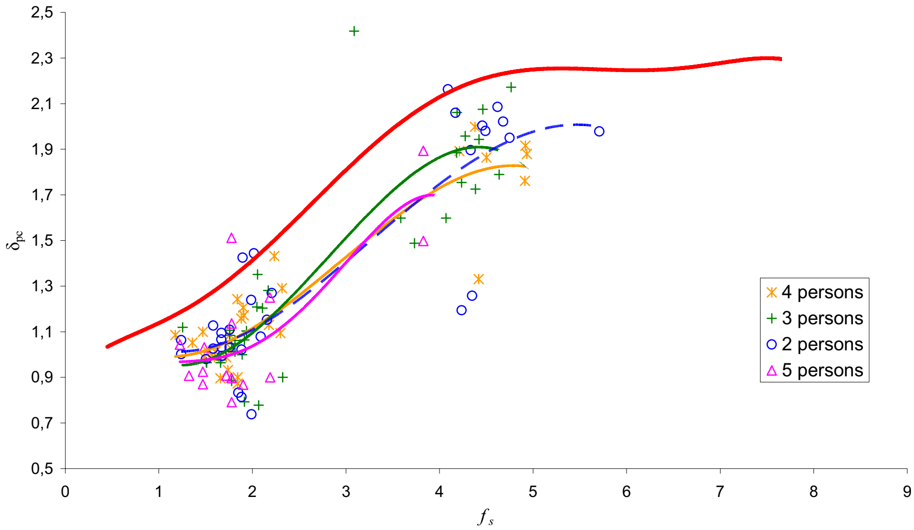

Figure 9. The dependencies of the dynamic coefficient δ_{pc} on the step frequency f_s for 2, 3, 4 and 5 people side by side, respectively.

2.1.2 Sensors of the vertical stride strength

The sensor is a steel plate, 5 mm thick, 395 mm long and 150 mm wide, supported on short sides as a simple beam. The deflection stress was measured by a strain gauges. The eigenfrequency of an unloaded sensor is 143 Hz and the logarithmic decrement of amplitude is $\vartheta \doteq 0.3$; also its eigenfrequency is sufficiently different from a step frequency. The deflection stress did not exceed 120 MPa. The positions of sensors are depicted in the Figure 10.

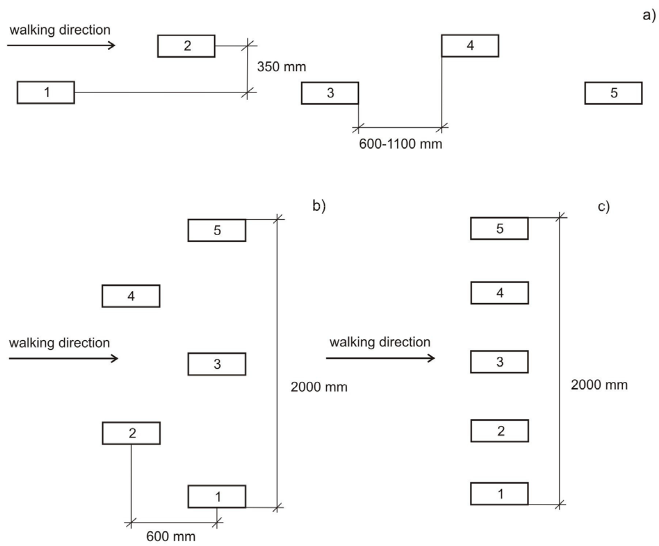

Figure 10. The positions of sensors: a) in the case of a single-person-walk; b) and c) in the case of five people walking.

2.2. Lateral horizontal load

The lateral horizontal force depends on the weight of pedestrian on the speed of the walking and on the length of the stride.

Typical time histories of horizontal lateral loads (right, left, right leg) can be seen in Fig. 1; Harper (1962) described the shape with two peaks.

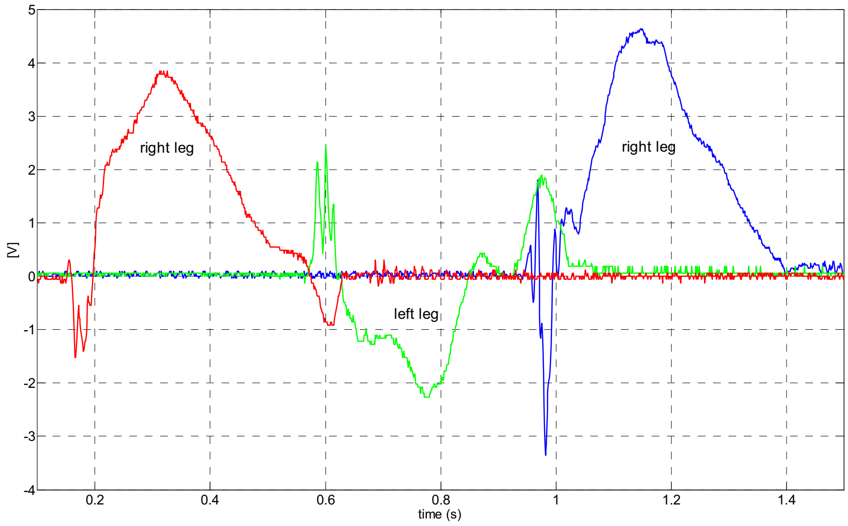

Figure 11. Typical time histories of horizontal lateral loads.

2.3. Sensors of the lateral horizontal load

The sensor is the series of three steel strips, 370 mm long, 30 mm wide, supported by boundary box. The defection stress of steel strips was measured by strain gauges. On the top of steel strips is the plate from soft material, which guarantees the participation of all strips. The sensor is shown on Fig. 12. On Fig. 13 is the position of sensors.

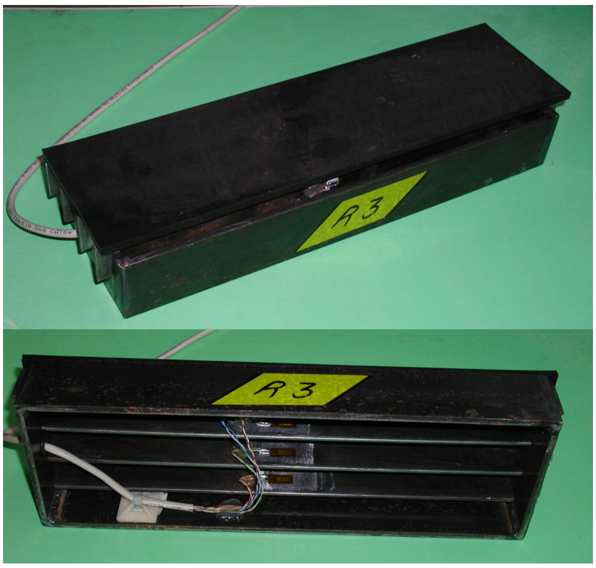

Figure 12. The sensor.

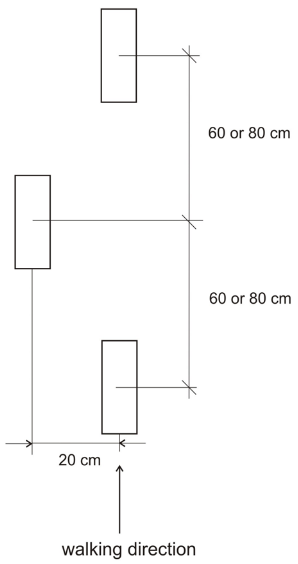

Figure 13. Sensors positions for lateral horizontal loads.

In Fig. 14 are results of our experiments. The weights of pedestrians was from 700 N up to 1125 N and the walking speed from 0,45 m/s up to 1,44 m/s (from 1,6 km/h up to 5,1 km/h)

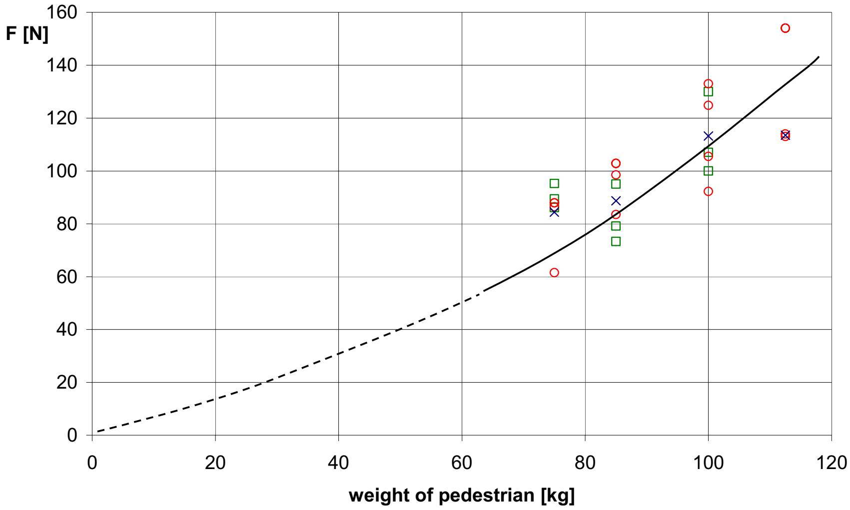

Figure 14. relation between weight of pedestrian and lateral horizontal load.
○– stride length 60cm, □– stride length 80cm, × –the mean value

3. Theory and empirical formula

3.1. The dynamic load in the vertical direction

3.1.1. Load exerted by a single pedestrian deterministically expressed

There is a reliable formula for the dynamic increment (Footbridge, 2002) for N = 20 ÷ 25

$$F_{dyn} = c_z \cdot N \cdot \alpha \cdot m_p \cdot g \tag{4}$$

where c_z is the correlation coefficient (≈ 0.2) expressing the synchronization of steps with footbridge movements,
N – the number of pedestrians
α – the dynamic coefficient of steps (α = 0.2 ÷ 0.5 for walking, α = 0.6 ÷ 1.4 for running)
$m_p{\cdot}g$ = gravity force of a pedestrian.

3.1.2 Load exerted by a continuous stream of pedestrians deterministically expressed

Measurements of the vertical response have confirmed that the vertical components of the strengths have themselves two components in time: nonstationary and a stationary one. In case of the damping value ζ = 0.015 (a common value) and the stride frequency f_k = 2 Hz, the maximal amplitude of the response occurs only after 60 steps (i.e., about 30 seconds after entering the footbridge), while 60% of the response occurs after 10 steps and 85% after 20 steps. Consequently, the nonstationary component is not important for long footbridges (Stoyanoff at al., 2002).

Let us assume that the continuous stream of pedestrians is formed by rows of 5 pedestrians (across the footbridge deck 3500 mm wide) which are d = 0.6 ÷ 1 m apart (it may be even more according to the step velocities). Let us moreover assume in agreement with Figures 9a and 9b that pedestrians' strides in a row are simultaneous. Time shifts of six rows are expressed by a phase shift φ, which may be chosen for example as six multiples of 30 degrees between 0 and 180 degrees randomly attributed while the phase shifts of the first six-pack are denoted by φ_1 to φ_6, of the next one φ_7 to φ_{12} etc.; the next rows of pedestrians follow till they fill up the whole footbridge deck. A scheme of the loading by six rows of pedestrians in time is drawn in Figure 15; for better comprehension we have used an axonometric projection and time functions are plotted in coordinate systems with time axes perpendicular to the axis of the footbridge – we have limited ourselves to three such functions, only.

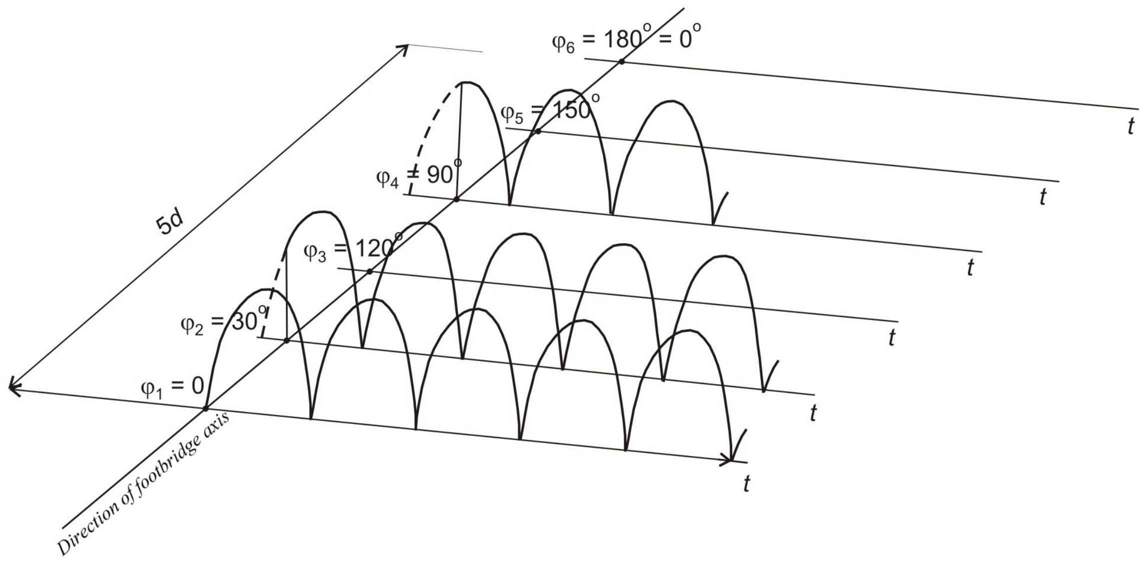

Figure 15. Six rows of pedestrians

The vertical component of the strengths of one row of pedestrians can be computed as

$$F_i(t) = 5 \cdot F_i \cdot \delta_{pc} \left| \sin(\omega t + \varphi_i) \right| \tag{5}$$

where ω_i is an angular step frequency of the i^{th} row of pedestrians.

The formula (5) means that we deal with a standing system of varying loads instead of a continuous moving stream of pedestrians.

Then solving the response of a footbridge is a matter of routine.

3.2 The dynamic load in the horizontal lateral direction

Stoyanoff (2002) gave the formula for the load

$$F(t) = c_R \cdot N \cdot \alpha \cdot w_P \cdot \cos \Omega t \text{ [1]} \tag{6}$$

Where c_R (correlation coefficient ≈ 1)

N number of pedestrians (20 ÷ 25 persons)

[1] Compare the value of $F(t)$ with results of our experiments for N = 1

α dynamic coefficient (0,125)
w_P weight of the pedestrian
Ω dominant walking circular frequency (commonly f = 1 Hz)

According to (footbridge, 2002) the force per unit length $f_P(x,t)$ can be expressed as

$$f_P(x,t) = \frac{\sqrt{N} \cdot \alpha \cdot w_P}{L} \cdot \cos \Omega t \qquad (7)$$

Where α = 0.04 (the footbridge without motion)
L = the footbridge length
Ω dominant walking circular frequency

4. Conclusions

The important results of measurements done in the ITAM laboratory and on the footbridges of various supportive systems follow:

1) Mutual relations among the stride frequency, step frequency, step length, dynamic coefficient and the striding velocity depend on individual body characteristics of a pedestrian.
2) The dynamic coefficient for a given pedestrian can be larger than for a group of pedestrians, if they do not move in a synchronous way.
3) The obtained dynamic coefficients are of use for computations of load exerted by a single pedestrian, a group of pedestrians, a connected stream of pedestrians and for the computation of responses of footbridges with different supportive systems.
4) The time histories of lateral forces of a pedestrian were registered and analyzed.

5. Acknowledgement

The support of the Grant Agency of the Czech Republic - grant No. 103/08/1340 and AV0720710524 research plan are gratefully acknowledged.

6. References

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