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KINEMATICAL ANALYSIS OF HEXASPHERE

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Summary: *The paper deals with the kinematical analysis of a new spherical mechanism Hexasphere. It is redundantly actuated parallel kinematical structure that enables spherical motion in large extent of +- 100 degrees. The main attention is paid to the dexterity that reaches very good values.*

1. Introduction

The mechanisms for realization of spherical motions are used for many purposes. Spherical mechanisms which enable to rotate and orientate an object in the space are used for many important operations. They are the mechanisms of swivel heads with spindle for machine tools that create the basis of absolute majority of machine tools for 5 axes machining. The assemblies of telescopes, i.e. the mechanisms for their motion, are also spherical mechanisms. Another group consists of mechanisms for rotation of different antennas. Many applications of spherical mechanisms are for pointing of optical beams.

Absolute majority of spherical mechanisms is based on the Cardan hinge. Its advantage is high movability, often $\pm 90^{o}$. The first basic disadvantage of Cardan hinge as serial kinematical structure is that it consists of a sequence of successive rotational motions. This leads to the necessity that the subsequent rotations must carry the drive with and thus increase the mass of the construction. Besides that the frame of the construction is loaded detrimentally by bending. The consequence is as well as the disadvantageous ratio between mass and stiffness and the smaller dynamic capabilities of the mechanism. Another consequence is the addition of errors in the chain of partial motions that leads to the higher resulting inaccuracy of mechanism positioning. The second basic disadvantage of Cardan hinge is that it includes the singular position in the zenith position causing that it is not possible to carry out a continuous trajectory between all positions in the workspace.

Therefore another solution for spherical mechanisms has been looked for. All of these problems were circumvented by the adoption of parallel kinematical structures (Valasek 2004) where the only form of loading is either compression or stress, all motors are situated on the machine frame, the length of error chains with summed up errors is significantly lower. The disadvantage of simple parallel kinematical structures is that their workspace is limited by singular positions and collisions, the mostly used spherical joints acquire lower stiffness when compared to sliding or rotational joints, nonlinear kinematic transformation between motors and the end-effector requires a short sampling period, in order to achieve required accuracy.

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Fig. 1 Swivel head for machine centers Sprint Z3 DS Technologie.

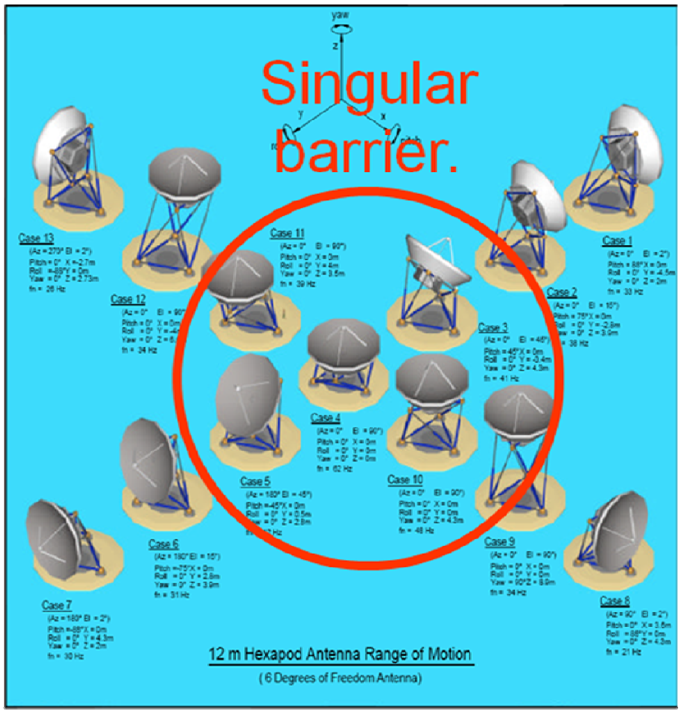


Fig. 2 Telescope HPT Cerro Armazones, Chile.

Fig. 3 Study of MMA antenna.

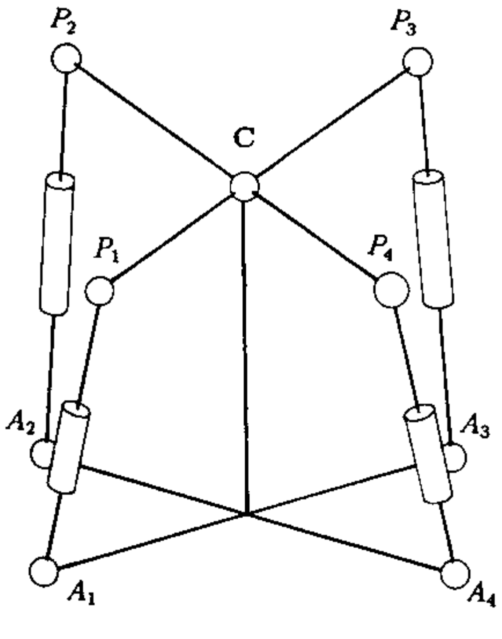
Examples of such parallel spherical mechanisms are in Fig. 1-3. The swivel head for machine tool centers is in Fig. 1. It has increased the productivity due to the removal of the Cardan zenith singularity but its rotational capability is still limited to ±30°. The Hexapod-based telescope has been built in Chile in 2007 (Fig. 2). Its mass was 1/5 of the mass of traditional telescope but it can tilt by only ±47° because then it reaches the singular positions and would collapse. The third example (Fig. 3) is a study of a mechanism for the spherical motion of an antenna based on classical Hexapod. The authors have not realized the existence of the singular barrier in the workspace (Fig. 3) that prevents the motion from the inner part of the workspace to the outer one.

Parallel kinematical structures are characterized by a platform carrying an object that is suspended by many parallel struts and redundant (redundantly actuated) one means that the number of parallel struts is higher than the number of degrees of freedom. The concept of redundant kinematics (Valasek 2004) significantly alleviates the problems associates with parallel kinematics: singularities do not occur, surprisingly the collisions can be limited, the stiffness and dynamics are significantly increased, kinematic accuracy is improved, the on-line calibration is possible. The result is that redundantly actuated parallel kinematical structures enable to create for the machines with serial kinematical structure the functional equivalents with significantly increased mechanical properties (stiffness, dynamics, accuracy). This has been successfully demonstrated on the machines Trijoint 900H and Sliding Star for Cartesian translational motions.

The remaining kinds of mechanisms with serial kinematical structure for finding fully equivalent parallel kinematical structures are exactly the spherical mechanisms. Although one of the most successful application of parallel kinematical structures is parallel swivel head for 5 axes machining but it reached only limited movability. The fully functional equivalent of Cardan hinge with movability ±90^o using parallel kinematical structures has been all the time an open challenge.

2. Concept of Hexasphere

The initial motivation came from (Kurtz & Hayward 1992) where the spherical mechanism in Fig. 4 has been proposed. It is claimed that the problem with singularities (dexterity) has been solved. This structure has been analyzed for the dexterity.



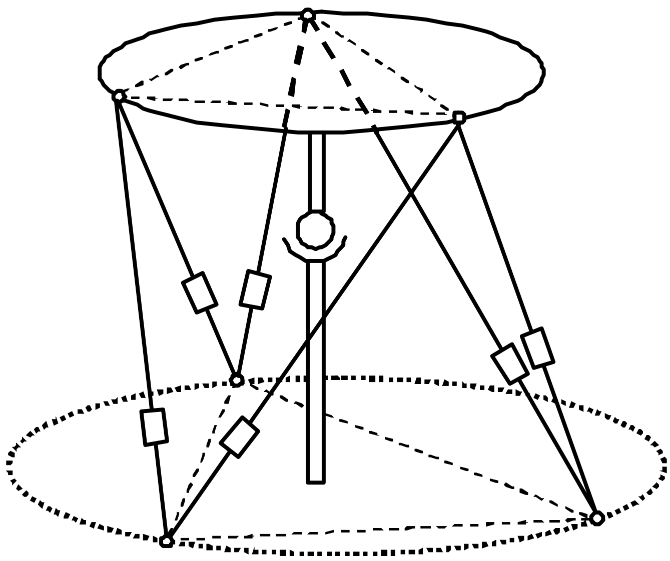


Fig. 4 Parallel spherical mechanism

Fig. 5 Hexasphere kinematical structure

The dexterity is defined as the quantity D

$$D = \frac{1}{cond\,(J_z^{-1}, J_q)} \tag{1}$$

ranging from 0 (the worse value corresponding to the singularity) and 1 (the best value). The Jacobian matrices J_z and J_q are determined from the kinematical constraints

$$\underline{f}(\underline{q}, \underline{z}) = \underline{0} \tag{2}$$

between the independent coordinates $\underline{q}$ describing the position of the platform (and end-effector) and the dependent coordinates $\underline{z}$ describing the position of the actuarors. It holds

$$J_{\underline{q}} = \frac{\partial \underline{f}}{\partial \underline{q}^T} \quad ; \quad J_{\underline{z}} = \frac{\partial \underline{f}}{\partial \underline{z}^T} \tag{3}$$

In the case of spherical mechanisms in Fig. 4 and Fig. 5 it is

$$\underline{T_{12}} = \underline{T_0} \cdot \underline{T_{q_z}(\psi)} \cdot \underline{T_{q_z}(\vartheta)} \cdot \underline{T_{q_z}(-\psi)} \tag{4}$$

where $\underline{T}_{12}$ is the transformation matrix (Stejskal & Valasek 1996) from the frame 1 to the platform 2 consisting of the transformation $\underline{T}_0$ from the frame 1 to the center of spherical motion followed by the rotation of Euler angles ψ, υ with the condition that the third Euler angle -ψ is opposite to the first one in order the platform realizes the rolling and does not interlace the struts. The actuators are described by the displacement coordinates s_i. However, the constraints (2) must be brought after time differentiation into the form

$$\frac{\partial \underline{f}}{\partial \underline{q}^T} \cdot \omega_{12} + \frac{\partial \underline{f}}{\partial \underline{z}^T} \cdot \dot{s} = \underline{0} \tag{5}$$

where $\underline{\omega}_{12}$ is the angular velocity vector of the platform and $\underline{s}$ is the displacement vector of the actuators. The constraints express the square of the distances between the end points of the struts at the platform and at the actuator carriages.

Using this approach the dexterity for the mechanism in Fig. 4 has been computed. The range of the dexterity is 0.0065 to 0.6307 (Fig. 6). It is nonzero and the workspace is free of singularities but the dexterity is changing in large interval (100 times) and the minimum values are very close to zero. It is disadvantageous because the dexterity describes the ratio between the driving force and the acting forces in the end-effector.

Therefore the concept of Hexasphere has been proposed (Fig. 5). The principle of redundant actuation (Valasek 2004) has been applied in order to improve the dexterity. It is a combination of Hexapod for actuation and a platform suspension on passive spherical joint. Hexasphere is three times redundantly actuated. The influence of the high degree of actuator redundancy is very positive on the dexterity. The dexterity of Hexasphere has been analyzed by the same approach. Its results are in Fig. 7. The dexterity is now ranging only in the interval 0.33 to 0.65. The change of the dexterity in the whole workspace is only twice and its values are quite high. The required actuation forces are just 2-3 times higher than the acting forces in the end-effector.

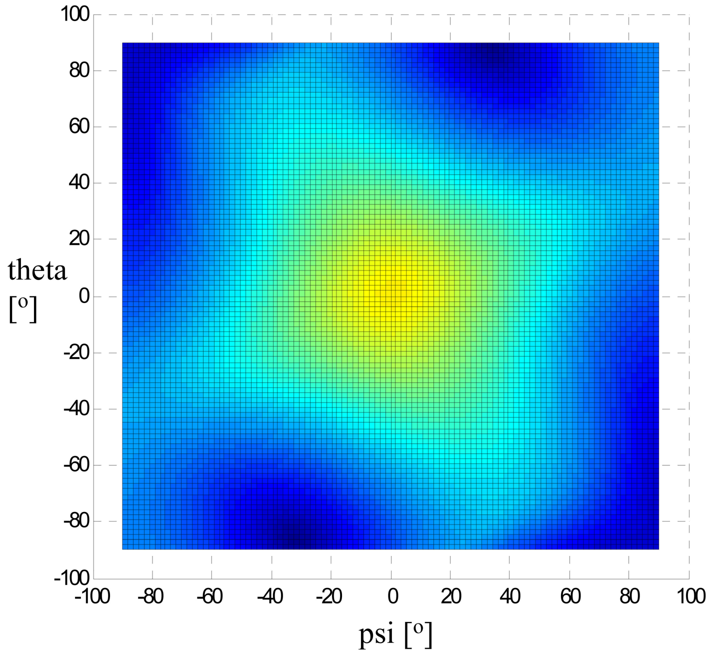


Fig. 6 The dexterity of spherical mechanism from Fig. 4

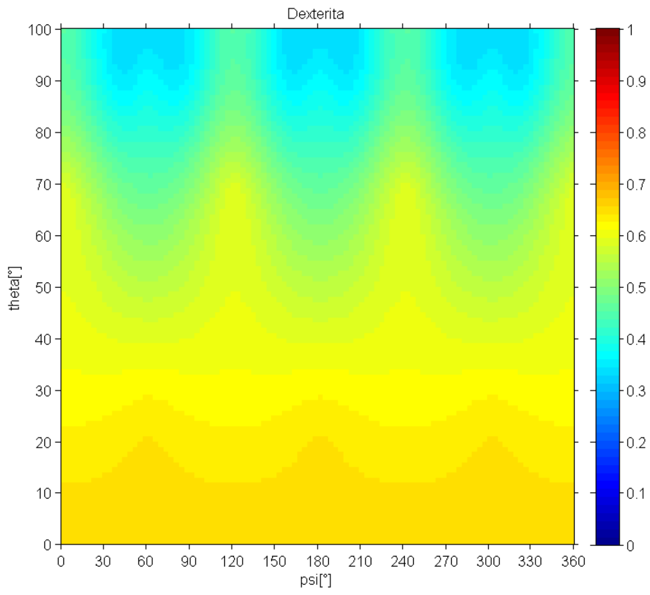


Fig. 7. The dexterity of Hexasphere

3. Design of Hexasphere

The mechanism HexaSphere has the open challenge of parallel spherical mechanism with large tilting angles positively closed. It demonstrates that the redundantly actuated parallel kinematical structure enables the spherical motion now with movability ±100^{o} and preservation of all advantages of parallel mechanisms. The new solution principles that enable to create HexaSphere are following. The platform is connected to the frame by a central spherical joint. Hence the mechanism has only 3 degrees of freedom and for the motion it would suffice just 3 actuators. However, they enable the motion just in small extent of angles because for large motions the singular positions occur when the platform acquires additional uncontrolled degree of freedom and collapses. Therefore the platform is suspended on 6 struts. The result is not only the removal of singularities but also very good dexterity in the whole workspace. Another important principle is that the struts are placed on shanks due to which the collisions between the struts and the platform do not happen for large rotations (Fig. 8). The other dimensions must be also adjusted accordingly.

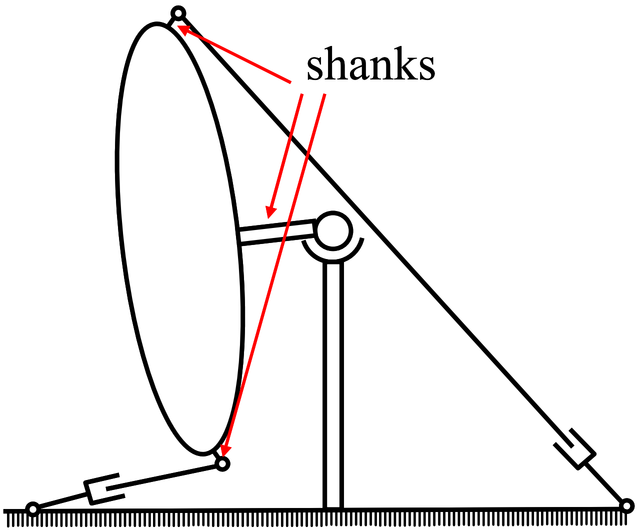


Fig. 8 Design features of Hexasphere

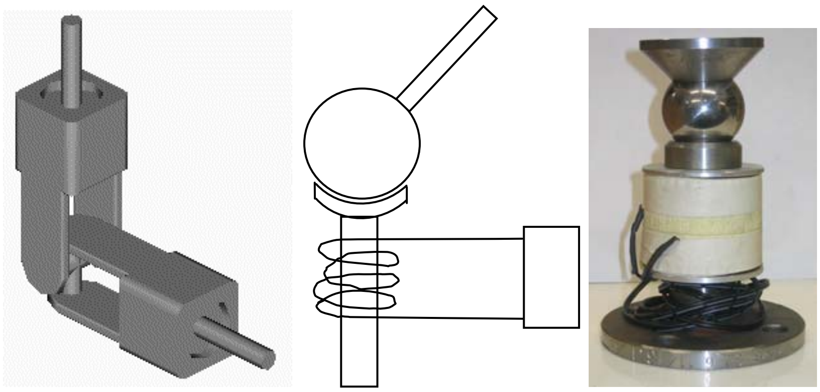


Fig. 9 Two variants of spherical joint with significantly increased movebility

Besides the mentioned solution principles of HexaSphere the usage of many innovative components was necessary. They are above all the spherical joints with substantially increased mobility. They are realized either purely mechanically (but at least with measurement of inner joint motion for calibration if not even with brakes) (Sulamanidze 2007) or by electromagnetic spherical joint (Fig. 9) (Valasek, Petru & Zicha 2008).

4. Possible applications of Hexasphere

The great advantage of HexaSphere is the removal of the zenith singularity, i.e. the possibility of fully continuous motion between any positions in the workspace. The HexaSphere mechanism can have large usage for all applications of spherical mechanisms. It is especially important the removal of zenith singularity of traditional Cardan hinge. It contributes to the increase of motion dynamics. HexaSphere can be the basis of swivel head of machine tools with large extent of motions ±100° with high stiffness, dynamics and with on-line determination and compensation of thermal deformations. It could be the basis of telescope assemblies that would be manytimes lighter than the present ones but simultaneously very agile e.g. for tracking of gamma flashes. It can move the antennas or cameras or mirrors with very high dynamics (Fig. 10).

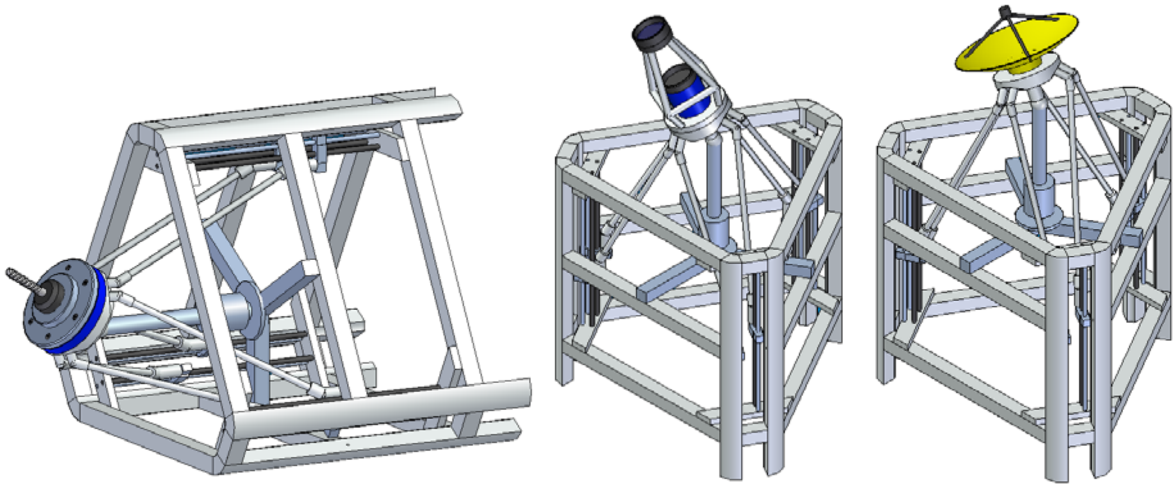


Fig. 10 Possible Hexasphere applications: swivel head for machine tools, telescope assembly, antenna orientation

5. Conclusion

The new proposed spherical mechanism Hexasphere has very promising properties. Its workspace is large as the tilting angles can be ±100°. Within this workspace the dexterity is changing just moderately. This is the basis for the full usage of other advantages of parallel kinematical structures like high stiffness, dynamics, accuracy. The great advantage of HexaSphere is the removal of the zenith singularity, i.e. the possibility of fully continuous motion between any positions in the workspace.

6. Acknowledgement

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