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FLOW BEHAVIOUR OF HIGHLY CONCENTRATED SLURRIES

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Summary: *The paper describes results of experimental investigation of the highly concentrated fine-grained slurry. The flow behaviour of the slurry was experimentally investigated for the laminar and turbulent regimes with respect to the effect of solid concentration and flow velocity. The re-circulation pipe loop with smooth stainless steel pipe of the inner diameter D = 17.5 mm was used. Stony dust from Mistrovice quarry (d_{50} = 8 μm, ρ_s = 2 701 kg/m^3) was used as a solid phase, water was used as a carrier liquid. Volumetric concentration and slurry velocity reached up to 44% and 6 m/s, respectively.*

1. Introduction

Pipeline transport has been one of the progressive technologies for conveying large quantities of bulk materials in the power engineering, mining and ore treatment, and the transport of minerals and raw materials, including the handling of wastes. Compared with the use of belt, rail, truck, or other mechanical means of transport, the hydraulic transport can bring several advantages. It is dust-free, easily surmounts ground obstacles, demands substantially less space, often needs lower investment and operational costs, makes possible full mechanisation and automation, and provides low dependence on the technological discipline and skilfulness of the operating staff (Vlasak & Kolarcik, 1995).

Pipeline transport or so called freight pipeline is defined as a pipeline whose main purpose is to convey bulk materials which are in a solid form. Solid form implies that freight is in the solid state, but it may be powdered, granulated, sintered, manufactured, and packed, and so on. Bulk implies an impressively large, heavy, or numerous materials such as coal, sand, gravel, phosphates, grain, mineral oils, gases, and so on. Freight pipeline is a transportation system where loads are moved totally enclosed by the pipe and where motive power is applied via a moving fluid – liquid or gas which is employed to entrap, fluidise, and convey the cargo through the pipeline. The freight is in a bulk form and is usually mixed with a carrier fluid (Zandi & Gimm, 1976).

The energy of the liquid carrier keeps the solid materials suspended in the liquid stream and conveys them in the direction of a liquid flow or pushes them along the pipe bottom in the form of a moving bed. The most important parameters for the safe and economic operation of

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a slurry pipeline are the operational velocity and pressure drop. The operational velocity together with solids concentration and pipe diameter determines the pipeline transport capacity.

Pressure drop, which determines the energy consumption and the technology of pumping, is produced by the internal friction in the conveyed slurry and the friction between the pipe and the slurry. The pressure drop depends on the flow velocity, solids concentration, density, shape, and size distribution of the conveyed solid material, the size and roughness of the pipe, and also the mutual particle-liquid, particle-particle, and particle-pipe interactions.

The presence of fine solid particles in a Newtonian liquid evokes a complex rheological behaviour of the slurry. The flow behaviour of highly concentrated fine-grained slurries depends on the particle shape, size distribution, density, solids concentration, and also on the contents of fine, especially colloidal particles. The flow behaviour is strongly affected by the mutual particle-particle and particle-liquid interaction and the slurry's physical-chemical environment. If the content of fine particles increases the slurry flow behaviour changes from Newtonian to non-Newtonian. The understanding of slurry flow behaviour makes possible to optimise energy and water requirements, to improve quality, safety, reliability and economy of transport and/or processing (Vlasak et al., 2006).

2. Slurry flow behaviour

The flow behaviour of concentrated fine-grained slurries (clay, coal, ash, tailings, ores) can be approximated by rheological models for the laminar flow regime, generally by the yield pseudo-plastic model. For the turbulent regime a turbulent model must be employed.

2.1. Laminar flow regime

The flow behaviour of the dense fine grained non-Newtonian slurries in the laminar regime can in general be modelled as the yield pseudo-plastic fluid, that is by the Bulkley-Herschel rheological model

$$\tau = \tau_y + K\ (-du/dr)^n, \qquad (1)$$

where τ_y, K and n are the yield stress, fluid consistency, and flow behaviour index, respectively; u and r are the local velocity of liquid and cylindrical co-ordinate, respectively. The Bingham model

$$\tau = \tau_B + K_B\ (-du/dr), \qquad (2)$$

is a special case of it and τ_B *and* K_B are the Bingham yield stress and fluid consistency, respectively (Govier & Aziz, 1972).

The analytical solution of a laminar flow in a circular tube can be obtained by substituting shear stress from Eq. (1) or (2) to the equation describing a laminar flow in a circular tube. Solving this equation it is easy to obtain a velocity distribution. Due to the yield stress a core part of the flow given by a region, where $0 < r < r_p$, is moving as a plug with uniform velocity

$$u_p = \left(\frac{1}{K}\right)^{\frac{1}{n}} \frac{n}{2(n+1)} \frac{D}{\tau_w} \left(\tau_w - \tau_y\right)^{\frac{n+1}{n}} . \tag{3}$$

In the gap between the plug and the pipe wall ($r_p < r < R$) the velocity distribution is

$$u = u_p \left[1 - \left(\frac{\tau - \tau_y}{\tau_w - \tau_y} \right)^{\frac{n+1}{n}} \right]. \tag{4}$$

In these equations τ_w is the wall shear stress, and $D = 2R$ is the pipe diameter and the radius of the plug is $r_p = R(\tau_y/\tau_w)$.

After integration of the velocity distribution profile the mean velocity v_s can be calculated as

$$\frac{8 v_s}{D} = \frac{4n}{K^{1/n} \tau_w^{\ 3}} \left(\tau_w - \tau_y\right)^{\frac{n+1}{n}} \left[\frac{\left(\tau_w - \tau_y\right)^2}{1+3n} + \frac{2 \tau_Y \left(\tau_w - \tau_y\right)}{1+2n} + \frac{\tau_Y^{\ 2}}{1+n} \right], \tag{5}$$

Eq. (5) may be rewritten for the friction factor

$$\lambda = \frac{64}{\mathrm{Re}_s} \frac{3n+1}{4n} (1-\xi)^{1/n-1} \left\{ 1 - \frac{\xi}{1+2n} \left[1 + \frac{2n\xi}{1+n} (1+n\xi) \right] \right\}^{-1} , \tag{6}$$

where $\xi = \tau_y / \tau_w$ and the slurry Reynolds number Re_s is given as

$$\mathrm{Re}_s = \frac{\rho v_s D (1-\xi)^{1/n}}{\tau_w^{\ 1-1/n} K^{1/n}} . \tag{7}$$

The rheological parameters τ_y, K, n (which generally depend on the particle size distribution and density of solids, carrier liquid properties and slurry concentration) control the slurry rheogram. They have to be determined experimentally using Couette viscometers or pipeline loops, where a laminar flow can be achieved, Vlasak & Chara (1999, 2001).

Eq. (5) can be directly used to obtain rheological parameters from viscometer data, but if the slurry is measured on a pipeline loop the parameters have to be estimated by fitting the values of τ_w and v_s measured in the laminar regime. The hydraulic gradient can be generally determined analogously as for the Newtonian liquid from the Darcy-Weisbach equation

$$i_s = \lambda \frac{1}{D} \frac{v_s^{\ 2}}{2g} = \frac{4 \tau_w}{\rho g D} \ . \mathrm{v} \tag{8}$$

The relationship between the hydraulic gradient and the slurry velocity can be determined also from Eq. (5) by substituting values of the wall shear stress τ_w (which should satisfy condition $\tau_w > \tau_y$). From the calculated values of the slurry velocity v_s and from Eq. (8) the hydraulic gradient i_s in given interval of shear stresses and slurry velocities can be determined. For the hydraulic gradient expressed in meters of water, we can substitute to Eq. (8) the density of water $\rho_w = 1000$ kg/m^3.

2.2. Turbulent flow regime

Operation of commercial pipelines is usually provided in the turbulent regime. For the turbulent regime of non-Newtonian fluids several models have been suggested, for example Metzner & Reed, Torrance, Ryan & Johnson, Hanks, Wilson & Thomas, or Slatter. Thomas and Wilson developed a new analysis for the turbulent flow of non-Newtonian slurries based on enhanced micro-scale viscosity effects (with small time and length scales for the dissipative micro-eddies). The model predicts the thickening of the viscous sub-layer by a factor referred to as an area ratio α, and of throughput velocity and thus promotes a drag reduction. They suggest for the slurry mean velocity v_s

$$v_s = V_N + V_*\left[11.6(\alpha - 1) - 2.5 \ln \alpha - \Omega\right], \tag{9}$$

where V_* is the friction velocity and V_N is the mean velocity for an equivalent Newtonian flow, based on a secant viscosity from the rheogram, that is a flow with the same τ_w as a Newtonian fluid with the same viscosity corresponding to the non-Newtonian value at $\tau = \tau_w$.

The term Ω represents the effect of possible blunting of the velocity profile in the logarithmic or core regions of the flow, caused by the yield stress

$$\Omega = -2{,}5 \ln(1-\xi) - 2{,}5\xi(1+0{,}5\xi), \quad \xi = \tau_y / \tau_w . \tag{10}$$

The area ratio α for the yield pseudo-plastic model is defined as the ratio of the integrals beneath the non-Newtonian and assumed Newtonian rheograms, under the identical shear stress conditions. The area ratio α is given as

$$\alpha = 2(1+\xi n)/(1+n) \text{ [26]}. \tag{11}$$

Slatter (1995) proposed a model for the turbulent flow regime. By analogy with the Newtonian approach he defined the roughness Reynolds number $\mathrm{Re_r}$ for the yield pseudo-plastic slurry

$$\mathrm{Re_r} = 8\, \rho\, V_*^2 / [\tau_y + K\,(8\, V_* / d_p)^n . \tag{12}$$

For the fine-grained slurries the $d_p \sim d_{85}$ was found to be a good representation of the turbulent roughness effect of the solid particles in the slurry and ρ is the slurry density. The mean velocity can be obtained by integration over the pipe cross- section

$$V_s / V_* = \kappa^{-1} \ln(R / d_p) + B - 3.75 , \tag{13}$$

where Karman´s constant $\kappa = 0.40$ and coefficient $B = B_R = 8.5$ for the fully developed rough wall turbulent flow ($\mathrm{Re_r} > 3.32$) or $B = B_s = 2.5 \ln \mathrm{Re_r} + 5.5$ for the smooth wall turbulent flow ($\mathrm{Re_r} < 3.32$). The Fanning friction factor for the rough wall turbulent flow is constant and can be expressed as

$$1/\sqrt{f} = 4.07 \log(3.34 D / d_p) \tag{14}$$

3. Experimental equipment and material

3.1 Experimental installation

The slurry was tested using an experimental re-circulation pipeline loop with the test section from the smooth stainless steel pipe of inner diameter D of 17.5 (see Figure 1). The slurry was forced by an EPS-125-6-60 screw pump from an agitated open storage tank to the transport pipe. A phase advancer was used to achieve the different slurry flow rates (Vlasak & Chara, 2001).

The loop can operate in the laminar as well as the turbulent regime up to the average slurry velocity v_s of about 6 m/s. The measurement section was equipped with three pressure taps connected to Hottinger-Baldvin PD-1 differential pressure transducers (measuring range up to 0.1 MPa, carrier frequency 5 kHz) monitored by computer.

The slurry flow rate and concentration were measured by the electromagnetic flow meter KROHNE-PROFILUX IFM 5080 K A and the mass flow meter KROHNE-CORIMASS-800 G+. The loop also allowed the direct measurement of mass flow rate and of slurry density. The temperature of the slurry was maintained at about 18 °C by the heat exchanger. Attention was paid principally to the effects of the slurry average velocity and concentration on the relationship i_s/v_s of the pressure gradient versus the average slurry velocity.

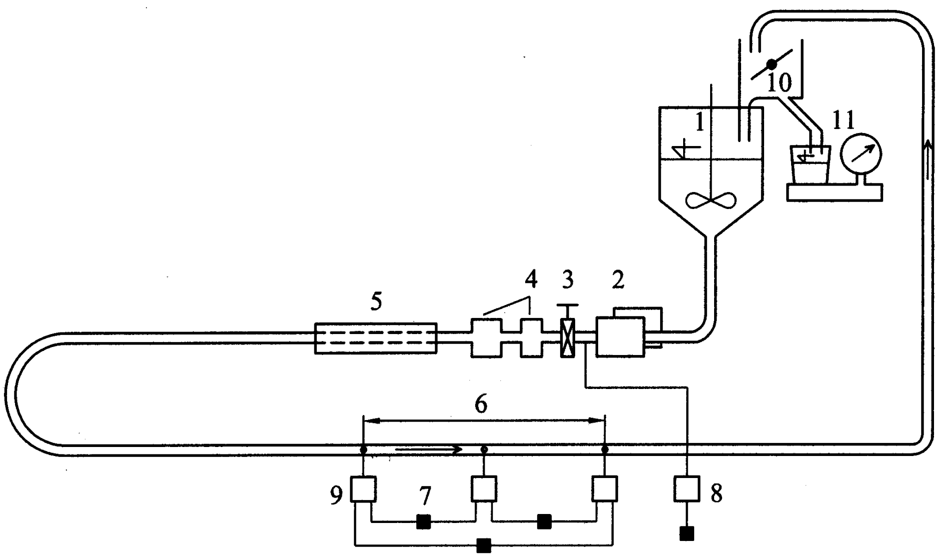

Figure 1 Layout of the experimental pipeline loop (1-slurry tank, 2-pumps, 3-control valve, 4-flow meters, 5-heat exchanger, 6-test section, 7-differential pressure transducers, 8-absolute pressure transducer, 9-pressure taps with sedimentation vessels, 10-flow divider, 11-density and discharge measurement)

3.2. Material used

Highly concentrated water mixtures of stony dust from Mistrovice quarry (d_{50} = 8 μm, d_{max} < 100 μm, ρ_s = 2 701 kg/m^3) were measured. Particle size distribution is shown in Figure 2. The used stony dust is a chemically resistant very fine material with high content of colloidal particles (nearly 20%) and it is suitable for highly concentrated slurries, maximal volumetric concentration was about 45%.

4. Results and discussion

Experimental values of the slurry velocity v_s, volumetric transport concentration c_v and hydraulic gradient i_s were determined from the measured values of mass flow-rate, pressure drops and slurry density. Attention was paid to the relationship i_s / v_s of the hydraulic gradient

i_s over the mean slurry velocity, which makes possible to compare the flow behaviour of slurries, which differ mutually by solids contents and concentrations.

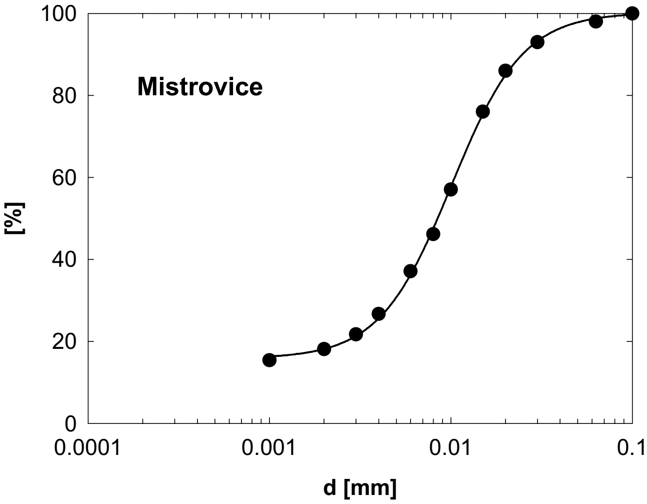

Figure 2 Particle size distribution of Mistrovice stony dust

Figure 3 documents the effect of the slurry concentration on the relationship i_s/v_s and the yield pseudo-plastic behaviour of measured slurries at concentrations higher than 20%. With increasing slurry velocity the hydraulic gradient i_s also gradually increases in the laminar region. Near the laminar/turbulent transition point, the hydraulic gradient i_s increases sharply and a marked instability and pressure pulsations are characteristic for this region. The increase in the hydraulic gradient i_s becomes higher than that of water alone i_w in the turbulent region.

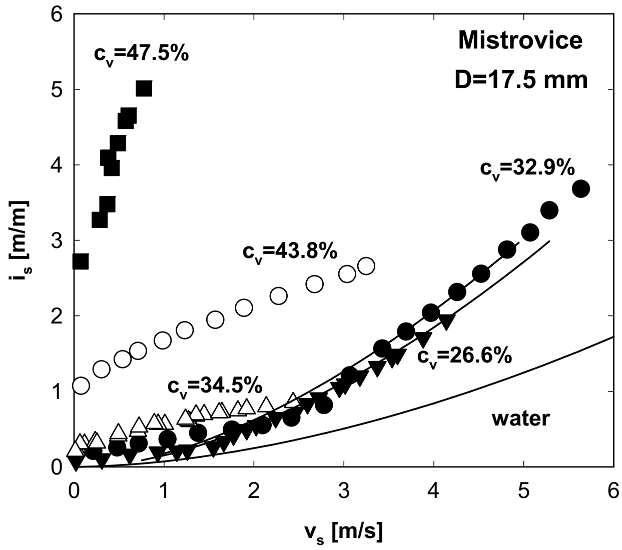

Figure 3 Hydraulic gradient i_s versus slurry velocity v_s for different slurry concentration c_v

It is obvious from i_s / v_s relationship is obvious that the flow behaviour of measured slurries depends mainly on the slurry concentration. The measured slurries contain a high percentage of very fine particles (clay and dust particles) and their flow behaviour in the laminar region is similar to the well-known kaolin or fly ash slurries. They are time independent yield pseudo plastic slurries for measured concentrations and their flow behaviour in the laminar regime can in general be modelled by the Bulkley-Herschel rheological model. The rheological parameters, i.e. the yield stress τ_y, fluid consistency K and flow behaviour index n depend on the slurry concentration.

For lower values of the slurry concentration (c_v = 26.6% and 32.9%) the turbulent regime was reached. Experimental points in Figure 3 are completed by graphs calculated by the Wilson turbulent model.

Many factors influence the accuracy of experimental data and thus a sensitivity analysis was used to find the effect of value of flow behaviour index n on the accuracy of laminar and turbulent flow models. The value of n given by the best fitting of laminar data by Eq. (4) represents quite well the laminar region. However, for turbulent data it is not valid. The best fitting value n for turbulent data varies not only with a kind of solid material, but depends also on concentration (Vlasak & Chara, 1999). Both Wilson's and Slater's turbulent models approximate the turbulent slurry flow well if the values of n are correctly pre-determined from the turbulent experimental data, and in this case, they represent well even the laminar flow regime. The rheological parameters evaluated from the turbulent experimental data and also computed for some concentration values are listed in Table 1.

Table 1 Mistrovice stony dust slurry rheological parameters

concentration	rheological parameters		
c_v [%]	K [Pa s^n]	n	τ_y [Pa]
26,6	0,02047	0,901	2,39
32,9	0,0636	0,821	6,16
34,5	0,0866	0,800	7,83
43,8	0,4659	0,680	31,7
47,5	3,376	0,632	55,3
25	0,0163	0,923	1,88
30	0,0366	0,858	3,98
35	0,0954	0,794	8,44
40	0,2417	0,729	17,90
45	0,8892	0,665	37,96

Dependence of the experimental values of the relative slurry hydraulic gradient i_s / i_w on the slurry velocity v_s for slurries containing Mistrovice stony dust with concentration from c_v = 26.6 % to 47.5% is illustrated in Figure 4, which is again completed by the lines calculated by Wilson turbulent model. The turbulent regime is characteristic by nearly constant values of the hydraulic gradient ratio i_s / i_w, independently on the slurry velocity v_s.

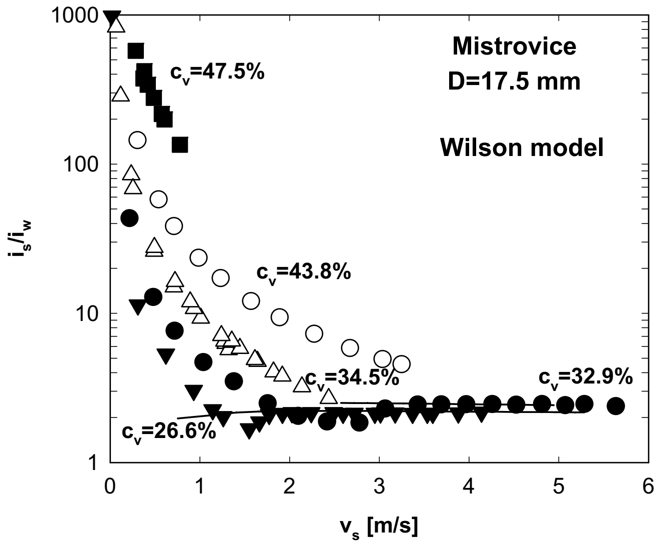

Figure 4 Hydraulic gradient ratio i_s/i_w versus slurry velocity v_s for the different slurry concentration c_v

Results of the slurry measurement in the hydraulically smooth pipe of inner diameter D = 17.5 mm can be used for the prediction of flow behaviour of the highly concentrated slurries in hydraulically smooth pipes of greater diameter. The calculated relationships of the hydraulic gradient i_s versus the slurry velocity v_s and the hydraulic gradient ratio i_s/i_w versus the slurry velocity v_s for pipe diameter D = 50 mm are presented in Figure 5 and Figure 6, respectively.

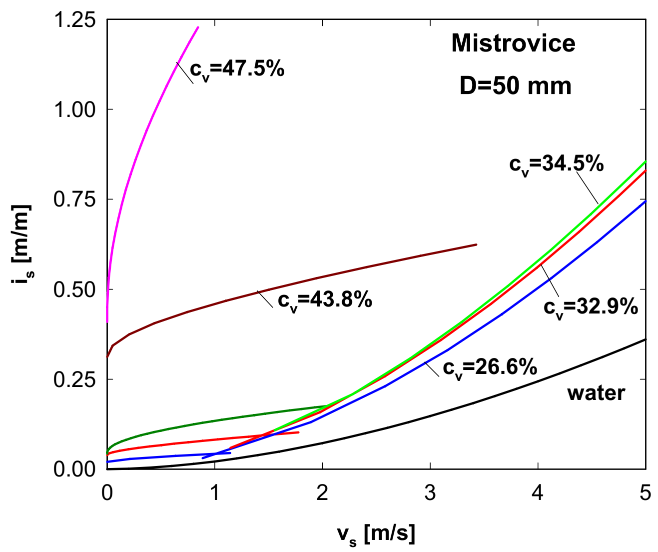

Figure 5 Calculated hydraulic gradient i_s versus slurry velocity v_s in the smooth pipe of diameter D = 50 mm for different concentration c_v

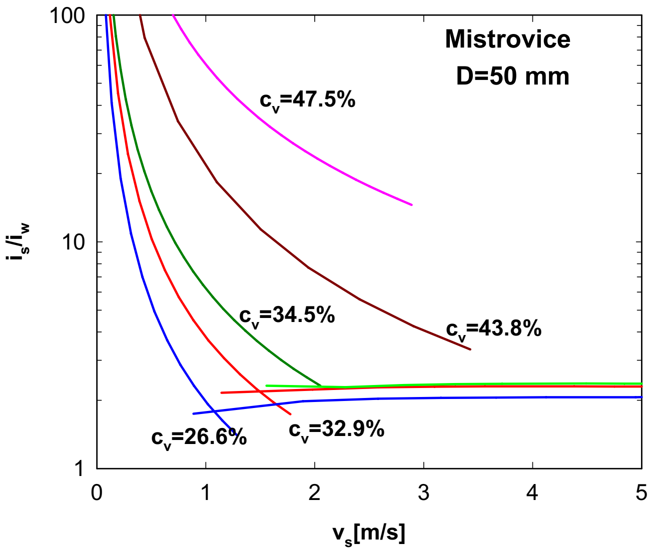

Figure 6 Calculated hydraulic gradient ratio i_s/i_w versus slurry velocity v_s in the smooth pipe of diameter D = 50 mm for different concentration c_v

It is evident for the values of volumetric concentration higher than c_v > 35 %, that transition of turbulent regime will be reached for the flow velocities that are substantially greater than 3 m/s even in relatively small pipe of D = 50 mm

5. Conclusion

The study revealed a time-independent yield pseudo-plastic behaviour of the studied highly concentrated fine-grained Mistrovice stony dust slurries.

Mistrovice stony dust contains high contents of clay and dust particles and is the suitable solid material for the highly concentrated slurry. The flow resistance increases with the increasing concentration of solids.

The flow behaviour of the fine-grained Mistrovice stony dust slurry can be approximated by Bulkley-Herschel model in the laminar region.

Wilson model can be used for prediction of the slurry flow behaviour in the turbulent region.

Both the above mentioned models are suitable even for the scale up.

Accuracy of both the rheological and turbulent models is strongly influenced by the flow behaviour index *n*, which should be determined from the experimental turbulent flow data.

The flow behaviour index n decreases and the values of both fluid consistency K and yield shear stress τ_y increase with the increasing slurry concentration c_v.

6. Acknowledgement

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