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LEARNING CONTROL FOR ROBOT MANIPULATORS

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Summary: *In this paper it is considered an iterative learning control algorithm for trajectory tracking of robots with unknown parameters, as masses of links or inertial momentums etc. The control schemes are based on using of a proportional derivative feedback , for which an iterative term is added to cope with the unknown parameters and disturbances. The control design is simple in the sense that only requirements on the PD and learning gains are the positive definiteness considerations. Arbitrary bounds of the robot parameters are not needed. The number of iterative variables in common algorithms are equal to the number of control inputs, but in this paper this one are defalcated only on two.*

1. Introduction

Researchers and industrials use classical linear controllers as PD or PID ones in control of robot manipulators. The reason is the sake of implementation simplicity. It is well known a PD controller with gravity compensation is globally asymptotically stable. This condition is not easy to satisfy, because the gravity influences depend on unknown parameters, for example time-varying payloads etc.

Without the gravity compensation the PD control leads to a steady-state error. This error can be reduced by increasing the proportional and derivative gains or by implementation an integral gain in feedback. Although for PD control with gravity compensation was proven a global asymptotic stability for PID control only local asymptotic stability was proven .

2. Description of the problem

It is well known that the rigid manipulator dynamics can be described in a form

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u + d \tag{1}$$

where $q \in \mathbb{R}^n$, $\dot{q} \in \mathbb{R}^n$, $\ddot{q} \in \mathbb{R}^n$ are the joint position, joint velocity and joint acceleration, respectively, n represents the number of degrees-freedom; $M(q) \in \mathbb{R}^{n\times n}$ is the (symmetric and positive definite) inertia matrix; $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is in a form

$$C(q,\dot{q}) = \frac{1}{2}\dot{M}(q) + S(q,\dot{q}) + M_0 \tag{2}$$

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where $S(q, \dot{q})$ is skew symmetric, so that $x^T S(q, \dot{q})x = 0$ for $\forall X \in \mathbb{R}^n$, M_0 is the non-negative definite matrix respectively dampnig factors; $g(q) \in \mathbb{R}^n$ is the vector resulting from the gravitational forces (the gradient of a potential energy).

Here $u \in \mathbb{R}^n$ represents the control input vector containing the torques and forces applied at each robot joint or currents or voltages on actuators, respectively; $d \in \mathbb{R}^n$ is the vector corresponding external disturbances and unmodeled dynamics. We suppose that $d = d(t)$ is bounded and integrable on $[0, T]$.

Let $\mathbb{N} = \{1, 2, 3, \ldots\}$ and $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$. The learning will be realized in steps, that we denote by $k \in \mathbb{N}$ the motion trajectory in the step k will be q_k, or $q_k(t)$ if we want to highlight the fact, that this one depends on time t. Similarly we denote

$$
\begin{aligned}
M_k &= M(q_k), \\
C_k &= C(q_k, \dot{q}_k), \\
g_k &= g(q_k), \\
e_k &= q_* - q_k \qquad \textit{is the trajectory error}, \\
\dot{e}_k &= \dot{q}_* - \dot{q}_k \qquad \textit{is the velocity error},
\end{aligned}
$$

where

$q_* = q_*(t)$ is the desired trajectory of the robot motion (end effector motion) and

$q_k = q_k(t)$ is the actual trajectory of time t in the step k.

The curves $q_*(t)$, $q_k(t)$ must satisfy the equation (1) so we suppose, that their first and second derivatives $\dot{q}_*, \ddot{q}_*, \dot{q}_k, \ddot{q}_k$ exist and are continuous. Assuming the joint positions and joint velocities are available for feedback, we want to design a control law $u_k = u_k(t)$ that ensures the boundedness of $q_k(t)$ for $\forall t \in [0, T], \forall k \in \mathbb{N}$ and the convergence of $q_k(t)$ to $q_*(t)$ for all $0 \le t \le T$, where $k \to \infty$.

Let the time interval $[0, T] = \{t \in \mathbb{R}; 0 \le t \le T\}$ be fixed. In this paper, we shall use L_{pe} norms:

$$
\begin{aligned}
\|f(t)\|_{pe} &= \left(\int_0^t \|f(\tau)\|^p \, d\tau \right)^{1/p} \quad \text{for} \quad 1 \le p \le \infty \\
\|f(t)\|_{\infty e} &= \sup_{0 \le \tau \le t} \|f(\tau)\|
\end{aligned}
$$

where $\|f(t)\|$ denotes arbitrary norm of $f(t)$, and $t \in [0, T]$. We define $f \in L_{pe}$ iff $\|f\|_{pe}$ exits and is finite. The matrices $M(q), C(q, \dot{q})$ and vector $g(q)$ are continuous with respect to their variables. We suppose, that there are constants k_c, k_g, such that

$$\|C(q, \dot{q})\| \le k_c. \|\dot{q}\| \text{ and } \|g(q)\| \le k_g$$

for $\forall t \in [0, T]$ and arbitrary $q = q(t)$ that suits equation (1). These constants are unknown.

In every step the solution start with the same initial condition

$$
\begin{aligned}
q_k(0) &= 0 \\
\dot{q}_k(0) &= 0
\end{aligned}
$$

for every $k \in \mathbb{N}_0$.

Let us denote

$$sign\left(\dot{e}_k\right) = \left(sign\left(\dot{e}_k^1\right), sign\left(\dot{e}_k^2\right), \ldots, sign\left(\dot{e}_k^n\right)\right)^T$$

for the vector

$$\dot{e}_k = \left(\dot{e}_k^1, \dot{e}_k^2, \ldots, \dot{e}_k^n\right)$$

where $\dot{e}_k^i = \dot{e}_k^i\left(t\right)$ is the i-component of the vector $\dot{e}_k$ in the step k.

Theorem

Consider the system (1) with the overhand defined presumptions.
Let the control law is defined

$$u_k\left(t\right) = Ae_k(t) + B\dot{e}_k + \varepsilon(\dot{e}_k)\widehat{Z}_k(t) \tag{3}$$

with

$$\widehat{Z}_k(t) = \widehat{Z}_{k-1}(t) + \Gamma\varepsilon^T(\dot{e}_k)\dot{e}_k(t) \tag{4}$$

where $\widehat{Z}_0(t) = 0$. *Let the matrices* $A, B \in \mathbb{R}^{n\times n}$ *and* $\Gamma \in \mathbb{R}^{2\times 2}$ *be symmetric positive definite, the matrix* ε *is defined as the block matrix*

$$\varepsilon_k = \varepsilon(\dot{e}_k) = [\dot{e}_k, sign(\dot{e}_k)] \tag{5}$$

Then $e_k\left(t\right) \to 0$ *and* $\dot{e}_k\left(t\right) \to 0$ *as* $k \to \infty$ *for every* $t \in [0, T]$ *and moreover* $e_k\left(t\right), \dot{e}_k\left(t\right)$ *and* $u_k\left(t\right)$ *are member of* L_{pe} *for all* $k \in \mathbb{N}_0$ *and every* $1 \le p \le \infty$.

Proof.

From our presumption follows that there is γ, such that $\|M_k\ddot{q}_* - d_k\| \le \gamma$. Let us define

$$V_k = \frac{1}{2}e_k^T Ae_k + \frac{1}{2}\dot{e}_k^T M_k\dot{e}_k, \tag{6}$$

$$W_k = V_k + \frac{1}{2}\int_t^0 \tilde{Z}_k^T\Gamma^{-1}\tilde{Z}_k d\tau, \tag{7}$$

With $\tilde{Z}_k\left(t\right) = Z - \hat{Z}_k\left(t\right)$, where $Z = \left(\alpha, \beta\right)^T$, $\tilde{Z} \in \mathbb{R}^2$ and α,β are unknown parameters defined in the following derivation

$$\begin{aligned}\dot{e}_k^T u_k = \dot{e}_k\left(M_k\ddot{q}_* + C_k\dot{q}_* + g_k - d_k\right) &\le \|\dot{e}_k\| \,.\, \left(\gamma + \|C_k\| \,.\, \|\dot{q}_*\| + \|g_k\|\right) \\ &\le \|\dot{e}_k\| \,.\, \left(\gamma + k_g + k_c\|\dot{e}_k\|\,\|\dot{q}_*\| + k_c\|\dot{q}_*\|^2\right)\end{aligned}$$

Since $\dot{q}_*$, as a function of t, is bounded on $[0, T]$

$$\begin{aligned}\dot{e}_k^T u_k \le k_c\|\dot{e}_k\|^2 \cdot \|\dot{q}_*\| + \|\dot{e}_k\| \cdot \left(\gamma + k_g + k_c\|\dot{q}_*\|^2\right) \le \|\dot{e}_k\|^2 \cdot \alpha + \|\dot{e}_k\| \cdot \beta = \\ = \dot{e}_k^T \cdot \alpha \cdot \dot{e}_k + \dot{e}_k^T \cdot \beta \cdot sign\left(\dot{e}_k\right) = \dot{e}_k^T \cdot \varepsilon\left(\dot{e}_k\right) \cdot Z,\end{aligned}$$

so we have

$$\dot{e}_k^T\left(M_k\ddot{q}_* + C_k\dot{q}_* + g_k - d_k\right) \le \dot{e}_k^T \cdot \varepsilon_k . Z, \tag{8}$$

where

$$\alpha = k_c \cdot \sup_{0\le t\le T} |\dot{q}_*(t)| \quad , \quad \beta = \gamma + k_g + k_c \cdot \sup_{0\le t\le T} |\dot{q}_*|^2$$

Let us compute $\dot{V}_k = \frac{\mathrm{d}V_k}{\mathrm{d}t}$

$$\dot{V}_k = \dot{e}_k^T \left(M_k \ddot{e}_k + \frac{1}{2}\dot{M}_k \dot{e}_k + Ae_k \right) = \dot{e}_k^T \left[M_k \ddot{q}_* + \frac{1}{2}\dot{M}_k \dot{q}_* + Ae_k - \left(M_k \ddot{q}_k + \frac{1}{2}\dot{M}_k \dot{q}_k \right) \right].$$

If we use (1) then

$$\dot{V}_k = \dot{e}_k^T \left[M_k \ddot{q}_* + \frac{1}{2}\dot{M}_k \dot{q}_* + Ae_k + S_k \dot{q}_k + g_k + M_0 \dot{q}_k - \tau_k - d_k \right]$$
$$= \dot{e}_k^T \left[M_k \ddot{q}_* + C_k \dot{q}_* + g_k + Ae_k - \tau_k - d_k \right] - \dot{e}_k^T S_k \dot{e}_k - \dot{e}_k^T M_0 \dot{e}_k$$

The matrix S_k is skew-symmetric, and if we use (8) we can obtain

$$\dot{V}_k \le \dot{e}_k^T \left(\varepsilon_k . Z + Ae_k - \tau_k - M_0 \dot{e}_k \right) \tag{9}$$

By substitution (3) into (9) we get

$$\dot{V}_k \le \dot{e}_k^T \left[\varepsilon_k \left(Z - \hat{Z}_k \right) - (B + M_0)\, \dot{e}_k \right] \tag{10}$$

Because $V_k(0) = V_k(t)|_{t=0} = 0$, we can derive from (10) by integration

$$V_k \le \int_0^t \dot{e}_k^T \left(\varepsilon_k \tilde{Z}_k - (B + M_0)\, \dot{e}_k \right) d\tau \tag{11}$$

The reader can persuade of

$$\tilde{Z}_k^T \cdot \Gamma^{-1} \tilde{Z}_k - \tilde{Z}_{k-1}^T \cdot \Gamma^{-1} \cdot \tilde{Z}_{k-1} = -2\Delta\hat{Z}_k^T \cdot \Gamma^{-1} \tilde{Z}_k - \Delta\hat{Z}_k^T \cdot \Gamma^{-1} \cdot \Delta\hat{Z}_k \tag{12}$$

where $\Delta\hat{Z}_k = \hat{Z}_k - \hat{Z}_{k-1}$. By using (4),(11) and (12) we obtain

$$\Delta W_k = W_k - W_{k-1} = V_k - V_{k-1} - \int_0^t \left(\frac{1}{2}\Delta\hat{Z}_k^T \cdot \Gamma^{-1} \cdot \hat{Z}_k + \Delta\hat{Z}_k^T \cdot \Gamma^{-1} \cdot \tilde{Z}_k \right) d\tau = -V_{k-1} +$$
$$+ \int_0^t \left[\dot{e}_k^T \left(\varepsilon_k \tilde{Z}_k - (M_0 + B)\, \dot{e}_k \right) - \frac{1}{2} \left(\Gamma \varepsilon_k^T \dot{e}_k \right)^T \Gamma^{-1} \cdot \Gamma \varepsilon_k^T \cdot \dot{e}_k - \left(\Gamma \varepsilon_k^T \cdot \dot{e}_k \right)^T \Gamma^{-1} \tilde{Z}k \right] d\tau$$

and so

$$\Delta W_k \le -V_{k-1} - \int_0^t \dot{e}_k^T \cdot \left(M_0 + B + \frac{1}{2}\varepsilon_k \cdot \Gamma \cdot \varepsilon_k^T \right) \cdot \dot{e}_k d\tau \le 0 \tag{13}$$

The function in the integral is positive semidefinite, hence

$$\Delta W_k \le -V_{k-1} \le 0 \tag{14}$$

From (14) follows

$$W_k \le W_{k-1} \qquad \text{for } k = 1, 2, \ldots$$

The sequence$\{W_k\}_{k=0}^{\infty}$is non-increasung, therefore if we shall prove that $W_0 = W_0(t)$ is bounded on $[0,T]$ we can derive the consequences $\dot{e} \to 0$ and $e_k \to 0$ as $k \to \infty$, as follows.

$$\sum_{j=0}^{k-1} V_j \le -\sum_{j=0}^{k-1} \Delta W_j = -\sum_{i=1}^{k} \Delta W_i = \sum_{i=1}^{k} (W_{i-1} - W_i) = W_0 - W_k \le W_0$$

that is,

$$\sum_{j=0}^{k-1} V_j \le W_0 \tag{15}$$

From (15) and (6) follows

$$\sum_{j=0}^{k-1} \dot{e}_j^T \cdot M_j \cdot \dot{e}_j + \sum_{j=0}^{k-1} e_j^T \cdot A \cdot e_j \le 2W_0 \tag{16}$$

and from (16)

$$\sum_{i=0}^{\infty} \dot{e}_i^T M_i \dot{e}_i \le 2W_0 \tag{17}$$

$$\sum_{i=0}^{\infty} e_i^T A e_i \le 2W_0. \tag{18}$$

These series (17) and (18) are convergent, hence

$$\dot{e}_i^T M_i \dot{e}_i \to 0 \quad \text{and} \quad e_i^T A e_i \to 0 \tag{19}$$

as $i \to \infty$ for $t \in [0,T]$. But M_i is regular, bounded positive definite and symmetric, hence

$$0 < \dot{e}_i^T \cdot M_i \cdot \dot{e}_i \quad \text{for} \quad \dot{e}_i \neq 0,$$

so from (19) follows that

$$\dot{e}_i \to 0 \quad \text{as} \quad i \to \infty \quad \text{for} \quad \forall t \in [0,t]\,.$$

Similarly follows

$$e_i \to 0 \quad \text{as} \quad i \to \infty \quad \text{for} \quad t \in [0,T]\,.$$

So $e_i, \dot{e}_i \in L_{pe}$ for $p = \infty$. But $e_i(t), \dot{e}_i(t)$ are continuous on compact interval $[0,T]$, so for arbitrary $1 \le p < \infty$ is

$$\int_0^T \left|\dot{e}^i\right|^p d\tau < \infty \tag{20}$$

and

$$\int_0^T \left|e^i\right|^p d\tau < \infty, \tag{21}$$

so $\dot{e}_i, e_i \in L_{pe}$ for $1 \le p \le \infty$.

It remains to prove $W_0(t)$ is bounded on $[0,T]$.

$$W_0 = V_0 + \frac{1}{2}\int_0^t \tilde{Z}_0^T \Gamma^{-1} \tilde{Z}_0 d\tau \tag{22}$$

If we define $\hat{Z}_0(t) = 0$, then $\tilde{Z}_0 = Z - \hat{Z}_0(t) = Z = (\alpha,\beta)^T$ The first part is bounded because every solution of equation (1) is bounded. The integral part is bounded too, because it is only a linear function of time on the interval $0 \le t \le T$, T is fixed, hence we see $W_0(t) < \infty$. From (4) follows $\hat{Z}_k \in L_{pe}$ for all $k \in \mathbb{N}_0$ and from (5) and (3) follows $u_k(t) \in L_{pe}$ for all $k \in \mathbb{N}_0$.

3. References

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