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STUDY OF SEMIACTIVE SPRING AND DAMPER BEHAVIOR IN A VEHICLE SUSPENSION

M. Zuščík*, M. Musil**

Summary: *The application to improve vehicle dynamics requires an instant change of stiffness and damping coefficient. In the paper, a method is shown which enables to create an extended quarter car model which is enough trustworthy and sufficient for the vertical dynamics analysis. The developed model is compared to an extended full car model in with four persons and engine. The silent blocks and the tire model are designed be means of a parallel combination of Maxwell element and a linear spring. The frequency analysis of the car model has been split in to a three frequency ranges: low frequency of the excitation, medium and high. In this paper is shown that the performance index in first range depends mostly on stiffness, in the second range on the damper properties.*

1. Introduction

The motivation for an automotive suspension system with independent control of stiffness, damping and ride-height comes from the trade-offs involved for the conflicting requirements of comfort and handling. The authors have previously proposed systems like: electromechanical suspension or double-volume air suspension system capable of independent control of stiffness, damping and ride-height and discussed the application of this suspensions to improved vehicle dynamics. The primary function of a vehicle suspension system is to isolate the road excitations experienced by the tires from being transmitted to the passengers. In this paper, a suitable optimizing component is applied at the suspension design to obtain the suspension parameters of a passive suspension and active suspension for a passenger car which satisfies with the performance as per ISO 2631 standards. A number of objectives such as maximum bouncing acceleration of seat and sprung mass, root mean square (RMS) weighted acceleration of seat and sprung mass as per ISO2631 standards, jerk, suspension travel, road holding and tire deflection are minimized subjected to a number of constraints. The constraints arise from the practical kinetic and comfort considerations, such as limits of the maximum vertical acceleration of the passenger seat, tire displacement and the suspension working space. A new method of constitution, of an extended quarter car model is presented in the paper. This extended model is enough trustworthy and sufficient for the vertical dynamics analysis. The developed model is compared to an extended full car model,

* Ing. Marián Zuščík, Institute of applied mechanics and mechatronics, STU in Bratislava, Nam. slobody 17, 812 31 Bratislava 1; tel.: +421 212 572 96 404, fax: +421 212 5249 7890; e-mail: zuscik@gmail.com

** Doc. Ing. Milos Musil, PhD., Institute of applied mechanics and mechatronics, STU in Bratislava, Nam. slobody 17, 812 31 Bratislava 1; e-mail: milos.musil@stuba.sk

which has fifty degrees of freedom. The extended quarter car model is created so that one can obtain, vary similar result to the extended full car model if it is used for the passive and active optimization process. The frequency analysis of the car model has been split in to a three frequency ranges. Low frequency of the excitation: (0-3) Hz, medium (3-10) Hz and high (10-25) Hz. It is shown in the paper that the performance index in first range depends mostly on stiffness (low stiffness coefficient is required), in the medium range on the damper (lower damping coefficient is required). The solution for the first sector is the double volume spring with auxiliary volume. The stiffness of this spring depends on the frequency. The stiffness property is changing from a specified frequency which can be chosen by setting the specified rate of valve opening. The problem of the second frequency range solves a semi-active Magneto-Rheological damper or a controlled viscous damper.

2. Extended full car model

The extended full car model consists of two masses with three degrees of freedom (the mass of the car body and the mass of the engine) and twelve masses with one degree of freedom (the unsprung masses, the masses of the roods of the dampers, and the masses of the passengers). A steady state model (1) of the extended full car has fifty degrees of freedom and is quite complicated.

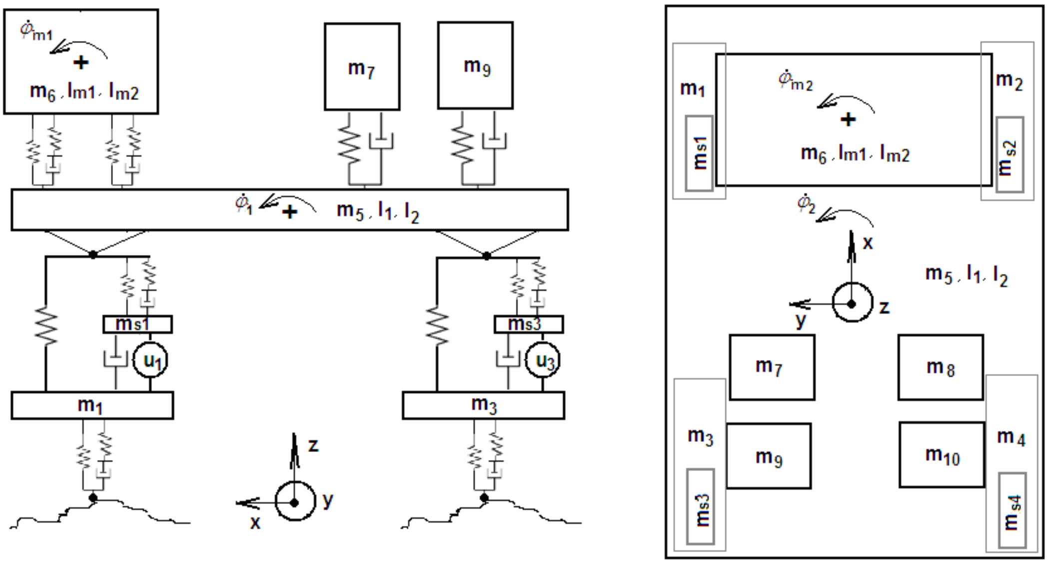

Figure 1 Extended full car mechanical model.

The equations (1) and (2) are the steady state equations for the passive and active suspension of the extended full car suspension model.

$$\dot{\mathbf{x}}(t) = \mathbf{A}\ \mathbf{x}(t) + \mathbf{G}\ \mathbf{w}(t) \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C}\ \mathbf{x}(t)$$

$$\dot{\mathbf{x}}(t) = \mathbf{A}\ \mathbf{x}(t) + \mathbf{B}\ \mathbf{u}(t) + \mathbf{G}\ \mathbf{w}(t) \tag{2}$$

$$\mathbf{y}(t) = \mathbf{C}\ \mathbf{x}(t) + \mathbf{D}\ \mathbf{u}(t)$$

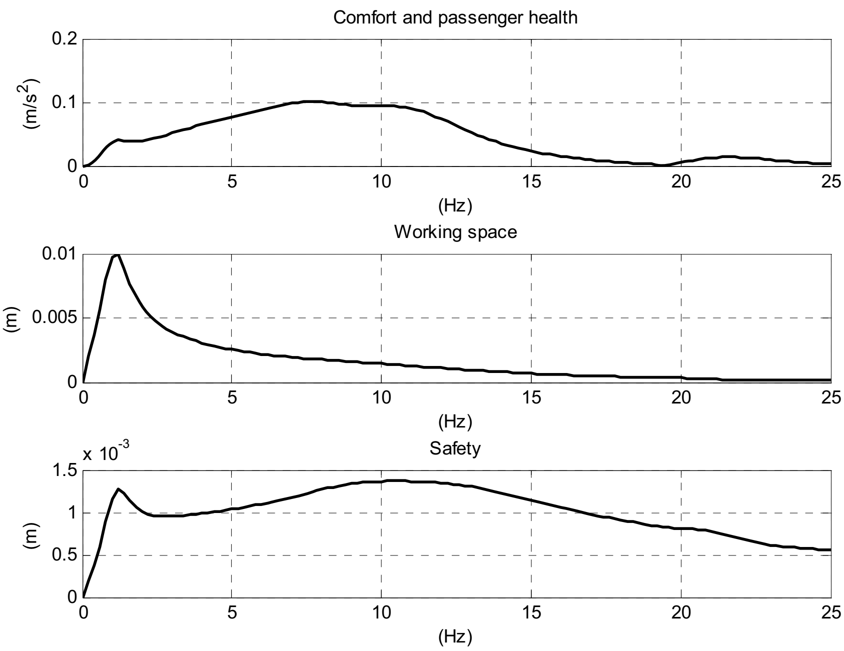

Figure 2 Amp-freq. characteristic of the extended full car performance indexes.

3. Extended quarter car model

The extended car model is created so that all of the main eigenvalues of the extended full car and the extended quarter car are similar, or with a very small deviation. It consists only of masses with one degree of freedom (the mass of the car body, the unsprung mass, the mass of the engine, the mass of the damper rood and the masses of one passenger). A steady state of this model has only thirteen degrees of freedom, is quite simple and allows as study the car suspension behavior more complex.

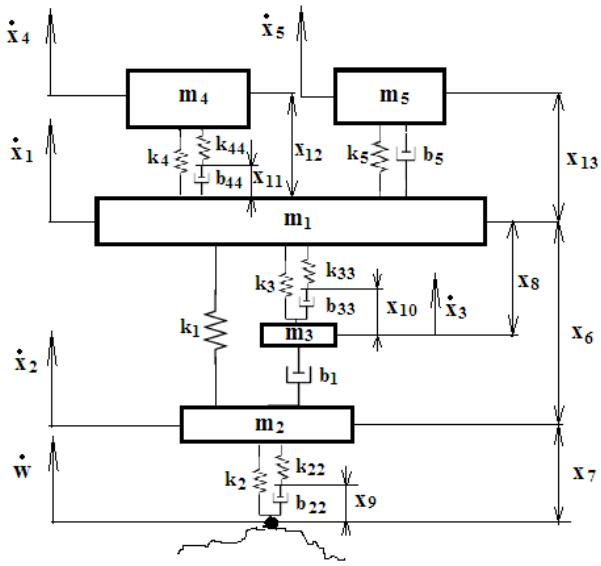

Figure 3 Extended quarter car mechanical model.

Figure 4 Amp-freq. characteristic of the extended quarter car performance indexes.

4. Ideal Semi-active damper

In a cases where in the model semi-active components are used, is as the semi-active damper an ideal semi-active damper considered. This damper can change its damping coefficient between the minimal and maximal value in an infinitely short time.

5. Pneumatics spring with auxiliary volume

The model of the double volume pneumatics spring (DVPS) is described by means of a set of nonlinear differential equations.

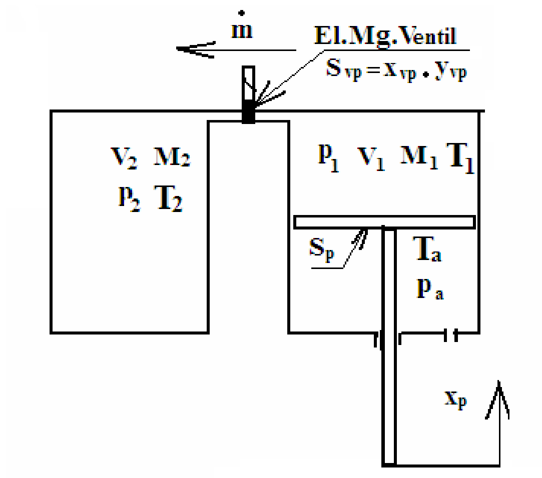

Figure 5 Pneumatics spring with auxiliary volume.

In the equations below the following denominations were used.

- $V_1, T_1, M_1, \rho_1, p_1$ are, the volume, temperature, mass of the air, density and pressure in primary volume of the DVPS,
- $V_2, T_2, M_2, \rho_2, p_2$ are, the volume, temperature, mass of the air, density and pressure in secondary volume of the DVPS,
- d_p, S_p, are the piston diameter and the piston area,
- $\dot{m}, S_{vp}$ are the mass flow though the valve and cross flow area of the valve,
- T_{int}, is the temperature of the incoming mass and for the flow from V_1 to V_2 it is equal T_1 an for the flow from V_2 to V_1 it is equal T_2,
- E_1, Q_1, W_1, Q_{m1} are, the internal energy of the system, the energy from the surroundings, the mechanical work and the energy which has bean brought with the incoming mass, in the volume V_1.
- E_2, Q_2, W_2, Q_{m2} are, the internal energy of the system, the energy from the surroundings, the mechanical work and the energy which has bean brought with the incoming mass, in the volume V_2,

The equations (3) and (4) are the constitutive relation for the gasses in volumes V_1 and V_2 in the algebraic and differential form.

$$p_1\, V_1 = M_1\, r\, T_1 \text{, or } \frac{p_1}{\rho_1} = r\, T_1 \qquad p_2\, V_2 = M_2\, r\, T_2 \text{, or. } \frac{p_2}{\rho_2} = r\, T_2 \tag{3}$$

$$\frac{\dot{p}_1}{p_1} + \frac{\dot{V}_1}{V_1} = \frac{\dot{M}_1}{M_1} + \frac{\dot{T}_1}{T_1}, \qquad \frac{\dot{p}_2}{p_2} = \frac{\dot{M}_2}{M_2} + \frac{\dot{T}_2}{T_2} \quad . \tag{4}$$

$$\dot{M}_1 = \dot{m}_1 = \dot{m}, \qquad \dot{M}_2 = \dot{m}_2 = -\dot{m}\,. \tag{5}$$

The first law of thermodynamics for V_1 and V_2 is,

$$\frac{dE_1}{dt} = \dot{Q}_1 - \dot{W}_1 + \dot{Q}_{m1} \tag{6}$$

$$\frac{dE_2}{dt} = \dot{Q}_2 - \dot{W}_2 + \dot{Q}_{m2}\,. \tag{7}$$

Also an important part of the solution is to solve the unstationary flow between the bought volumes. The equation of the conservation of the energy in the system is

$$c_p (T_1 - T_2) + \left(\frac{v^2}{2} \right) = 0\,. \tag{14}$$

From the last equation we can express the formula for the speed of the fluent gas

$$v = \sqrt{2 c_p (T_1 - T_2)}\,, \quad \text{or} \quad v = \sqrt{\frac{2\kappa}{\kappa - 1} \left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} \right)}\,. \tag{15}$$

The maximal possible speed can by solved as follow

$$v_{krit} = \sqrt{\kappa\, r\, T_{\text{int}}}\ . \tag{16}$$

And finally we can express the equation for the mass flow through the valve

$$\dot{m} = \mu\, S_v\, \rho_{out}\, v\ . \tag{17}$$

By using a Churchill formula (19) to obtain a Nusselt number for the heat convection between the wall and the gas we can further obtain a heat transfer coefficient for this case.

$$\alpha = \frac{\lambda}{l} Nu \text{ .pričom } Nu = f(Ra, \text{Pr})\ . \tag{18}$$

$$Nu = 0.68 + \frac{O.67 Ra^{1/4}}{[1+(0.492/\text{Pr})^{9/16}]^{4/9}} \text{ pre } Ra \le 10^9\ . \tag{19}$$

where $$Ra = \frac{g\beta}{\nu a}(T_s - T_a) l^3, \qquad \text{Pr} = \frac{c_p \eta}{\lambda}\ . \tag{20}$$

where g is the gravitational constant, T_s is the temperature of the wall and T_a is the temperature of the air in the spring and l is the characteristic dimension,

η and ν, are the dynamics and kinematics viscosity of the air, β is the thermal expansivity of the air and a is the heat conductivity coefficient. These properties could by obtained by means of the next equations,

$$\nu = \frac{\eta}{\rho}\ , \qquad \frac{1}{\rho} = \frac{\nu}{\eta} = \frac{rT_m}{p_a}, \qquad \beta = \frac{1}{T_m}\ , \qquad T_m = \frac{(T_s + T_a)}{2} \tag{21}$$

$$a = \frac{\nu\lambda}{\eta c_p} = \frac{\lambda}{\rho c_p} = \frac{r\lambda T_m}{c_p p_a} \tag{22}$$

and then the Rayleigh and Prantl number could by expressed as follow,

$$Ra = \frac{g c_p p_a}{\nu r \lambda T_m^2}(T_s - T_a) l^3, \qquad Pr = \frac{r c_p T_m \nu}{\lambda p_a}\ . \tag{23}$$

By means of the previous equations, we cam expressed a set of nonlinear differential equation for the volume V_1 (25-30), and V_2 (32 - 36), where we assume that the initial conditions are expressed by (24) for V_1 and (31) for V_2. The sum $\sum_{i=1}^{i\max} m_i$ is the whole mass exposed to the pneumatics spring.

$$T_{10} = T_a\,, \quad p_{10} = \frac{(\sum_{i=1}^{i\max} m_i\ g)}{S_p} = \frac{4\ (\sum_{i=1}^{i\max} m_i\ g)}{\pi d_p^2}\,, \qquad V_{10} = S_p\ x_0\,. \quad M_{10} = \frac{p_{10}\, V_{10}}{r\, T_{10}} \tag{24}$$

$$\dot{p}_1 = \frac{p_1}{c_v}\left(\frac{\alpha_1 S_1 (T_a - T_1)}{M_1 T_1} - c_p \frac{\dot{V}_1}{V_1} + c_p \left(\frac{T_{\text{int}}}{T_1}\right)\left(\frac{\dot{m}_1}{M_1}\right)\right) \tag{25}$$

$$p_1 = p_{10} + \int_0^t \dot{p}_1 dt \tag{26}$$

$$\dot{T}_1 = \frac{T_1}{c_v}\left(\frac{\alpha_1 S_1 (T_a - T_1)}{M_1 T_1} - r\frac{\dot{V}_1}{V_1} + c_p\left(\frac{T_{\text{int}}}{T_1}\right)\left(\frac{\dot{m}_1}{M_1}\right) - c_v\frac{\dot{m}_1}{M_1}\right) \tag{27}$$

$$T_1 = T_{10} + \int_0^t \dot{T}_1 dt \tag{28}$$

$$M_1 = M_{10} + m_1 = M_{10} + \int_0^t \dot{m}\, dt \tag{29}$$

$$V_1 = V_{10} + S_p\, x_p,\ \dot{V}_1 = S_p\, \dot{x}_p \tag{30}$$

$$T_{20} = T_a, \qquad p_{20} = p_{10}, \qquad V_{20} = V_2 = konst \Rightarrow \dot{V}_2 = 0. \qquad M_{20} = \frac{p_{20}\, V_{20}}{r\, T_a} \tag{31}$$

$$\dot{p}_2 = \frac{p_2}{c_v}\left(\frac{\alpha_2 S_2 (T_a - T_2)}{M_2 T_2} + c_p\left(\frac{T_{\text{int}}}{T_2}\right)\left(\frac{\dot{m}_2}{M_2}\right)\right) \tag{32}$$

$$p_2 = p_{20} + \int_0^t \dot{p}_2 dt \tag{33}$$

$$\dot{T}_2 = \frac{T_2}{c_v}\left(\frac{\alpha_2 S_2 (T_a - T_2)}{M_2 T_2} - r\frac{\dot{V}_2}{V_2} + c_p\left(\frac{T_{\text{int}}}{T_2}\right)\left(\frac{\dot{m}_2}{M_2}\right) - c_v\frac{\dot{m}_2}{M_2}\right) \tag{34}$$

$$T_2 = T_{20} + \int_0^t \dot{T}_2 dt \tag{35}$$

$$M_2 = M_{20} + m_2 = M_{20} - \int_0^t \dot{m}\, dt \tag{36}$$

The nominal stiffness of a pneumatics spring is given by the equation (37). By means of these equation we can estimated the minimal and maximal stiffness of the double volume spring (38).

$$k = \frac{\kappa\, S_p^2 p}{V} \tag{37}$$

$$k_{\min} = \frac{\kappa\, S_p^2 p}{V_1}, \qquad k_{\max} = \frac{\kappa\, S_p^2 p}{(V_1 + V_2)} \tag{38}$$

For a concrete double volume spring with a concrete cross flow area on the valve we can created a linear model Fig. 6. This mode is accurate only for a specified range of the excitation frequency. For other range of excitation frequency we have to change the damping coefficient of the linear damper, which represents the flow resistance of the air passing through the valve.

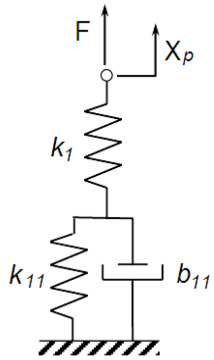

Figure 6 Linear model of pneumatics spring.

The stiffness k_1 is the stiffness of the gas in the primary volume and the stiffness k_{11} is the stiffness of the gas in the secondary volume. Parameter b_{11} represents the flow resistance through the valve. The value of the damping parameter can by estimate by means of the thermodynamics modeling. It is obvious that if $b_{11} = 0$, than the spring has a minimal stiffness and if $b_{11} = \infty$, the spring reaches the maximal stiffness. The stiffness can by solved by means (39) and (40) is the transfer function of the mechanical model of the spring.

$$k_1 = \frac{\kappa\, S_p^2 p}{V_1}, \qquad k_{11} = \frac{\kappa\, S_p^2 p}{V_2} \tag{39}$$

$$H(s) = \frac{F(s)}{X_p(s)} = \left[\frac{k_1(k_{11} + b_{11}s)}{k_1 + k_{11} + b_{11}s}\right] \tag{40}$$

6. Control policy hybrid

The hybrid control policy is a combination of a Skyhook and Groundhook controller policy. This controller can provide the suspension system with almost the some quality of regulation as the full LQR controller. Its mechanical model is shown in Fig. 7.

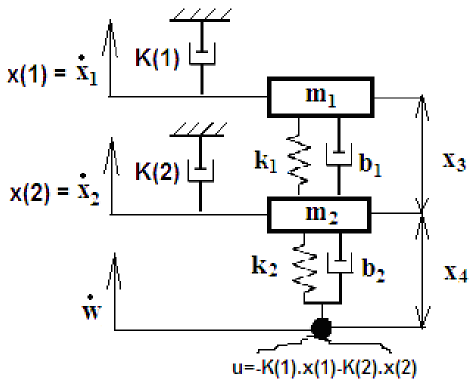

Figure 7 Mechanical model of a hybrid controller.

7. Numerical simulation

In the figures below, the characteristic of performance indexes of the different suspensions types considered in this work are shown.

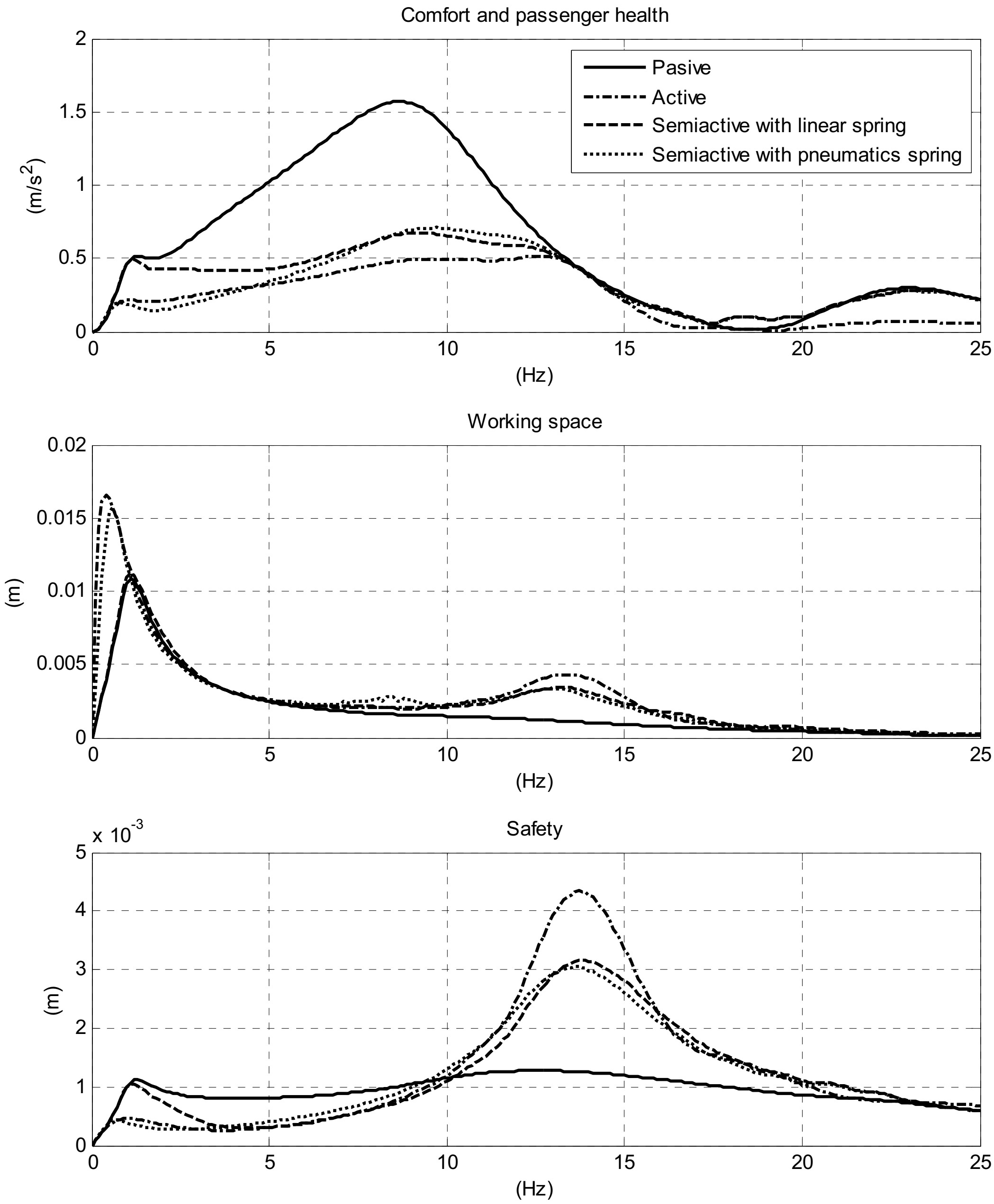

Figure 8 Amp-freq. characteristic of performance indexes of various suspension systems

8. Conclusion

In the Fig. 8 we can clearly see that the best performance indexes has the active suspension and the worst has the passive suspension. The other two curves represent the semi-active suspension with linear spring (dashed) and the semi-active suspension with the pneumatics spring with auxiliary volume (dotted). In bought of the semi-active suspensions has been an ideal semi-active damper considered. From the simulation result we can make a conclusion that the performance indexes of the suspension can be optimized in a lower range of the excitation frequency by means of pneumatics spring with auxiliary volume and that also, if it is whit out a continuous controlled valve. It is because the frequency dependent stiffness parameter of the pneumatics spring with auxiliary volume, agrees with the requirements on the suspension at this frequency of excitation and they are: a lower stiffness and a higher damping coefficient. In the middle range of frequency excitation we can see that the pneumatics spring can make the behavior of the suspension better only at the beginning, but it makes not the system worse at the end of this frequency excitation range. The controlled damper is a necessary component to improve the suspension performance indexes in this frequency range. The performance indexes in the range of high frequency excitation, is more then less a matter of optimizing the silencer parameters.

9. Acknowledgement

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10. References

Deo, H.V. & Suh, N.P. (2004) Axiomatic Design of a Costumatizable Automotive Suspension, *Proceedings of the third International Conference on Axiomatic Design*, Seoul, Korea, pp. 21-24.

Stein, G. & J., Ballo, I. (1991) Vehicle System Dynamics, 20, p. 51

Tener, D.R. (2005) Overcoming the Ride/Handling Compromise – A Cockpit Adjustable suspension System, *Transactions of the SAE 2005 World Congress,*. Detroit, MI.

Suh, N.P., (2001) *Axiomatic Design: Advances and Applications,*. Design Book Series, Oxford University Press, Oxford, NY, 0-19-504345-6.

Karnop, D.C. (1983) Active Damping in Road Vehicle Suspension System, *Vehicle System Dynamics,* 12, 291-316.

Zuščík, M. & Musil, M. (2008) PNEUMATIC SUSPENSION SYSTEM WITH OPTIMAL CONTROL OF DAMPING, STIFFNESS AND RIDE-HEIGHT, Engineering Mechanics, 2008, Svratka, Czech Republic.

ISO: 2631-1, (1997), "Mechanical vibration and shock – Evaluation of human exposure to whole-body vibration".

Bartko R., Musil, M. & Úradníček J. (2007) LOADING OF TRUCK TYRE WITH SEMIACTIVE SUSPENSION, The 12th International Conference on Problems of Material Engineering, Mechanics and Design, Trenčianska Univerzita.