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# EXPERIMENTAL INVESTIGATION OF DRAG FORCE, MAGNUS FORCE AND DRAG TORQUE ACTING ON ROUGH SPHERE MOVING IN CALM WATER

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**Summary:** *The paper describes the results of experiments with a rotating golf ball moving quasi-steadily in calm water. The motion of the ball was recorded on a digital video camera. The dimensionless drag force, Magnus force, and drag torque coefficients were determined from the comparison of the calculated translational and angular velocities and trajectory with experimental ones for the rough particle. The proper value of the correction coefficients were established from condition of the best fitting of the experimental trajectory by the calculated one.*

## 1. Introduction

The investigation of forces acting on spheres with rough surface is mostly connected with games as baseball or golf, where the lateral deflection due to the rotation of the ball allows to obtain the higher and longer trajectories. The translational movement with simultaneous rotation of a solid particle in fluid is also important for many natural and industrial processes, for example, for the bed load sediment transport in rivers, hydraulic pipeline transport of slurries etc. The lateral deflection of a particle is caused by the lateral force due to the ball rotational movement, known as Magnus force, which is defined as

$$\boldsymbol{F}_M = C_M \Omega_p \rho_f \left[ \boldsymbol{\omega} \times \boldsymbol{V} \right], \tag{1}$$

where $\Omega_p$ is the particle volume, $\rho_f$ is the fluid density, $\boldsymbol{\omega}$ is the vector of the particle instantaneous angular velocity, $\boldsymbol{V}$ is the vector of the instantaneous translation velocity of the particle centre of mass and $C_M$ is the Magnus force coefficient. The goal of the paper is to determine relationships describing the effect of the particle surface roughness on the force acting on the spherical particle moving translationally with simultaneous rotation in calm water, the effect of translational and rotational Reynolds numbers on the coefficients of the forces, acting on the particle.

The theoretical analysis of the Magnus force was performed by Rubinov & Keller (1961) for $Re_p << 1$ and $Re_\omega << 1$. They deduced the value of $C_M = 3/4$.

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Davies (1949) investigated rotating golf balls in a wind tunnel and from his results the formula for Magnus force coefficient depending on rotational Reynolds number only was derived

$$C_M = \frac{5355}{\mathrm{Re}_\omega}\left(1-\exp\left(-8.25\cdot 10^{-5}\cdot \mathrm{Re}_\omega\right)\right). \tag{2}$$

The particle Reynolds number $\mathrm{Re}_p = Vd/v$ was based on air velocity $u$ in the wind tunnel and the ball diameter $d$, rotational Reynolds number is $\mathrm{Re}_\omega = \omega R_p^2/v$, where $R_p$ is the radius of the ball, $\omega$ is the module of the vector of the instantaneous angular velocity, and $v$ is the kinematical viscosity of the fluid. From the drift of the ball he calculated the drag and Magnus force and noticed that for golf balls Magnus force was much greater than that observed for smooth balls. His results are valid for the high Reynolds numbers: $\mathrm{Re}_p \leq 9.10^5$ and $\mathrm{Re}_\omega \leq 2.5.10^5$.

Oesterle & Dinh Bui (1998) measured the lift force acting on a rotating sphere moving in viscous fluid with the constant linear and angular velocities for the intermediate Reynolds numbers: $10 \leq \mathrm{Re}_p \leq 140$ and $5 \leq \mathrm{Re}_\omega \leq 420$. They examined the trajectory of the sphere moving upwards in a liquid at rest. The sphere was equipped with two very thin cylindrical axles. The motion was induced by means of two suspension threads, which were coiled on the axles, yielding a rotational velocity. Their investigation yields the following expression for Magnus force coefficient:

$$C_M = \frac{1}{12}\frac{\mathrm{Re}_p}{\mathrm{Re}_\omega}\left(1-\exp\left(-0.1\cdot \mathrm{Re}_\omega{}^{0.4}\cdot \mathrm{Re}_P{}^{0.3}\right)\right)+\frac{3}{4}\exp\left(-0.1\cdot \mathrm{Re}_\omega{}^{0.4}\cdot \mathrm{Re}_P{}^{0.3}\right). \tag{3}$$

Sawatzki (1970) established the dependence of the dimensionless drag rotation coefficient $C_\omega$ on the rotational Reynolds number ($\mathrm{Re}_\omega$). This dependence was developed based on the thorough experimental investigation conducted with the sphere rotating in fluid around its centre of mass without any translational movement. It is unfortunately only in the graphical form. Lukerchenko et al. (2008) found experimentally that the drag torque coefficient $C_\omega$ can be calculated using the data of Sawatzki (1970) according to the relationship Eq. (4) and drag force coefficient according to the relationship Eq. (5)

$$C_\omega = C_{\omega 0}\left(1+0.0044\, Re_p^{0.5}\right), \tag{4}$$

$$\mathrm{C_d} = \mathrm{C_{d0}}\left(1+0.065\, Re_\omega^{0.3}\right). \tag{5}$$

In the present study we deal with the translational movement of the rough sphere without rotation, and also with the rough sphere, which moves translationally with simultaneous rotation in calm water. The aim of the work is to find how the drag force, Magnus force and drag torque coefficients already known for a smooth sphere vary due to the particle surface roughness, i.e. to study the effect of the sphere roughness on these coefficient, evaluate the mutual effect of the translational and rotational particle movements, and to find the real value of the mentioned coefficients. The main difficulty is that it is impossible to get solely the Magnus force. Other forces, such as drag force and drag torque, can be measured individually.

More detailed review of drag force and Magnus force coefficient can be found for instance in Lukerchenko et al. (2005, 2008).

## 2. Experimental procedure

The experiments were carried out in the rectangular glass vessel of the inner sizes: 0.780 m long, 0.600 m wide and 1.030 m high. The water depth was kept on the level about 0.840 m. Spherical particle with rough surface, i.e. the golf ball was used as a particle model; the mean diameter of the golf ball was determined equal $d$ = 42.80 mm, mass $m$ = 0.0458 kg, and density of the balls $\rho_p$ = 1120 kg/m$^3$.

According to the generally used definition the particle relative roughness is $k_\Omega = (D_{max} - D_\Omega)/D_\Omega$ , where $D_{max}$ is the maximum diameter of the rough sphere, $D_\Omega$ is the reduce diameter, e.g. the diameter of the smooth sphere with the same volume as that of the rough sphere. For the used golf ball the relative roughness was $k_\Omega$ = 0.0026.

Each measured particle was speeded up in a special device that ensured the required particle rotation in the given plane. In all experiments the axis of the rotation of the device was situated at the height 35 mm above the water surface what determined the initial translational velocity with which the ball enters water. The starting ball rotation of 500, 1000, 1500, 2000, 2500, 3000, and 3500 rpm (revolutions per minute) were measured. Rotation was carried out in the clockwise direction and also in the opposite direction. An unsteady entrance region can be observed when the particle enters water. Therefore, only the experimental data outside this region were used for the data analysis. The ball movement in water was recorded using the digital video camera NanoSenze MKIII+ with frequency up to 1000 frames per second.

## 3. Numerical simulation

The system of equations describing the particle motion in calm water may be written as

$$\frac{4}{3}\pi R_P^3 \rho_P \frac{d\boldsymbol{V}}{dt} = \boldsymbol{F}_g + \boldsymbol{F}_d + \boldsymbol{F}_m + \boldsymbol{F}_M , \tag{6}$$

$$J\frac{d\boldsymbol{\omega}}{dt} = \boldsymbol{M}, \tag{7}$$

where $J$ is the particle momentum of inertia, $\boldsymbol{M}$ is the torque acting on a sphere rotating about its diameter in fluid, $\boldsymbol{V}$ is the module of the vector of the instantaneous translational velocity of the particle centre of mass.

The first term of the right-hand side of Eq.( 6) is the gravity force acting on the particle in water

$$\boldsymbol{F}_g = \frac{4}{3}\pi R_P^3 \left(\rho_P - \rho_f\right)\boldsymbol{g}, \tag{8}$$

where $\boldsymbol{g}$ is the vector of the gravitational acceleration. The second term is the drag force

$$\boldsymbol{F}_d = -\frac{1}{2} C_d \rho_f \pi R_P^2 |\boldsymbol{V}| \boldsymbol{V}. \tag{9}$$

where $C_d$ is the drag force coefficient. For the particle translational motion in fluid without rotation (Nino & Garcia, 1994) the drag force coefficient is given as a function of the particle Reynolds number

$$C_{d0} = \frac{24}{\mathrm{Re}_p}(1 + 0.15(\mathrm{Re}_p)^{\frac{1}{2}} + 0.017\,\mathrm{Re}_p) - \frac{0.208}{1 + 10^4\,\mathrm{Re}_p^{-0.5}} \tag{10}$$

The third term in Eq.(5) is the added mass force

$$\boldsymbol{F}_m = -\frac{4}{3}\pi R_P^3 \rho_f C_m \frac{d\boldsymbol{V}}{dt}. \tag{11}$$

The coefficient $C_m$ for the particle motion in a viscous fluid has the value $C_m$ = 0.5. The last term of the right-hand side of Eq. (6) is the Magnus force which is expressed by Eq. (1).

The torque of the force acting on the rotating particle in the fluid is

$$\boldsymbol{M} = C_\omega \frac{\rho}{2} \boldsymbol{\omega} |\boldsymbol{\omega}| R_P^5, \tag{12}$$

where $C_\omega$ is the dimensionless drag rotation coefficient expressed e.g. by Eq.(4).

The above mentioned system of equations was solved numerically. The values of the individual coefficients were found by the method of the best fitting of experimental data and compared with values valid for the same particle with smooth surface and correction coefficients were introduced to describe the effect of roughness.

## 3. Results and discussion:

In the common case the drag force, drag torque and Magnus force coefficients of the rough sphere moving translationally with simultaneous rotation in fluid can be defined as functions of the Reynolds number, rotational Reynolds number and the relative roughness:

$$C_d = C_d(\, k_\Omega\, , Re, Re_\omega), \tag{13}$$

$$C_\omega = C_\omega(\, k_\Omega\, , Re, Re_\omega), \tag{14}$$

$$C_M = C_M(\, k_\Omega\, , Re, Re_\omega). \tag{15}$$

The aim of the present study is to evaluate the effect of the particle roughness on these three coefficients. The problem is very complex and the effect of different $k_\Omega$ values and/or shape of so called dimples (roughness elements on the ball surface) were solved separately, e.g. by Aoki (2004). Therefore, in the present paper only one value of the relative roughness is considered that corresponds to the used golf ball roughness ($k_\Omega = 0.0026$).

The Magnus force acts in normal direction to the velocity vector, the particle trajectory deviates from the rectilinear direction. According to Eq. (1) the larger angular velocity corresponds to the larger deviation. This effect of the angular velocity on the experimental ball trajectories for five different values of the angular velocity is shown in the Figure 1.

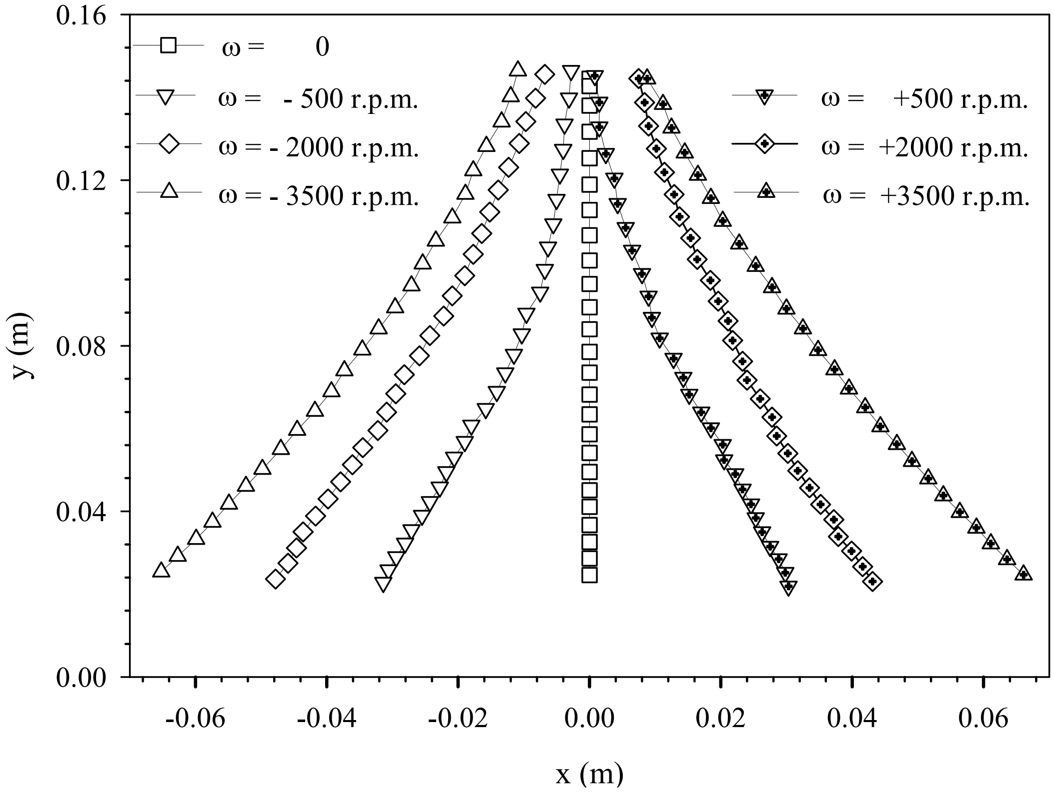


Figure 1 The golf ball trajectories for different values of the ball starting angular velocity *ω*

The drag force coefficient of the smooth sphere (i.e. when $k_\Omega = 0$) which moves translationally without rotation (i.e. when $Re_\omega = 0$) in fluid depends on particle Reynolds number $C_{d0} = C_{d0}\,(Re_p)$, see Eq. (10). Lukerchenko et al. (2008) found experimentally for the translational and simultaneously rotational motions of the smooth sphere the dependence of the drag force and drag torque coefficients on both the Reynolds number and rotational Reynolds number, which are expressed by Eqs. (4) and (5). The value of the Magnus force coefficient $C_M$ was found to be in range $0.023 \le C_M \le 0.048$.

In case of the translational motion of a rough sphere without rotation, the relationships for the drag coefficient will be $C_d = \lambda_d\,.C_{d0}$ , where $\lambda_d$ is the correction factor, which can be determined by comparison of the relationships valid for smooth and rough sphere of the same parameters. Achinbach (1974) found that the drag coefficient of rough ball falls compared to the smooth ball if the flow around the ball changes from laminar to turbulent. The transition velocity falls with increasing roughness of the ball, for golf ball the transition is in the Reynolds number region from about $3.10^4$ to $6.10^4$. Similar effect was found for tennis, squash or football balls. The drag coefficient of golf ball or squash ball is about $C_d = 0.45$ for

Re = 5.10[4], tennis ball is about $C_d = 0.65$ for Re = 5.10[4], and football is about $C_d = 0.65$ for Re = $10^5$ and about $C_d = 0.20$ for Re = $2.10^5$.

Let us suppose that effect of the surface roughness can be described by a correction factor and the Eqs. (13) and (14) can be found in the form

$$C_d = \lambda_\omega C_{d0} \left(1 + 0.065\, Re_\omega^{0.3}\right), \tag{16}$$

$$C_\omega = \xi\, C_{\omega 0} \left(1 + 0.0044\, Re_p^{0.5}\right). \tag{17}$$

The surface roughness correction factors of the smooth sphere $\lambda_\omega = \xi = 1$; the values of these correction factors greater than unit mean that due to the ball roughness the drag force or drag torque increases, for the values less than unit the opposite is valid.

The experimental data of the golf ball movement in water with the starting angular velocity varying from 500 to 3 500 rpm were used for the evaluation of the correction factors $\lambda_\omega$ and $\xi$. The Reynolds number ranged from 13 300 to 28 700, the rotational Reynolds number – from 16300 to 114100.

The above mentioned system of Eqs. (6-7), which describes the particle motion in fluid, together with Eq. (1), Eqs. (8-9), and (11 – 12)) and relationships for drag, torque and Magnus coefficient (Eqs.2-5), Eq. (10) and (Eqs.16-17) were used for the simulation of the golf ball motion.

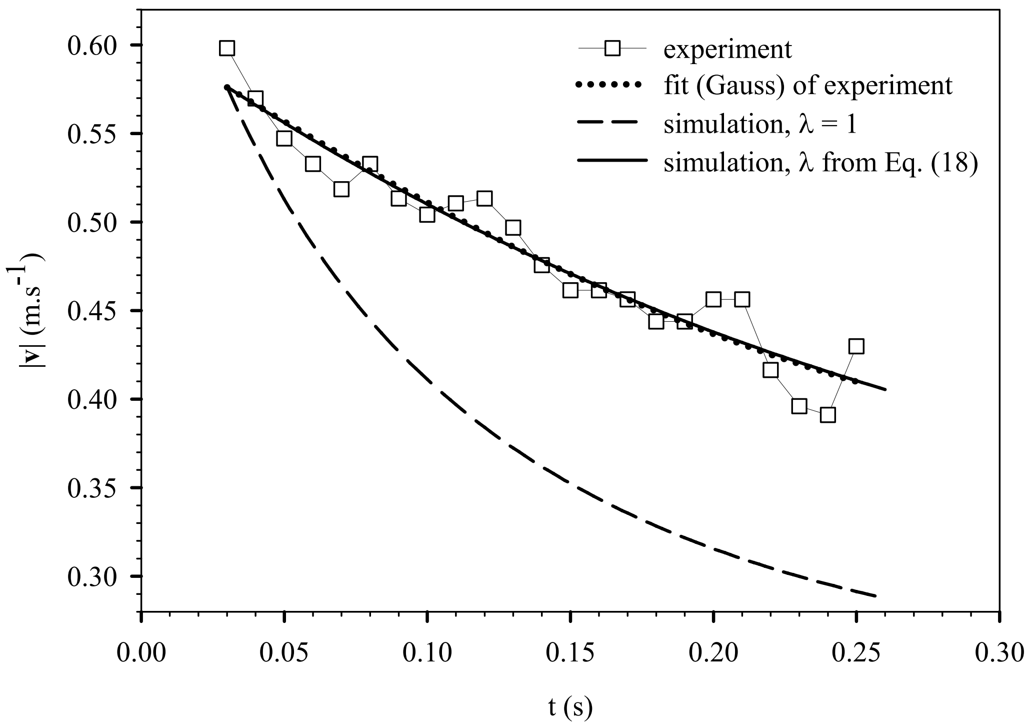


Figure 2 The translational velocity versus time (golf ball in water, $\omega = 2000$ rpm)

The correction factors $\lambda_\omega$ and $\xi$ correlations were found from the condition of the best agreement between numerical simulation and experimental data. The initial values of the correction factors $\lambda_\omega = \xi = 1$ were chosen, it corresponds to the smooth ball. In this case the calculated translational velocity modulus is less than the experimental one, the calculated angular velocity is greater than the experimental one, see Figures 2 and 3, respectively. It means that the proper values of the correction factors $\lambda_\omega$ and $\xi$ should be found from the comparison of experimental and calculated values of the translational velocity modulus and angular velocity modulus.

The final values of the correction factors determined by the simulation process satisfy the proper values of the drag force and drag torque coefficients. It can be confirmed by agreement of the calculated and experimental translational and angular velocities, see Figures 2 and 3, respectively, where the dependence of translational and angular velocities on time is presented for the ball movement with starting angular velocity $\omega$ = 2000 rpm. The agreement of the calculated and experimental ball trajectories satisfies the correct value of Magnus force coefficient, see Figure 4. The mutual influences of the correction factors one the another are small and can be reduced by using a few iterations. The Magnus force coefficient was established as constant for individual experiments, determined by the value of angular velocity, see Table 1.

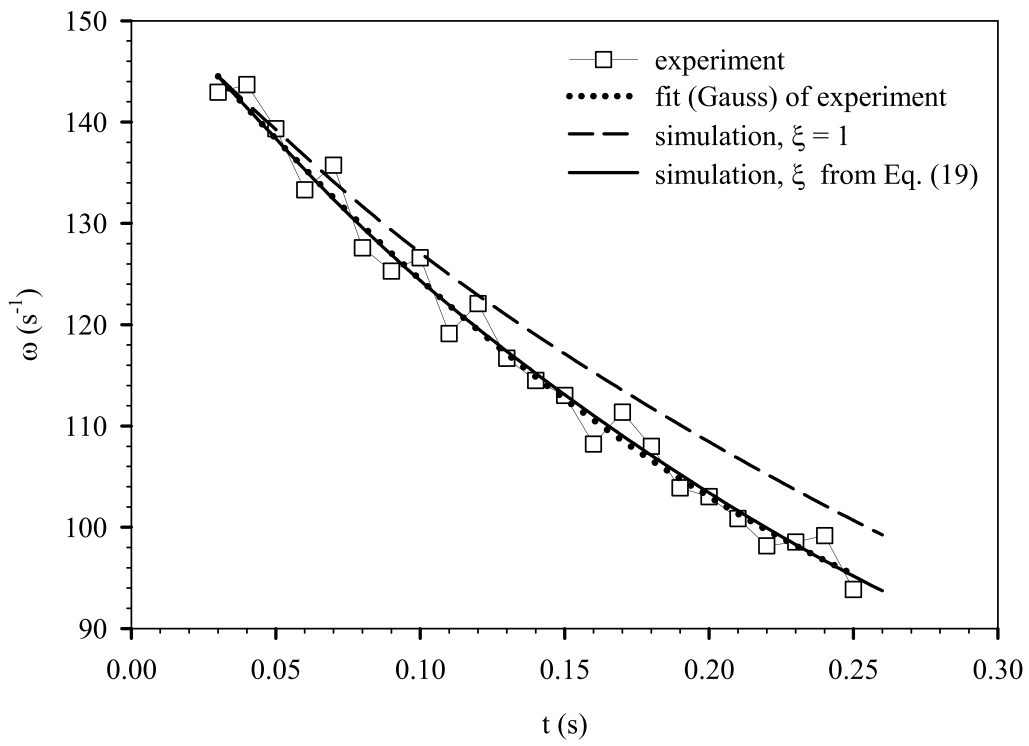


Figure 3 The angular velocity versus time (golf ball in water, $\omega$ = 2000 rpm)

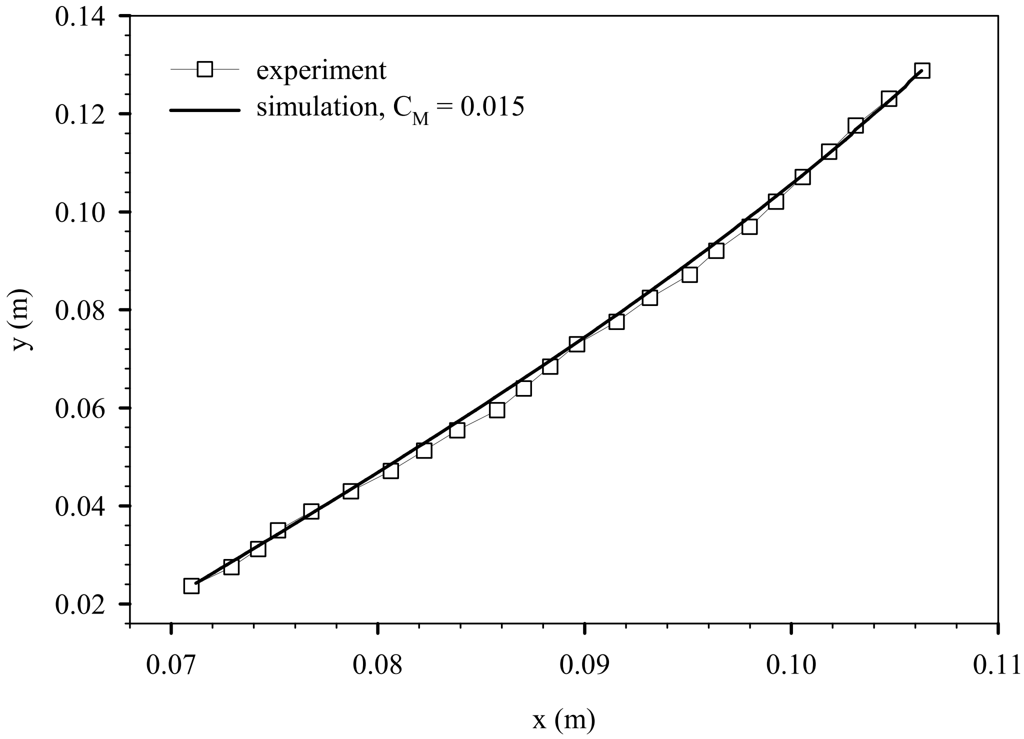


Figure 4 The ball centre trajectory (golf ball in water, $\omega$ = 2000 rpm)

Table 1 Magnus force coefficient $C_M$ for golf ball in water

| No. | $\omega$ [rpm] | *Re* | $Re_\omega$ | $\Gamma = 2\ Re_\omega / Re$ | $C_M$ |
|---|---|---|---|---|---|
| 1 | 500 | 13 300 – 28 700 | 16 300 – 21 200 | 1.55 – 2.22 | 0.080 |
| 2 | 1000 | 14 600 – 24 700 | 29 400 – 38 400 | 3.11 – 4.03 | 0.025 |
| 3 | 1500 | 16 500 – 26 100 | 37 600 – 57 600 | 4.41 – 4.57 | 0.021 |
| 4 | 2000 | 18 700 – 26 000 | 43 600 – 66 400 | 4.67 – 5.11 | 0.015 |
| 5 | 2500 | 17 400 – 25 800 | 48 700 – 84 100 | 5.60 – 6.52 | 0.017 |
| 6 | 3000 | 19 500 – 27 600 | 55 700 – 98 900 | 5.71 – 7.16 | 0.017 |
| 7 | 3500 | 19 400 – 27 300 | 59 500 – 114 100 | 6.12 – 8.35 | 0.010 |

The dependences of the drag force and drag torque correction factors on rotational Reynolds number can be expressed by Eqs. (18) and (19), see Figure 5

$$\lambda_\omega = 1.02 - 1.23*10^{-5}*Re_\omega + 4.5*10^{-11}*Re_\omega^2 \qquad (18)$$

$$\xi = 1.21 - 0.834*\exp(-4.2*10^{-5}*Re_\omega) \qquad (19)$$

These correlations were determined for the experiment with starting angular velocity $\omega$ = 2000 rpm. Verification of the correlations was carried out for the other six experiments and confirms a good agreement of the found relationships. The effect of the ball roughness on the drag torque and on the drag force is different for the studied ranges of the particle Reynolds number and rotational Reynolds number. The drag force coefficient, and of course also the drag force decreases with the increasing rotational Reynolds number. The opposite is valid for the drag torque coefficient. For the rotational Reynolds number less than about 32 800 the drag torque coefficient correction factor is less than unit, it means that the roughness reduces the drag torque for the low angular velocity. However, with increasing angular velocity the drag torque coefficient increases, for rotational Reynolds number exceeding values of about $10^5$ the drag torque correction factor is nearly constant, it reach value $\xi \approx 1.20$.

From the comparison of the Magnus force coefficient values of the rough ($0.010 \le C_M \le 0.080$) and smooth ball ($0.023 \le C_M \le 0.048$) the following conclusion can be made: the ball roughness increases the Magnus force for the relatively low values of the rotational Reynolds number, and decreases the Magnus force for the medium and high values of the rotational Reynolds number (i.e. over above1000 rpm).

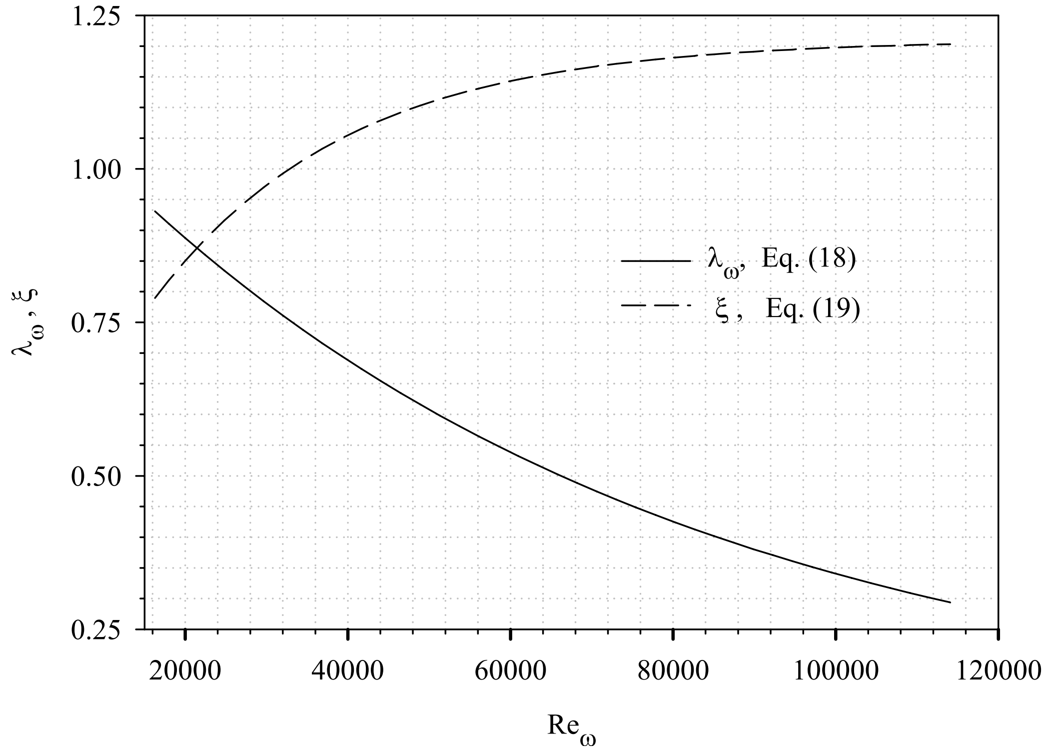


Figure 5 The drag force and drag torque correction factors $\lambda_\omega$ and $\xi$ versus the rotational Reynolds number $Re_\omega$

## 4. Conclusions

The effect of the rough surface of the spherical particle on the drag force, drag torque and Magnus force coefficients was studied experimentally and evaluated using the numerical simulation. The golf ball was used as the model particle, the Reynolds number ranged from 13 300 to 28 700, the rotational Reynolds number from 16300 to 114100.

It was revealed that

- the drag force coefficient of the rough ball decreases with increase of the rotational Reynolds number $Re_\omega$,
- the drag torque coefficient of the rough ball increases with increase of the rotational Reynolds number $Re_\omega$, however for $Re_\omega$ less than about 32 000 the correction factors of the drag torque coefficient is less than unit
- the ball roughness increases the Magnus force coefficient of the rough ball for the relatively small values of the rotational Reynolds number $Re_\omega$, for the relatively high values of $Re_\omega$ (i.e. over above1000 rpm) the Magnus force coefficient is less than that of the smooth ball.

## 5. Acknowledgements

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## 6. Notation

$C_d$ - drag force coefficient;
$C_{d0}$ - drag force coefficient for the particle moving in fluid without rota- tion;
$\lambda_\omega$ - roughness correction factor for the drag force coefficient $C_d$;
$C_m$ - added mass coefficient;
$C_M$ - Magnus force coefficient;
$C_\omega$ - drag rotation coefficient;
$C_{\omega 0}$ - drag rotation coefficient for the particle rotating in calm fluid;
$D_{max}$ - maximum diameter of the rough sphere;
$D_\Omega$ - reduce diameter of the rough sphere;
$\xi$ - roughness correction factor for the drag rotation coefficient $C_\omega$;
$\boldsymbol{F_d}$ - drag force;
$\boldsymbol{F_g}$ - submerged gravitational force;
$\boldsymbol{F_m}$ - added mass force;
$\boldsymbol{F_M}$ - Magnus force;
$g$ - gravitational acceleration;
$J$ - particle momentum of inertia;
$k_\Omega$ - the relative surface roughness of the rough sphere;
$M$ - moment of the force acting on the rotating particle in fluid;
$R_p$ - particle radius;
$Re_p$ - particle Reynolds number;
$Re_\omega$ - rotational particle Reynolds number;
$\Gamma$ - dimensionless angular velocity;
$\boldsymbol{V}$ - vector of the translational particle velocity;
$t$ - time;

$\rho_f$ - fluid density;
$\rho_p$ - particle density;
$\boldsymbol{\omega}$ - particle angular velocity.

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