

# SIMULATION OF THE FRACTURE PROCESS IN QUASI-BRITTLE MATERIALS USING A SPRING NETWORK MODEL

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**Abstract:** This paper deals with numerical simulation of the fracture process in cementitious composite specimens. A technique based on physical discretization of continuum is employed. The study supplements the verification of a technique for estimation of the extent (size and shape) of the fracture process zone in quasi-brittle silicate-based specimens/structures during tensile failure. The analysis presented in the paper shows that the extent of the cumulative failure zone simulated by the considered model agrees very well with experimental results reported in the literature.

### Keywords: Quasi-brittle fracture, physical discretization, nonlinear dynamic system, process zone.

## 1. Introduction

Today, numerical simulations are often used to investigate failure processes in quasi-brittle cementitious composites. In order to better understand and verify the fundamental issues involved, numerous simulation techniques have been proposed based on different approaches (for a review see e.g. Bažant and Planas, 1998, Jirásek, 2009). In this paper, simulations by means of the physical discretization of continuum technique (Frantík, 2007) are presented. This discretization technique, described in detail later below and successfully applied in e.g. Frantík et al. (2009), is similar to lattice modelling and uses a procedure developed for rigid body spring networks (RBSN) (Bolander et al., 1999). The computational code developed for the simulations by the authors is based on a nonlinear dynamical description of the problem. The results of such numerical dynamical simulations are considered to answer questions regarding the evolution of the fracture process zone (FPZ), the amount of energy dissipated within it, etc.

# 2. Numerical simulation of experiments

Several experimental techniques which deal with the estimation of the zone of tensile failure in quasibrittle cementitious composites are reported in the literature (summarized in e.g. van Mier 1997, Shah et al. 1995). The simulations presented in this paper are compared to records of experimental techniques based on acoustic emission scanning (AES). This technique was considered here under the assumption that the sources of AE phenomena are similar in nature to the failure mechanisms simulated within the used model and also because AES is the method most widely employed in such research.

Mihashi & Nomura (1996) tested two sets of wedge-splitting test (WST) specimens made of concrete and mortar differing in strength and aggregate size. The experimental set-up with an indication of the specimens' dimensions is depicted in Fig. 2 left. Two specimens (marked as C10 and C20) were selected for simulation of the fracture process since the results of the AE scanning of fracture phenomena (i.e. the extent of the damage zone) were also reported in the paper.

# 2.1. Numerical modelling via the physical discretization of continuum approach

The modelled specimen is substituted by a set of mass points with the given coordinates  $(x_i, y_i)$  in Cartesian space, mutually connected by simple translational springs. The springs can be generally

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defined as fully nonlinear and their inertial properties are neglected (concentrated at the mass points). The model is defined as a nonlinear dynamical system described by the following equations:

$$\frac{dx_{i}}{dt} = v_{xi}, \quad \frac{dv_{xi}}{dt} = \left(R_{xi} - c_{1}m_{i}v_{xi}\left(1 + c_{2}|v_{xi}|(1 + c_{3}v_{xi})\right)\right), \\
\frac{dy_{i}}{dt} = v_{yi}, \quad \frac{dv_{yi}}{dt} = \left(R_{yi} - c_{1}m_{i}v_{yi}\left(1 + c_{2}|v_{yi}|(1 + c_{3}v_{yi})\right)\right), \quad (1)$$

where  $v_{xi}$  and  $v_{yi}$  are the velocity vector components of the mass point with index *i*,  $m_i$  is its mass,  $c_1$ ,  $c_2$ ,  $c_3$  are coefficients of nonlinear viscous damping and  $R_{xi}$ ,  $R_{yi}$  are vector components of the resultant force  $R_i$  caused by connected translational springs. A single translational spring with indices *ij* loads the mass points *i* and *j* via the force  $F_{ij} = f_{ij}(u_{ij})$ , where  $f_{ij}$  is the stress function shown in Fig. 1 left. This function is described using four parameters: stiffness  $k_{ij}$ , initial critical tensile elongation  $u_{crit,0ij}$ , initial 'zero' elongation  $u_{zero,0ij}$  (i.e. the value of the tensile elongation for which the value of the carrying force drops to zero), and initial 'flac' elongation  $u_{flac,0ij}$  (i.e. the value of the carrying force reaches a constant value).



Fig. 1: Scheme of the stress function used within the FyDiK simulation technique.

As was mentioned above, the discretization technique used here is based on a procedure developed for the RBSN model (Bolander et al. 1999). This means that the mass of the mass points and the parameters of the translational springs are computed based on Delaunay triangulation and Voronoi tessellation. Mass points are generated to cover an area larger than that of the specimen. This area serves as a virgin material from which the specimen area is cut. The generation of mass points is provided in equilateral triangular mesh of size *s* (triangle side length) with an added random 'noise' vector of size 0.8rs, where *r* is a random number

from the rectangular distribution over an interval of unit length. Every mass point has its own Voronoi cell, which specifies its mass according to cell area, material density  $\rho$  and thickness *b*. Similarly, every translational spring has an edge of a Voronoi cell which specifies its 'cross-sectional' area. This area,  $A_{ij}$ , is then used for calculation of the spring parameters (more details in Frantík et al., 2011).

The FyDiK model enables simulation of the progress of failure of the selected WST specimen. The numerical model of the test was created from a homogeneous elastic isotropic material with parameters corresponding to the concrete tested by Mihashi & Nomura (1996), i.e. specific gravity  $\rho = 2400 \text{ kg/m}^{-3}$ , modulus of elasticity E = 33.01 GPa, tensile strength  $f_t = 2.8$  MPa, and compressive strength  $f_c = 31.45$  MPa. The specimen was modelled free of any boundary conditions except the fixation of the velocities of two points, which ensures loading. An illustrative example of the discretization of a WST specimen is shown in Fig. 2. Dynamical effects were regulated by nonlinear viscous damping optimized for this loading speed by the coefficients:  $c_1 = 1000 \text{ Nskg}^{-1}\text{m}^{-1}$ ,  $c_2 = 0$ ,  $c_3 = 100$ . Loading was stopped after reaching the time t = 0.5 s.

#### 2.2. Results

During the simulation, the amount of dissipated energy caused by exceeding the critical elongation of the individual translational springs was recorded, see Fig. 3 and 4. The locations of the individual failure events are indicated by spots in the images and coloured according to the amount of dissipated energy. The size of the spots corresponds with the length of the associated spring (where the energy is dissipated). The term 'dissipated energy' refers to the energy released by the failure of particles (being at the softening branch of the stress function, see Fig. 1) creating the model in this paper. The evolution of a simulation of the FPZ is depicted via a sequence of six stages of the fracture process in Fig. 4. The FPZ is represented here as a union of failure events which took place during the 0.01 s preceding the time step when the FPZ was evaluated and depicted. The considered stages of the simulated failure are emphasized on the corresponding loading curve in the graph in Fig. 2 right.



Fig. 2: Experimental set-up and dimensions of a selected WST specimen (after Mihashi & Nomura 1996) (left); an illustrative example of the FyDiK model showing triangulation and tessellation; load–crack opening displacement (P–COD) diagrams from the experiment and FyDiK simulations (right).

#### 3. Discussion of results and conclusions

A comparison of the considered numerical approach to the experimental estimation of the FPZ extent (particularly its cumulative representation) is performed for the stage corresponding to the peak load and for the end of the fracture in Fig. 3 left and right, respectively. The region where the acousticemission-like events are located that was provided by the FyDiK fracture model is in good agreement with the considered experimental example taken from Mihashi & Nomura (1996). However, a more detailed experiment is necessary for a comparison of the intensities of energy dissipation. Due to the lack of sound and complete experimental data available in the literature the authors are planning to conduct its own AE experiments.

The paper presents a numerical study of the formation and evolution of failure zones at propagating crack tips during the fracture testing of concrete WST specimens. This analysis was motivated by the need for verification and validation of the (semi-)analytical technique for estimation of the extent (size and shape) of the fracture process zone in quasi-brittle silicate-based composites during tensile failure which has been recently developed by the authors (termed the ReFraPro technique, Veselý & Frantík, 2008, Veselý & Frantík, 2010). The author's own implementation of the computational model based on the physical discretization of continuum was successfully employed for a partial verification of the developed ReFraPro method. The used FyDiK model provided a fairly good approximation of a volume where AE events have been localized experimentally. Moreover, it also enabled the investigation of the energy consumption of such events. Therefore, the results of the simulations can serve as a significant source of new knowledge which can be obtained regarding the energy dissipation mechanisms in the fracture process zone in the case of quasi-brittle materials.



Fig. 3: A comparison of the cumulative damage zone extents for two stages of the fracture estimated experimentally using AES (Mihashi & Nomura, 1996), and numerically using the FyDiK model.



Fig. 4: A WST specimen with a depiction of the evolution of the FPZ extent with an indication of the energy dissipation intensity (from light – low intensity – to dark – high intensity) simulated by the FyDiK model for selected stages of the fracture process (see the P–COD diagram in Fig. 2 right).

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