17th International Conference
ENGINEERING MECHANICS 2011
Svratka, Czech Republic, 9 – 12 May 2011

SENSITIVITY ANALYSIS OF STABILITY PROBLEMS OF STEEL COLUMNS USING SHELL FINITE ELEMENTS AND NONLINEAR COMPUTATION METHODS

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Abstract: *The paper analyzes the influence of initial imperfections on the load-carrying capacity of a slender strut, applying the ANSYS programme. The geometrical and material nonlinear finite element method was applied for the theoretical analysis. Modelling of the steel structure was performed using SHELL elements. The effect of input imperfections on the load-carrying capacity is evaluated by Sobol' sensitivity analysis. The computation model elaborated is unique with regard to its numerically demanding character. The Latin Hypercube Sampling method was applied for the evaluation of sensitivity indices.*

Keywords: ***Sensitivity analysis, shell elements, steel, stability, buckling.***

1. Introduction

The sensitivity analysis is a study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation, and how the given model depends upon the information fed into it (Saltelli et al., 2004). If we mean the computation models of building structures, the sensitivity analysis can be defined as a study of relationships between information flowing in and out of the model. Finite element models (FEM) of steel structures usual for the computation and analysis of output quantities necessary for an assessment of limit states of the structures mentioned, see, e.g., (Gottvald et al., 2010). The output quantities usually are load-carrying capacity, stress state, and deformation. The input quantities usually are material and geometrical characteristics of structures the histograms and statistic characteristics of which are determined by experimental research (Melcher et al., 2004; Kala et al., 2009; Kala et al., 2010). Let us note that not all the uncertainties are of stochastic character, see, e.g. (Kala, 2008). The epistemic uncertainty does not need to be studied in case that the input random quantities have been determined by experimental measurements including a considerable number of observations. Then there can be applied purely stochastic approaches. The stochastic sensitivity analysis determines which input random characteristics have the greatest influence on the random output. In general, there can be distinguished the local sensitivity analysis (Kala, 2005) and the global sensitivity analysis, see, e.g., (Kala, 2009). The local sensitivity analysis (e.g., correlation coefficients approach) does not provide any instruments for an analysis of the influence of higher order interaction effects which occur in systems consisting of more members (Kala, 2011a; Kala, 2011b0). In spite of this fact, it is applied to numerically demanding computation models, see, e.g., (Melcher et al., 2009). The global sensitivity analysis is much more demanding on the CPU time of the computer but in connection with nonlinear computational FEM, it provides importantly more information on the system studied. The detailed computation model is the basic prerequisite for obtaining exact results. The application of nonlinear FEM is necessary when, e.g., ultimate limit state of steel hot-rolled members is studied the load-carrying capacity of which is influenced by residual stress. The presented paper deals with the application of global sensitivity analysis to the study of influence of imperfections on load-carrying capacity of a steel hot-rolled member under compression. The results are unique because the influence of all the imperfections together with residual stress has been include into the solution.

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2. Sensitivity analysis

Within the scope of modelling, the notion "sensitivity analysis" has different meaning to different people, see, e.g., (Okazawa et al., 2002). The sensitivity analysis enabling an analysis of the influence of arbitrary subgroups of input factors (doubles, triples, etc.) on the monitored output was worked out by the Russian mathematician Ilja M. Sobol (Sobol, 1993; Saltelli et al., 2004). The sensitivity analysis of load-carrying capacity (random output Y) to input imperfections (random inputs X_i) was performed in the presented study. Sobol's first order sensitivity indices may be written in the form:

$$S_i = \frac{V\left(E\left(Y \middle| X_i\right)\right)}{V(Y)} \tag{1}$$

S_i measures the first order (e.g., additive) effect (so-called main effect) of X_i on the model output Y. The second order sensitivity index S_{ij} is the interaction term (2) between factors X_i, X_j. Analogously, the second order sensitivity indices may be rewritten:

$$S_{ij} = \frac{V\left(E\left(Y \middle| X_i, X_j\right)\right)}{V(Y)} - S_i - S_j \tag{2}$$

Sensitivity index *ij* expresses the influence of doubles on the monitored output. Other Sobol' sensitivity indices enabling the quantification of higher order interactions are expressed similarly.

$$\sum_i S_i + \sum_i \sum_{j>i} S_{ij} + \sum_i \sum_{j>i} \sum_{k>j} S_{ijk} + \ldots + S_{123\ldots M} = 1 \tag{3}$$

The number of members in (3) is 2^M-1, i.e., for $M = 3$, we obtain 7 sensitivity indices; for $M = 10$, 1023 sensitivity indices; it is excessively large for practical use. The main limitation in the determination of all members of (3) is the computational demanding character.

3. Computation model and input random variables

The strut was meshed in the programme ANSYS, being modelled of thin-walled elements, type SHELL 181. The symmetry was used with regard to the very demanding character of the problem solved. In the bar half in the symmetry plane, we supposed the translation fixed in all cross-section nodes in direction of axis *X*, and the rotation around axes *Y* and *Z*. On the second edge of the bar half solved, we fixed the translation of nodes in direction of the axis *Y* on the flange of profile IPE220. On the lower flange of that edge, we fixed the translations in the direction of axis *Z*. The upper flange was left free. The Euler method was applied, based on proportional loading in combination with the Newton-Raphson method in the geometrical and material non-linear FEM solution.

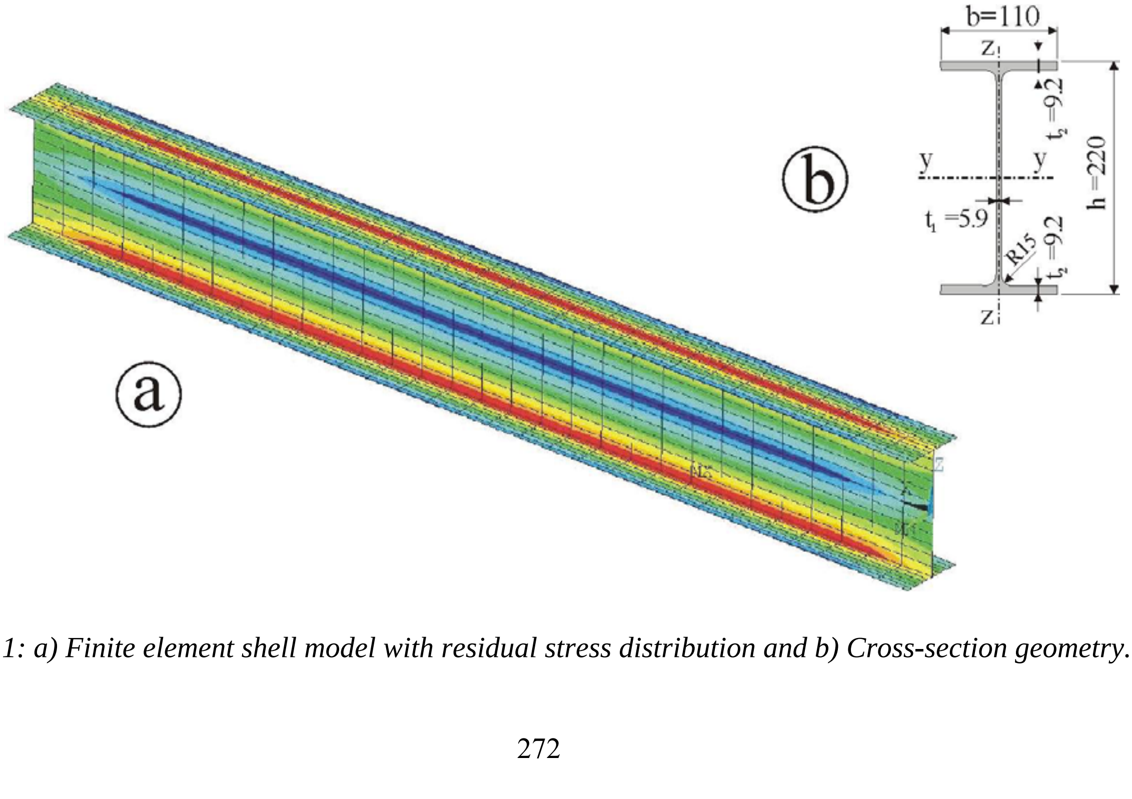

Fig. 1: a) Finite element shell model with residual stress distribution and b) Cross-section geometry.

The load-carrying capacity was determined as the loading constant during which the matrix determinant of tangential stiffness K_t of the structure would approach zero with certain accuracy. Since an accuracy of 0.1% was required for the determination of load-carrying capacity, it was necessary to use automatic control of the loading step with the Euler method. Bilinear kinematic material strengthening was assumed. We also assumed that the initial steel plasticization occurred when the Mises stress exceeded yield strength.

Theoretical models for expression of load carrying capacity should be always based on assumption that a real structure member contains various imperfections which influence the load carrying capacity. Generally, all input imperfections are of random character. Relatively sufficient information on material and geometrical characteristics of mass produced members of steel structures is available in comparison to other engineering structures (Soares, 1988; Melcher et al., 2004; Kala et al., 2009). Residual stress was introduced with mean value 80 MPa and standard deviation 40 MPa, with triangular distribution both on flanges and web. All the input random quantities were considered with the Gauss density function. All the input characteristics, synoptically given in Table 1, are statistically independent of one another.

Tab. 1: Input random quantities.

Random variables		Mean value	Standard deviation
Yield strength	f_y	297.3 MPa	16.8 MPa
Cross-sectional depth	h	220 mm	0.975 mm
Cross-sectional width	b	110 mm	1.093 mm
Web thickness	t_1	5.9 mm	0.247 mm
Flange thickness	t_2	9.2 mm	0.421 mm
Initial crookedness	e_0	0	0.00077 L
Young's modulus	E	210 GPa	12.6 GPa
Residual stress	rs	80 MPa	40 MPa

4. Sensitivity analysis results and discussion

The LHS method was applied to calculation of sensitivity indices (McKey et al., 1979; Iman and Conover, 1980). The model output Y is the load-carrying capacity calculated in each run of the LHS method. The conditional random arithmetical mean $E(Y|X_i)$ was evaluated for 3000 simulation runs; the variance $V(E(Y|X_i))$ was calculated for 3000 simulation runs, as well. The variance $V(Y)$ of load carrying capacity has been calculated for 10000 runs. The second-order sensitivity indices S_{ij} were calculated analogously. The influence of imperfections on the load carrying capacity changes with the strut increasing nondimensional slenderness, see Fig. 2.

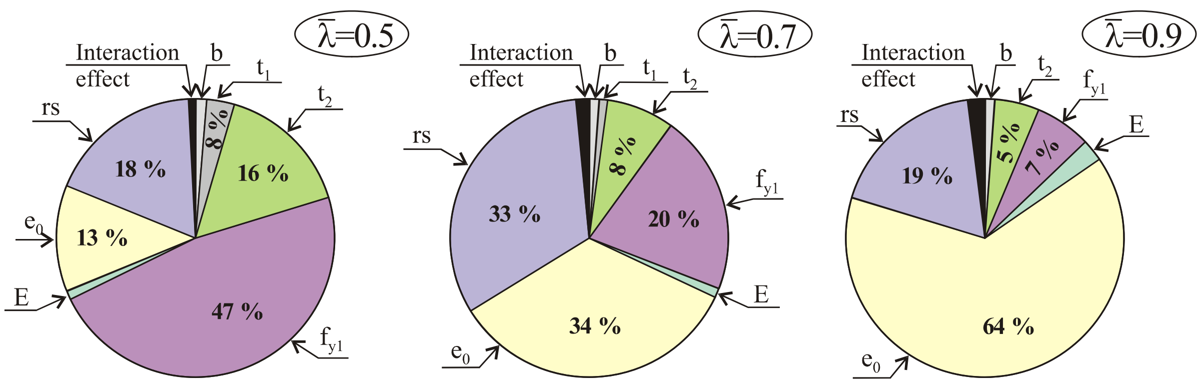

Fig. 2: Sensitivity analysis results.

5. Conclusions

The results presented in this paper quantify the influence of material and geometrical characteristics on the load-carrying capacity. The Sobol' sensitivity analysis in connection with nonlinear shell FEM enables to quantify the influence of residual stress, too. It is evident that the influence of residual stress is not the same as the influence of initial crookedness. From the point of view of standards for design of steel structures, it rises up the question whether the influence of residual stress can be reliably substituted by increase of the amplitude of bow imperfections. One of approaches how to answer this question is the elaboration of probabilistic reliability analyses of the ultimate limit state of steel members under compression (Kala, 2007). Further on, the results of the Sobol' sensitivity analysis have shown that the higher order interaction effects are relatively low.

Acknowledgement

The article was elaborated within the framework of projects of AVČR IAA201720901, MSM0021630519 and P104/11/0703.

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