17th International Conference
ENGINEERING MECHANICS 2011
Svratka, Czech Republic, 9 – 12 May 2011

INFLUENCE OF ECCENTRICITY ON THE AXIAL FLOW IN LEAKAGE JOINTS

A. Kubis[*], P. Navrátil[**]

Abstract: *This article is concerned with comparison of velocity field size depending on size of the eccentricity in leakage joints of centrifugal pumps. Eccentricity is created by gravitation and radial force from impeller. This report describes approach for calculating the straight seal without rotation. An eccentricity dependence of the axial velocity field in the article is numerically verified by using FLUENT software.*

Keywords: ***Leakage joint, seal, velocity field, centrifugal pumps, eccentricity.***

1. Introduction

It is very important, that centrifugal pumps works at high efficiency. Efficiency can be modified by minimization of hydraulic, mechanical and volume losses in the pump. Volume losses are generated by fluid flow through sealing rings back to suction, they are dependent on shape of sealing rings, whereas cannot be equal to zero. It could happen to seize of sealing rings, that's why there always must be precisely defined gap δ between sealing rings (tenths of mm). Flow q is circulating through the leakage joint, volume efficiency is defined by (1), (Fig. 1). (Nechleba & Hušek, 1966)

$$\eta_v = \frac{Q}{Q+q} \tag{1}$$

2. Theory and principle of sealing ring modeling

In this case are leakage joints (Karassik & Carter, 1966) straight without labyrinths and cells. They are replaced by cylinder annular area for simplification. In real leakage joint happens due to radial and gravitation forces from impeller to eccentricity and displacement of inner leakage joint cylinder axis (displacement of whole shaft in bearings) (Fig. 1). In the flow simulation is modelled eccentricity by axial displacement of the inner cylinder, whereas angle displacement is neglected.

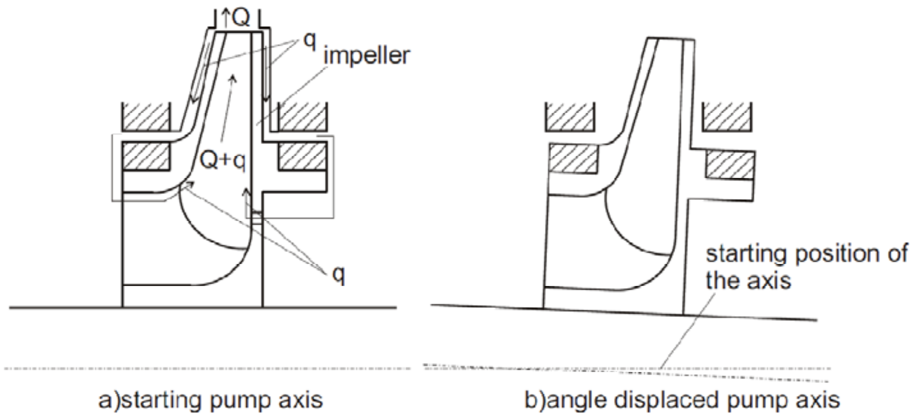

Fig. 1: Narrow and displaced pump axis.

[*] Ing. Aleš Kubis.: Institute of Fluid Mechanics Viktora Kaplana, Faculty of Mechanical Engineering, Technická 2896/2; 616 69, Brno; CZ, e-mail: ykubis03@stud.fme.vutbr.cz

[**] Ing. Petr Navrátil.: Institute of Solid Mechanics, Mechatronics and Biomechanics, Faculty of Mechanical Engineering; Technická 2896/2; 616 69, Brno; CZ, e-mail: ynavra26@stud.fme.vutbr.cz

Fig. 2 describes cylindrical annular area even with axes schematics, these schematics are important to derive formulas which describe rising flow processes in sealing joint. Kinetic equation is needed to describe processes inside leakage joint for direction r, ϕ, z (*Navier – Stokes equation*) and *continuity equation* (2), (3) (Brdlička & Sopko & Samek, 2005).

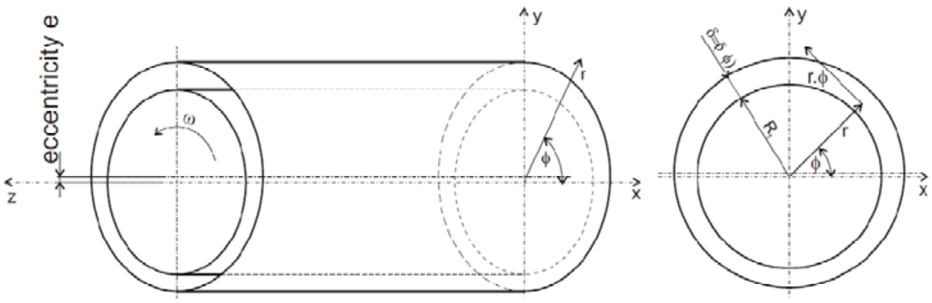

Fig. 2: Cylindrical annular area geometry.

$$r:\ \rho\left(\frac{\partial c_r}{\partial t}+\frac{\partial c_r}{\partial r}c_r+\frac{1}{r}\frac{\partial c_r}{\partial \emptyset}c_\emptyset+\frac{\partial c_r}{\partial z}c_z-\frac{{c_\emptyset}^2}{r}\right)+\frac{\partial p}{\partial r}-\mu\begin{bmatrix}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c_r}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 c_r}{\partial \emptyset^2}+\\+\frac{\partial^2 c_r}{\partial z^2}-\frac{c_r}{r^2}-\frac{2}{r^2}\frac{\partial c_\emptyset}{\partial \emptyset}\end{bmatrix}=\rho.g_r \qquad (2)$$

$$\emptyset:\ \rho\left(\frac{\partial c_\emptyset}{\partial t}+\frac{\partial c_\emptyset}{\partial r}c_r+\frac{1}{r}\frac{\partial c_\emptyset}{\partial \emptyset}c_\emptyset+\frac{\partial c_\emptyset}{\partial z}c_z-\frac{c_r c_\emptyset}{r}\right)+\frac{1}{r}\frac{\partial p}{\partial \emptyset}-\mu\begin{bmatrix}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c_\emptyset}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 c_\emptyset}{\partial \emptyset^2}+\\+\frac{\partial^2 c_\emptyset}{\partial z^2}-\frac{c_\emptyset}{r^2}+\frac{2}{r^2}\frac{\partial c_r}{\partial \emptyset}\end{bmatrix}=\rho.g_\emptyset$$

$$z:\ \rho\left(\frac{\partial c_z}{\partial t}+\frac{\partial c_z}{\partial r}c_r+\frac{1}{r}\frac{\partial c_z}{\partial \emptyset}c_\emptyset+\frac{\partial c_z}{\partial z}c_z\right)+\frac{1}{r}\frac{\partial p}{\partial z}-\mu\begin{bmatrix}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 c_z}{\partial \emptyset^2}+\\+\frac{\partial^2 c_z}{\partial z^2}\end{bmatrix}=\rho.g_z$$

$$continuity:\quad \frac{\partial \rho}{\partial t}+\frac{\partial(\rho.c_r)}{\partial r}+\frac{1}{r}\frac{\partial(\rho.c_\emptyset)}{\partial \emptyset}+\frac{\partial(\rho.c_z)}{\partial z}=0 \qquad (3)$$

These equations describe complex flow in straight seal, inside of seal happens to superposition of two processes, to flow through the leakage joint in z direction (axial flow) and to instabilities which are generated inside of leakage joint due to rotation of inner cylinder by angle speed ω - dependence of these instabilities are Taylor vortexes. Starting, ending and size of Taylor vortexes are given by Taylor number (Reynolds number function).

3. Simulation and numerical verification of axial flow in dependence of eccentricity size

In this chapter is described dependence of eccentricity size on axial flow in leakage joint, solving separately flow in z axis (with neglecting rotation of inner cylinder). Numerical calculation will be done with help of Fluent software, which can be compared with analytically calculated values. Formulas which is needed for calculation is modified Navier – Stokes equation for z axis (4) (for e = 0). Values added to formula (4) are displayed on Fig. 3.

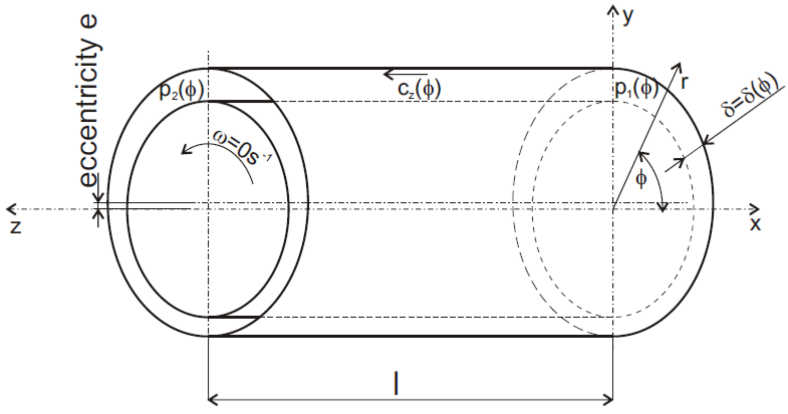

Fig. 3: Input values in axial flow calculation.

$$\overline{c_z} = \frac{1}{\sqrt{\lambda.\frac{l}{2.\delta}+\xi_{vytok}+\xi_{vtok}}}.\sqrt{\frac{2.(p_1-p_2)}{\rho}} \quad (4)$$

Fig. 3 displays variating width of leakage joint on angle φ, pressures $p_1(\phi)$ and $p_2(\phi)$ are generally considered as constant by ϕ, because leakage joint flow is generated with pressure difference and difference in flow is compensated by decreasing axial speed. Elements ($\xi_{vtok} = 0.5$) and ($\xi_{vytok} = 1$) in formula (4) mean inlet and outlet dissipation.

3.1. Numeric calculation

Mesh was made in Gambit software before beginning the calculation. The task in Fluent is axisymmetric by y axis, Fig. 2. Due to calculation simplification is sufficient to do only half of geometry. In formula (4) figure components $\xi_{vytok} + \xi_{vtok}$, they are modelled on large input and output area, Fig. 4. In the figure, there are also displayed boundary conditions used with solving, for all models. Tab. 1 contains eccentricity values, for which the calculation was solved and pressure boundary conditions, which are similar for all geometries. Geometry contains 1 200 000 3D elements, most of them (605 000) contains the part of leakage joint. Before calculation it must be also estimated, which type of flow will be inside of leakage joint. Reynolds number Re will be suitable for this purpose, for its calculation we need to know fluid speed inside of leakage joint, formula (5), surface roughness factor is equal to λ = 0.04. Basic estimation is that speed will not change with eccentricity variation. Re is calculated only once, eccentricity e = 0 (area is constant by ϕ coordinate and speed is steady), formula (6), (Paciga, 1967).

$$\overline{c_z} = \frac{1}{\sqrt{1{,}5+\frac{0{,}04.0{,}015}{2.0{,}0004}}}.\sqrt{\frac{2.(500000-200000)}{1000}} = 16.33\ m/s \quad (5)$$

$$Re = \frac{c_z.2.\delta}{\nu} = \frac{16{,}33.2.0{,}0004}{0{,}000001} = 13064 \quad (6)$$

Turbulence model is set for calculation in Fluent. For calculation has been used realizable k-ε model.

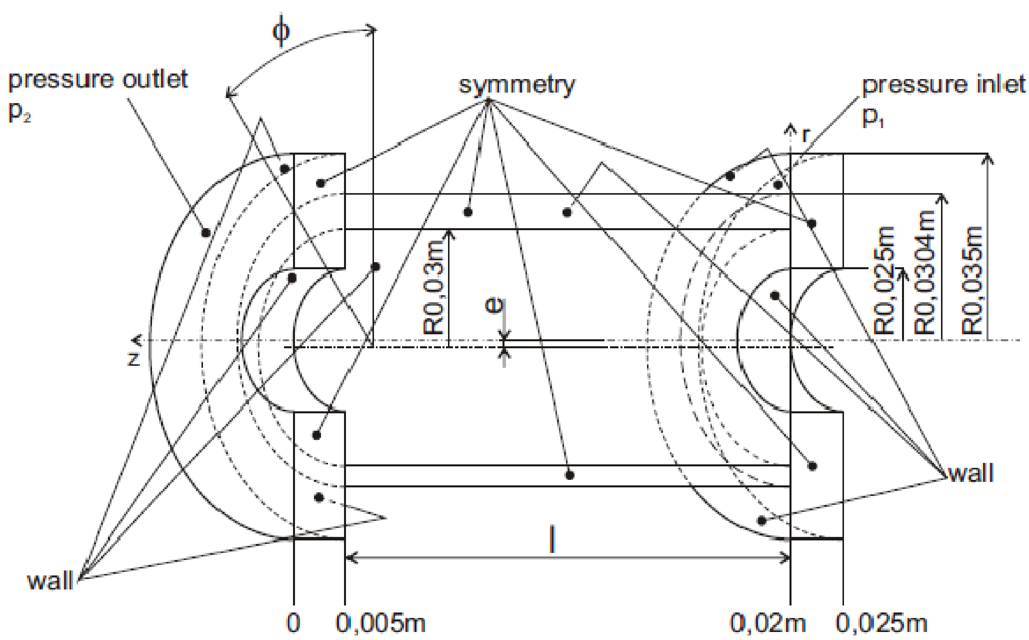

Fig. 4: Calculation geometry with boundary conditions.

Tab. 1: Size of eccentricity, pressure boundary conditions and flow through the leakage joint.

eccentricity e[mm]	pressure b.c. p_1[Pa]	pressure b.c. p_2[Pa]	flow through the leakage joint [m^3/s]
0			$1.168x10^{-3}$
0.075			$1.164x10^{-3}$
0.15	**500000**	**200000**	$1.167x10^{-3}$
0.225			$1.164x10^{-3}$
0.3			$1.166x10^{-3}$

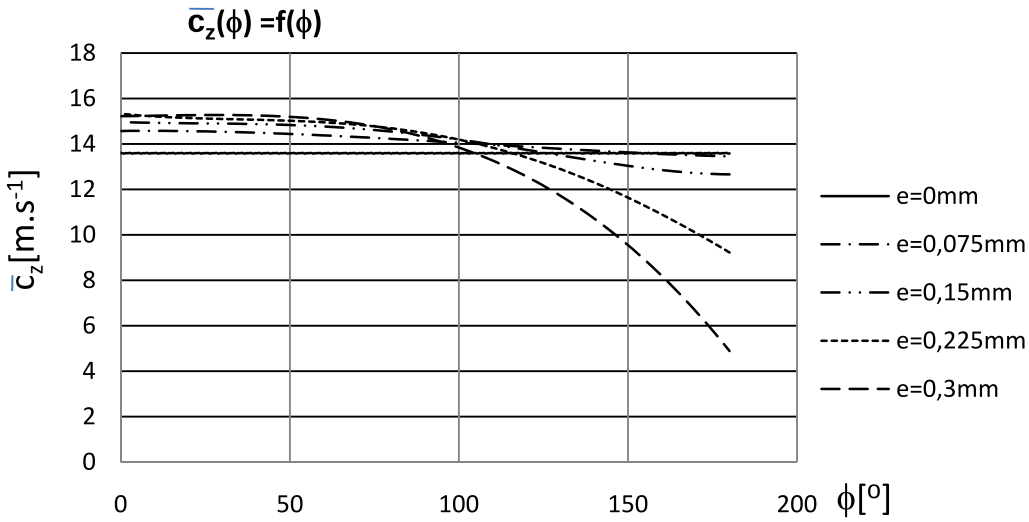

Fig. 5: Graph of axial speed dependence on ϕ coordinate for different eccentricity values.

Fig. 5 describes behavior of average axial speed by ϕ angle for different eccentricity sizes. These speeds are derived from Fluent by numerical integration in each area by trapeze rule. In the end is checked numerical calculation validity. Average speed c_z is calculated for case of zero eccentricity. Equation (4) is used and values p_1, p_2 are determined by Fluent). These pressures generally needn't be the same with boundary conditions on boundaries of calculation area. From numerically calculated data - pressures on input and output are $p_1 = 200\,000$ Pa and $p_2 = 415\,000$ Pa. Theoretical flow speed in straight seal (4) is nearly the same as calculated one from numerical simulation (7). Numerical calculation can be considered as correct.

$$\overline{c_{z\,analytical}} = \frac{1}{\sqrt{1,5+\frac{0,04.0,015}{2.0,0004}}}\cdot\sqrt{\frac{2.(415000-200000)}{1000}} = 13.82\ m/s \quad (7)$$

4. Conclusions

The objective of this article was to numerically describe progress of axial speed with variable eccentricity. Results are in graph on Fig. 5, where are each speeds displayed by ϕ coordinate, see Fig. 5. Variable size of axial speed is very important, because it corresponds with stiffness of fluid layer. It has been verified that, axial flow doesn´t correspond on eccentricity size (see Tab. 1 - flow through the leakage joints - calculated from Fluent software). Objective of next articles will be to describe instabilities, that are generated in leakage joint by rotation and description of fluid layers stiffness.

References

Brdička, M., Sopko,B. & Samek, L. (2005) Continuum mechanics, Academia, pp.426-610.

Nechleba, M. &, Hušek, J. (1966) Hydraulic machines, Prague, pp. 81-86.

Karassik, I. &, Carter, R. (1960) Centrifugal pumps, Mc Graw-Hill book company, pp. 42-44.

Paciga, A. (1967) Projektovanie zariadení čerpacej techniky, Slovenské vydavatelstvo technickej literatůry, pp. 40-47.