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DYNAMICS OF A VIBRATION DAMPERWORKING ON A PRINCIPLE OF A HEAVY BALL ROLLING INSIDE A SPHERICAL DISH

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Abstract: *Wind excited vibrations of slender structures such as towers, masts or certain types of bridges can be reduced using passive or active vibration absorbers. If there is available only a limited vertical space to install such a device, a ball type of absorber can be recommended. In general, it is a semi-spherical horizontal dish in which a ball of a smaller diameter is rolling. Ratio of both diameters, mass of the rolling ball, quality of contact surfaces and other parameters should correspond with characteristics of the structure. The ball absorber is modeled as a holonomous system. Using Lagrange equations of the second type, governing non-linear differential system is carried out. The solution procedure combines analytical and numerical processes. As the main tool for dynamic stability investigation the 2nd Lyapunov method is used. The function and effectiveness of the absorber identical with those installed at the existing TV towers was examined in the laboratory. The response spectrum demonstrates a strongly non-linear character of the absorber.*

Keywords: ***Vibration ball absorber, dynamic stability, non-linear vibration.***

1. Introduction

Passive vibration absorbers of various types are very widely used in civil engineering, especially when wind induced vibration should be suppressed. TV towers, masts and other slender structures exposed to wind excitation are usually equipped by such devices. Conventional passive absorbers are of the pendulum type. Although they are very effective and reliable, they have several disadvantages limiting their application. First of all, they have certain requirements to space, particularly in a vertical direction. These requirements cannot be satisfied any time when an absorber should be installed as supplementary equipment. Also horizontal construction, like foot bridges, cannot accept any absorber of the pendulum type. Another disadvantage represents a need of a regular maintenance.

Both above shortcomings can be avoided using the absorber of ball type. The basic principle comes out of a rolling movement of a metallic ball of a radius r inside of a metallic rubber coated dish of a radius $R > r$. This system is closed in an airtight case. Such a device is practically maintenance free. Its vertical dimension is relatively very small and can be used also in such cases where a pendulum absorber is inapplicable due to lack of vertical space or difficult maintenance. First papers dealing with the theory and practical aspects of ball absorbers have been published during the last two decades, see Pirner (1994) and Pirner and Fischer (2000). The first paper dealing with the problem on the basis of the rational dynamics has been published some years ago, see Náprstek and Pirner (2002).

Dynamics of the ball absorber is more complicated in comparison with the pendulum one. Its movement can be hardly described in a linear state although for the first view its behavior is similar to the pendulum absorber type. A number of problems are still open being related with movement stability, bifurcations, auto-parametric resonances and at least but not last with the spherical dish and ball surface imperfections. This paper should be the first attempt to present basic mathematical model in 2D together with its numerical evaluation and practical application as far as to the state of the realization including some results of long-term measurements.

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2. Mathematical model

The dish is fixed to a vibrating structure. Their dynamic character is represented by a linear SDOF system represented by a mass *M*. Inside of a dish an internal ball m in a vertical plane is moving, i.e. 2DOF system should be investigated, as it is outlined in the Fig. 1. It follows from geometric relations:

$$R\cdot\varphi = r(\psi+\varphi) \Rightarrow r\psi = \varrho\varphi, \quad \text{where } \varrho = R - r \tag{1}$$

It holds for both components of a displacement and velocity of the internal sphere center:

$$\left.\begin{array}{lll} \text{horizontal:}\; u + \varrho\cdot sin\varphi & \Rightarrow & \dot{u} + \varrho\dot{\varphi}cos\varphi \\ \text{vertical:} \qquad \varrho\cdot cos\varphi & \Rightarrow & -\varrho\dot{\varphi}sin\varphi \end{array}\right\} \tag{2}$$

Kinetic energy of a moving system of dish and ball *m*, *M* can be written in a form:

$$T = \frac{1}{2}m[(\dot{u}+\varrho\dot{\varphi}\cos\varphi)^2 + \varrho^2\dot{\varphi}^2\sin^2\varphi] + \frac{1}{2}J\dot{\psi}^2 + \frac{1}{2}M\dot{u}^2 = \frac{1}{2}(m+M)\dot{u}^2 + m\varrho\dot{u}\dot{\varphi}\cos\varphi + \frac{m}{2\kappa}\varrho^2\dot{\varphi}^2 \tag{3}$$

$$\frac{m}{\kappa} = m + \frac{J}{r^2} \;\Rightarrow\; \kappa = \frac{5}{7}$$

while the potential energy is given by an expression:

$$V = mg\varrho(1-cos\varphi) + \frac{1}{2}Cu^2 \tag{4}$$

The damping should be introduced in a form of a simple Rayleigh function:

$$B = \frac{1}{2}(M\cdot b_u\dot{u}^2 + m\cdot b_\varphi\varrho^2\dot{\varphi}^2) \tag{5}$$

m – mass of the ball m;
J – inertia moment of the ball m;
b_u, b_φ – damping coefficients (logarithmic decrements);

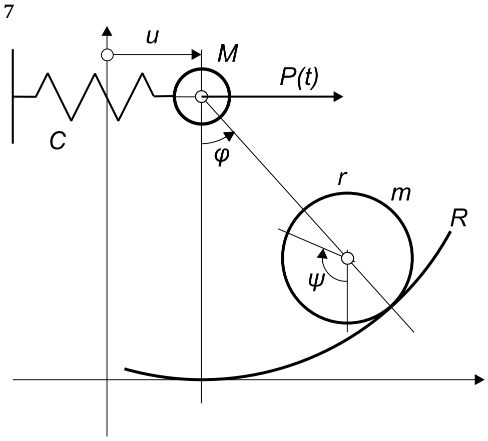

Fig. 1: Basic scheme of a system.

Expressions (3), (4), (5) should be put into the Lagrange equations of the second type, see any of the basic monographs, e.g. Hamel (1978):

$$\sum_{r=1}^{n}\left\{\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \frac{\partial T}{\partial q_r} + \frac{\partial V}{\partial q_r} + \frac{\partial B}{\partial \dot{q}_r}\right\}\delta_{q_r} = P_r(t) \tag{6}$$

$$q_1 = u = \zeta\cdot\varrho; \quad q_2 = \varphi; \quad P_u(t) = p(t)\cdot M\varrho; \quad P_\varphi(t) = 0$$

which give the governing equations of the system:

$$\left.\begin{array}{lr} \ddot{\varphi} + \kappa b_\varphi\dot{\varphi} + \kappa\omega_m^2\sin\varphi + \kappa\ddot{\zeta}\cos\varphi = 0 & (a) \\ \mu\ddot{\varphi}\cos\varphi - \mu\dot{\varphi}^2\sin\varphi + (1+\mu)\ddot{\zeta} + b_u\dot{\zeta} + \omega_M^2\zeta = p(t) & (b) \\ \mu = \frac{m}{M}; \quad \omega_M^2 = \frac{C}{M}; \quad \omega_m^2 = \frac{g}{\varrho} & (c) \end{array}\right\} \tag{7}$$

Eq. (7) describes 2D movement of a ball absorber under excitation by the force *P(t)* at any arbitrary deviation amplitudes including incidental transition through a limit cycle towards an open regime.

3. Basic properties of the absorber

Theoretical efficiency of the absorber will be assessed using its frequency characteristics for excitation of the mass *M* by harmonic force $P(t) = p_0\sin\omega t$ simulating influence of external loading or kinematic excitation of the same mass *M*. In the later case the movement of the ball *m* rolling inside of the dish is fully described by Eq. (7a). Should we solve the deviation *φ(t),* Eq. (7b) can serve us subsequently for an evaluation of the force *P(t),* which is necessary when the deviation $u(t) = \varrho\cdot\zeta(t)$ should be achieved.

To obtain frequency characteristics the harmonic excitation $\zeta(t) = \zeta_0\cos(\omega t)$ should be introduced into Eq. (7), which yields:

$$\left.\begin{array}{lr} \ddot{\varphi} + \kappa b_\varphi\dot{\varphi} + \kappa\omega_m^2\sin\varphi - \kappa\omega^2\cos\varphi\cdot\zeta_0\cos\omega t = 0 & (a) \\ \mu\ddot{\varphi}\cos\varphi - \mu\dot{\varphi}^2\sin\varphi + (-(1+\mu)\omega^2 + \omega_M^2)\zeta_0\cos\omega t - b_u\omega\cdot\zeta_0\sin\omega t = p(t) & (b) \end{array}\right\} \tag{8}$$

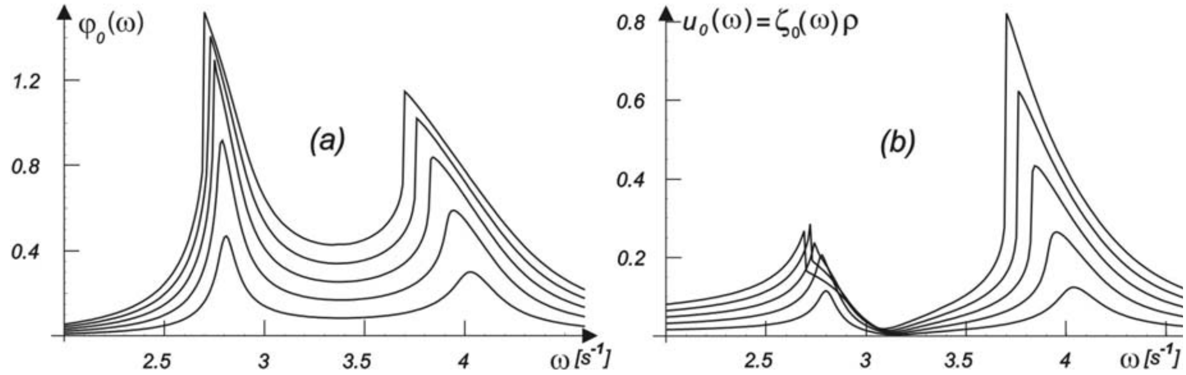

Fig. 2: Frequency characteristics of a ball absorber.

Eq. (8a) corresponds to the equation of a mathematical pendulum excited in a point of suspension. Its effective mass is increased due to a moment of inertia of the ball m by the factor $1/\kappa = 7/5$. Even in practice the movement amplitudes of this ball doesn't admit to linearize the Eq. (8a). At least a simple Duffing non-linear form should be retained:

$$\ddot{\varphi} + \kappa b_{\varphi}\dot{\varphi} + \kappa\omega_m^2\left(\varphi - \tfrac{1}{6}\varphi^3\right) - \kappa\omega^2\left(1 - \tfrac{1}{2}\varphi^2\right)\cdot\zeta_0\cos\omega t = 0 \tag{9}$$

Nevertheless the aim of this study is a basic engineering approach demonstrating the problem as a whole from the theoretical background until realization in practice. For this reason a strong analytical investigation working with the simplified Eq. (9) is postponed to a next paper which will be oriented predominantly to mathematical aspects of the problem. This time numerical analysis has been preferred as it leads the most quickly to a basic overview about dynamic properties of a ball absorber. For numerical analysis the original form of Eq. (8) is more suitable for further investigation. Indeed, it does not introduce any limitations of the response amplitudes without any complication in the analysis itself. Excitation by harmonic force $P(t) = p_0\sin\omega t$ has been applied and response in a form of *u(t)*, *φ(t)* computed.

With respect to actual experiences, following reference input data have been introduced:

$$M = 10.0;\ m = 2.0; \Longrightarrow \mu = 0.2\quad \varrho = 0.71;\ b_{\varphi} = 0.1;\ b_u = 0.2;\ p_0 = 1\ \div\ 5$$

A wide parametric analysis has been done. Sample results obtained have been summarized and plotted in Fig. 2. It is obvious for the first view the non-linear character manifesting oneself by a dependence of a position of extreme points on an amplitude of excitation force. This effect is visible predominantly in a neighborhood of a conventional "linear" natural frequency of the absorber although also the second natural frequency related with the original first natural frequency of the structure is affected. The resonance curves are typical for a system with "softening" non-linearities. It turns out that the non-linear element represented by a ball absorber can be more effective when broad band random response should be reduced. Even better results can be expected in case of non-stationary excitation when amplitude spectrum is significantly variable in time. In such a case no doubt non-linear absorber should be preferred, while the linear one works better in cases of strong narrow band excitation mostly of deterministic character.

The effective damping should not drop below a certain limit, as the stability loss of the ball movement inside the dish can occur. Then the damper can exhibit negative influence as long as the stable régime is regained. However, it is of interest to note that the increase of instable chaotic response domains under random excitation can act for the sake of the structure, as the effective response amplitudes are decreasing under these circumstances due to the rapid increase of the entropy of the response probability density. Nevertheless the sub-critical regime of the system is highly recommended.

On the other hand it is necessary to remain realistic. During testing in laboratory many effects corresponding to various critical and post-critical effects have been observed which are not yet described and quantified theoretically. As regards the damping, the use of the logarithmic decrement as the measure of damping does not correspond very well to the non-linear nature of the phenomenon. However, a comparison of the behavior of different physical models which were examined is very useful. Fig. 3 shows the value of the logarithmic decrement $b\varphi$ of the model plotted against the absorber - mass ratio μ, see Eq. (7c). The model was put into vibration by initial deflection from its

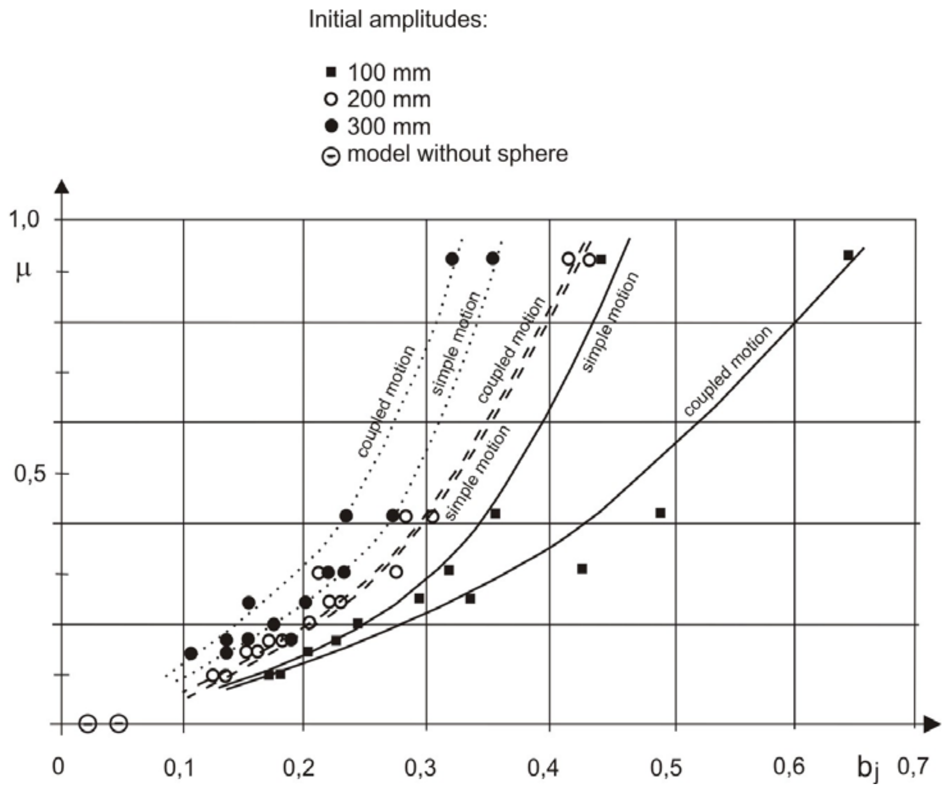

Fig. 3: Logarithmic decrement b_φ plotted against the mass-ratio μ for different initial amplitudes.

equilibrium position. In this figure diagrams for several values of initial displacement have been plotted. It can be seen that the model without ball (μ = 0) has the damping nearly 0.02 (the point on the horizontal axis μ = 0), while adding the ball absorber the damping reaches 0.17 - 0.25, i.e. nearly 8 times more. Similar effect appears also using conventional pendulum absorbers, see Pirner (1994).

4. Conclusions

The mathematical model of the ball type vibration absorber has been outlined. The basic Lagrangian analytical theory of non-linear behavior has been done. Very wide numerical investigation reveals that the non-linear character of this device is an important factor influencing significantly its dynamic properties and practical efficiency. It turns out, that the non-linear character making the form of resonance curves dependent on the excitation amplitude leads to better efficiency in comparison with linear mechanism. Laboratory tests of the vibration ball absorber with the dish without and with rubber coating have demonstrated several aspects of real operation of the damper. With respect to laboratory tests and long-term in situ measurements can be concluded that the vibration ball absorber is a simple nearly maintenance free low cost device with very small vertical dimensions. For these properties it is very convenient for application especially in cases when broad band excitation of random character prevails and when very limited vertical space is available.

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