

# VERIFICATION OF THE DESIGN RELIABILITY OF STEEL STRUCTURES

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**Abstract:** The verification of the design reliability of a steel element according to the concepts of standards EUROCODE 3 and EN1990 is presented in the article. Reliability analysis is used for the determination of the probability of failure, which is evaluated using the Monte Carlo methodology. Reliability analysis presents a valuable tool for the verification of reliability indices of the fore mentioned standards, generalization of evaluated results and further development of design methodology of structures according to the limit state theories. The article presents an overview of random input imperfections of steel structures whose histograms and statistical characteristics have been monitored and measured over a long time period. The completeness of statistical information on input data for the purpose of utilization in probabilistic studies is discussed. The quantification of fuzzy set is illustrated on a numerical example. The analysis of the influence of fuzzy uncertainty of random input variables on the fuzzy uncertainty of failure probability is presented. Uncertainty of computational models is discussed and modern instruments utilizable in the analysis of these uncertainties are listed. The analysis of the load carrying capacity of a steel plane frame with compression members is presented.

Keywords: Design reliability, failure probability, random, Monte Carlo, limit state, fuzzy, uncertainty.

### 1. Introduction

At present, the best method in practice for the verification of structural reliability of standards is according to the limit state methods. In standard procedures characteristic and design values, which guarantee design reliability, are used. Design values are obtained with the aid of partial safety factors, which present the basic indicators of the reliability of structural design. Reliability analysis may be performed using a number of methods, see e.g. (Kala, 2008; Gottvald, 2010). Probabilistic methods are also frequently used, see e.g. (Li et al., 1995; Kala et al., 2010). A prerequisite for the utilization of probabilistic methods is that ample statistical information is available from experimental research; see e.g. (Melcher et al., 2004; Strauss et al., 2006; Karmazínová et al., 2009). This article focuses on the analysis of the influence of partial safety factors of standards EUROCODE 3 and EN 1990 on the resulting structure reliability. The probabilistic analysis is supplemented with fuzzy analysis enabling the analysis of the effects of epistemic uncertainties (Kala, 2007; Kala, 2008). The fuzzy inputs were considered as model uncertainties in determining the load action and load-carrying capacity effects. The fuzzy analysis of output failure probabilities was evaluated according to the general extension principle (Zadeh, 1965; Möller & Reuter, 2007).

### 2. Verification of design procedures of structural stability

A simplified example of a compression member under the load effects of permanent load action G in combination with single variable load action Q is considered for the elaboration of a parametric study. The reliability design condition according to EC3 and EN1990 may be expressed as:

$$\gamma_G \cdot G_k + \gamma_Q \cdot Q_k \le R_{A\chi} \cdot f_{yk} / \gamma_M \tag{1}$$

where  $R_{A\chi} = \chi A$  is the product of buckling coefficient  $\chi$  and nominal cross-sectional area A,  $\gamma_M$  is the material partial safety factor, and  $G_k$ ,  $Q_k$ ,  $f_{yk}$  are characteristic values of load actions and yield strength

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respectively. Reliability of design is ensured by partial safety factors  $\gamma$ . The condition of design reliability (1) can be rewritten as the inequality of design load action  $F_d$  and design load-carrying capacity  $R_d$ . It was assumed in the reliability study that the design load action is equal to the design load-carrying capacity,

$$F_d = R_d \tag{2}$$

i.e., the structure is designed economically with maximum load carrying capacity. Characteristic values  $G_k$ ,  $Q_k$  are expressed using the ratio  $\delta$  of load action  $Q_k$  to the total load action  $G_k+Q_k$ :

$$\delta = \frac{Q_k}{G_k + Q_k} \tag{3}$$

Characteristic values  $G_k$  and  $Q_k$  are evaluated from the relation:

$$.35 \cdot G_k + 1.5 \cdot Q_k = 382.3 \,\mathrm{kN} \tag{4}$$

Equation (4) is derived from (1) for partial safety factors  $\gamma_G = 1.35$ ;  $\gamma_Q = 1.5$  (EN1990) and  $\gamma_M = 1.0$  (EC3). The value  $R_d = 382.3$  kN on the right-hand side of the equation is the design load carrying capacity of strut profile IPE 200 of length 2.1m and non-dimensional slenderness  $\overline{\lambda} = 1.0$  calculated acc. to EC3:

$$R_{\rm d} = \frac{\chi_b \cdot f_y \cdot A_n}{\gamma_{\rm M1}} = \frac{0.597 \cdot 235 \text{MPa} \cdot 2.7248 \cdot 10^{-3} \text{ m}^2}{1.0} = 382.3 \text{ kN}$$
(5)

where  $\chi_b$  is the buckling coefficient for the buckling strength curve *b*,  $A_n$  is the nominal crosssectional area evaluated from the nominal values of the cross section comprised of rectangular segments, and  $f_{yk}$  is the characteristic value of yield strength.

#### 2.1. Probabilistic verification of design procedures of structural stability

The random characteristics of load action effects G and Q are obtained from the characteristic values listed in (Kala, 2007). It was assumed for the dead load that the characteristic value  $G_k$  is also the mean value of the normal distribution. Furthermore a variation coefficient of 0.1 was assumed according to (Kala, 2007). Gumbel distribution with mean value  $m_Q = 0.6 Q_k$  and standard deviation  $S_Q = 0.21 Q_k$  was considered for the variable load in accordance with (Kala, 2007). According to EUROCODE 3, the failure probability of a strut occurs when the reliability condition (6), in which R is the random load-carrying capacity, and G, Q are random load action effects, is not satisfied.

$$G + Q < R \tag{6}$$

The variable quantifying reliability or unreliability is the probability that condition (6) isn't fulfilled during the life span of the structure with regard to structural, aesthetical, service, energetic, economic and ecological aspects. Attainment of the limit state or generally occurrence of failure cannot be absolutely eliminated due to technical and economical reasons and hence we try to design the structure so that the probability  $P_f$  that failure occurs is very small.

The failure probability  $P_f$  is the most important and objective indicator of reliability and is commonly related to a certain reference time (usually 50 to 100 years). Within this time interval the given degree of reliability should be maintained. The load carrying capacity R in (6) may be obtained from the response function described in (Kala, 2009):

$$R = -\frac{\sqrt{A^2 \cdot Q^2 + 2 \cdot A \cdot F_{cr} \cdot W_z} \cdot \left(\left|e_0\right| \cdot F_{cr} - f_y \cdot W_z\right) + F_{cr}^2 \cdot W_z^2 - A \cdot Q - F_{cr} \cdot W_z}{2 \cdot W_z}$$
(7)

where  $e_0$  is the amplitude of initial strut curvature formatively identical to a half-wave of the sine function, A is cross-sectional area,  $F_{cr}$  is Euler's critical force of a bilaterally hinged steel strut,  $W_z$  is the sectional modulus to axis Z (axis perpendicular to the flange around which the section bends during buckling), and  $f_y$  is the yield strength.

#### 2.2. Input random quantities

Statistical characteristics *h*, *b*,  $t_1$ ,  $t_2$ ,  $f_y$  were considered as histograms of experimentally obtained results (Melcher et al., 2004; Kala et al., 2009). Young's modulus was considered as a Gaussian probability distribution with statistical characteristics given acc. to (Fukumoto et al., 1976). According to results of experimental research (Fukumoto et al., 1976), the dominant shape of initial curvature is given as a half-wave of the sine function. A Hermite four-parametric probability distribution, which makes provision for skewness and kurtosis, was considered for the amplitude of initial imperfection  $e_0$ , see Fig. 1. We know with certainty that the mean value and skewness of symmetrical elements comprising of IPE profiles is equal to zero. Standard deviation of the Hermite density function is designated on the assumption that 95 % of the realizations of the amplitude of initial imperfection  $e_0$  are found within the tolerance limits (-3.15; 3.15) mm of the standard EN 10034. Kurtosis is given as a symmetric fuzzy number, see Figure 1. The support of the membership function is  $\langle 1.816; 4.184 \rangle$  and the kernel = 3.



Fig. 1: Hermite density distribution function and fuzzy number of kurtosis.

The parameters of the four-parametric Hermite density function include: mean value, standard deviation, skewness and kurtosis. The limit case arises for kurtosis of 1.816 corresponding to a rectangular density function. The kurtosis of the Gaussian density function has a value of 3.0. The maximum kurtosis was considered as 4.184, (i.e. 3.0 + 3.0 - 1.816) leading to symmetrical minimal and maximal support values around the kernel. The left side of Fig. 1 illustrates an example of a set of density functions. The functions vary in kurtosis values, which is listed for each function. The standard deviation of the amplitude of initial imperfection  $e_0$  is also a fuzzy number. Twenty cuts of the  $\alpha$ -cut method were utilized (Möller & Reuter, 2007). The standard deviation of the Gaussian density function is given by the kernel value of 1.607. The fuzzy number of standard deviation  $e_0$  was evaluated using the general extension principle (Zadeh, 1965). The support and kernel of the failure probability distribution is depicted in Figure 4. The main output of the study is represented by the dashed curve, which was obtained utilizing the COG (centre of gravity) defuzzification method.





#### 3. Conclusions

The presented study illustrates fuzzy uncertainty of failure probability emanating from the vague (fuzzy) uncertainty of the kurtosis of the random amplitude of initial member curvature  $e_0$ . Discrepancies between obtained results are considerable and point to the necessity of fuzzy analysis whenever input random variables are assigned subjectively. It is apparent that the defuzzified values (centroids) represented graphically by the dashed curves are higher than the values evaluated by means of purely stochastic analysis (kernels) represented by the full curves in all cases. Results of the application of probabilistic analysis point out the significant discrepancies of design reliability of steel structures acc. to the EUROCODE concept. Dominant input variables which have the greatest influence on the design reliability can be identified using sensitivity analysis methods (Kala, 2005; Melcher et al., 2009; Kala, 2011a; Kala, 2011b). In the future it is necessary to continue in the verification and calibration of the reliability indicators based on results of experimental research.

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