

PROBABILISTIC ANALYSIS FOR DESIGN ASSESSMENT OF COMPOSITE STEEL-AND-CONCRETE COMPRESSION MEMBERS

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Abstract: Steel-concrete composite columns are extensively used in modern buildings. Steel being of high tensile strength presents a high quality building material, concrete on the other hand carries the compressive stresses and protects the steel from high temperatures and corrosion. The theoretical analysis of the actual behaviour of modern concrete-steel structures is more frequently performed during design, realization and utilization of such structures. The main goal of the presented paper is the non-linear analysis of the load-carrying capacity of concrete encased steel columns. Ample information, obtained from experimental research, on the material characteristics of steel was available and was considered during analysis as random input variables. Statistical analysis of the ultimate limit state. Stresses in both concrete ad steel sections were evaluated using principles of elasticity and plasticity. Numerical LHS simulation methods were applied. These methods take into account the variability influences of input imperfections. Obtained output is the random load-carrying capacity, which quantifies the uncertainty due to the random character of material and geometrical characteristics. The important output of the statistical analysis is the design load-carrying capacity, which was evaluated according to EN1990 as 0.1 percentile.

Keywords: Steel, concrete, stability, strut, stochastic.

1. Introduction

Generally, two types of composite columns are commonly used in the building industry, i.e., those with steel section in-filled with concrete and those with steel section encased in concrete. Basic examples of composite column cross-sections are illustrated in Fig. 1.



Fig. 1: Types of composite columns.

Concrete-encased steel composite columns have become more popular for many resistant structures. Under severe flexural overload, the concrete encasement cracks which results in the reduction of stiffness, however, the steel core provides the shear capacity and the ductile resistance to subsequent cycles of overload.



Fig. 2: Theoretical decomposition and nominal values of steel-concrete cross section.

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2. Theoretical model

The subject of analysis is the ultimate limit state of a steel-concrete column of system length equal to its critical length, $L = L_{cr} = 3$ m, in compression. The column consists of steel profile HEA140 encased in high strength concrete, see Fig. 1. The load *F* acting on the column consists of load *F_s*, which is carried by the steel section, and of load *F_c*, which is carried by the concrete section, i.e., $F = F_s + F_c$. Let us assume that the strut is produced in the shape affine to eventual buckling, with deflection at mid length denoted as e_0 . The maximum deflection mid-span of the strut *e* which is loaded by axial force *F* in its elastic state may be determined according to (Timoshenko, 1961) as:

$$e = e_0 / (1 - F / F_{cr}) \tag{1}$$

where *F* is the load acting on the column, and F_{cr} is Euler's critical force $F_{cr} = \pi^2 EI / L_{cr}^2$. In accordance with article 6.7.3.1 (3) of standard EN 1994-1-1: 2006, the effective elastic flexural rigidity *EI* of the steel-concrete column which is given according to the formula listed below may be used for short term loading:

$$EI = E_s \cdot I_s + K_E \cdot E_C \cdot I_C \tag{2}$$

where I_s and I_c are the second moments of area in the plane of bending of structural steel and concrete (without consideration of cracking), E_s is the modulus of the steel elasticity, E_c is the tangent modulus of the elasticity of concrete, $K_E \cdot E_c \cdot I_c$ is the effective flexural rigidity of the concrete section, $E_s \cdot I_s$ is the effective flexural rigidity of the steel section.

Values of forces F_S , F_C and parameter K_E can be obtained from the following deformation conditions:

- (i) Bending around the z-axis: Deflection mid-span of the strut in the direction of y-axis e is given as the deflection of the steel section e_S which is equal to the deflection of the concrete section e_C ; i.e. $e = e_S = e_C$.
- (ii) Compression in direction of x-axis: The compression of the steel section by F_s equals the compression of the concrete section by F_c . The load carried by the steel section $F_s = F \cdot E_s I_s / (EI)$ and the load carried by the concrete section $F_c = F \cdot K_E \cdot E_c I_c / (EI)$ with parameter $K_E = I_s \cdot A_c / (I_c \cdot A_s)$ can be determined from the above listed mathematical dependencies (where A_c is the area of concrete and A_s is the area of steel cross section).

The elastic load-carrying capacity of the steel-concrete column is given as the minimum of the elastic load-carrying capacities of the steel and concrete sections, which are determined as follows:

- (i) The elastic load-carrying capacity of the steel member is given by the yield strength attained in the most stressed section.
- (ii) The elastic load-carrying capacity of the concrete section is given as the cubic strength in the most compressed section or as 10 % of the cubic strength in the most tensed part of the section. Detailed description of the calculation of both elastic and ultimate load-carrying capacities is described in the paper (Kala et al., 2010).

3. Input random variables

Generally, all input variables are of random character. Results obtained from a long-term experimental research were used for the input random characteristics (Melcher et al., 2004; Kala et al., 2009). Input random material characteristics of steel S420 were published in (Kala et al., 2010). The mean value of cubic strength f_{cc} of the concrete section in Fig. 2b was considered according to the results obtained from experimental research (Kala et al., 2010). The modulus of elasticity E_C of concrete was evaluated in compliance with standard Eurocode 2 according to the formula:

$$E_{C} = 22 \cdot \left(\frac{5}{6} \cdot \frac{f_{cc}}{10}\right)^{0.3} \cdot \Theta_{E_{C}}$$
(3)

Statistical characteristics of parameter Θ_{E_c} were identified so that maximum agreement between the theoretical analysis and experiment has been attained. The statistical characteristics of all input random variables were published in (Kala et al., 2010). All input random variables were considered with Gauss density distribution function.

4. Statistical analysis and discussion

The curves of "load action - deformation" determined under the presumption that all variables are considered as random ones are depicted in the left part of Fig. 3. The numerical simulation method LHS (McKey et al., 1979; Iman and Conover, 1980), which is a method of type Monte Carlo, was applied. Obtained curves of "load action - deformation" are one of the main outputs describing the random characteristics of the ultimate limit state of the examined strut.



Fig. 3: Theoretical decomposition of steel-concrete cross section.

The maximum possible load which the column is able to carry is defined as the ultimate load-carrying capacity; see Fig. 3. The dependence between random initial bar crookedness e_0 (bow imperfection) and 100 000 runs of load-carrying capacities is very interesting; see Fig. 3. Let us note that the ultimate load-carrying capacity is approximately 3% higher than the elastic load-carrying capacity. This is relatively insignificant. The theoretical model described in chapters 2 and in (Kala et al., 2010) is sufficiently accurate. This was verified using the geometrical and material nonlinear solution with SHELL 181 elements of the ANSYS software, see the stress state in Fig. 4.



Fig. 4: Theoretical decomposition of steel-concrete cross section.

From Fig. 3, large variance of the load - deformation curves is evident; it will manifest itself on the dispersion of ultimate load-carrying capacity and on its design value. In future, it will be necessary to get concentrated on the study of the ultimate load-carrying capacity on behalf of specialized sensitivity analysis methods. The local sensitivity measures determine the influence of parameters by varying one parameter at the time and keeping the other parameters constant (Kala, 2005; Melcher et al., 2009). The global sensitivity analysis identifies the influence of individual imperfections, and of their mutual interactions (Kala, 2011a; Kala, 2011b). In sophisticated systems, the application of advanced methods of the reliability analysis including the sensitivity analysis methods is indispensable, see, e.g., (Karmazínová et al., 2009; Gottvald et al., 2010). Let us note that, when creating a computation model, it is necessary to distinguish between epistemic uncertainty a stochastic uncertainty, see, e.g., (Kala, 2007; Kala, 2008). The generally given problematic leads to the applications of "soft computing" method, which form an appropriate unifying framework.

5. Conclusions

The output of the probability analysis is the set of random realizations of the load-carrying capacity mapping the influence of random uncertainties of geometrical and material characteristics on the load-carrying capacity. The output of the standards solutions is the design value of load-carrying capacity. The design load-carrying capacity was calculated, according to EN1990, as 0.1 percentile. The difference between standard design load-carrying capacities evaluated according to EN1990 and EUROCODE 4 solution is alarming. The basic probabilistic background of Eurocode is in EN 1990. If the design probability of failure is 7.2E-5 ($\beta_d = 3.8$), then the design load-carrying capacity evaluated according to EN 1990 should be approximately equal to 0.1 percentile; however, it is not fulfilled. The quantification of these differences would require a more detailed study with application of global sensitivity analysis. The results of sensitivity analyses in (Kala, 2009) have shown that the influence of higher order interaction effects is low for the steel member. This knowledge will have to be verified also for the steel-concrete composite column solved here.

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