

# CORRECTION OF FATIGUE PARAMETER VALUES OF CONCRETE USING APPROXIMATION OF MECHANICAL-FRACTURE PARAMETER VALUES IN TIME

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**Abstract:** Selected approximation curves of mechanical-fracture parameter values – modulus of elasticity, effective fracture toughness, specific fracture energy, compressive cube strength, splitting tensile strength, maximum load force – in time are used to determine the most accurate fatigue parameter values corresponding to the age of specimens when dynamic tests were performed.

Keywords: Fatigue, concrete, fracture test, approximation.

### 1. Introduction

Concrete is one of the most widely used materials in civil engineering structures. For concrete structures under cyclic loading – bridges, tunnels, concrete sleepers etc. – fatigue is a serious problem: a process of progressive and permanent internal damage in materials subjected to repeated loading. This is attributed to the propagation of internal micro-cracks that may result in the propagation of macro-cracks and unpredictable failure.

Fatigue phenomena related to the behaviour of reinforced/concrete structures under cyclic loading has been studied for only a few decades (see Lee & Barr, 2004, for a review; Seitl et al., 2010a,b; Pryl et al., 2010). Concrete is a highly heterogeneous material and the processes occurring in its structure and leading to its degradation under cyclic loading are more complicated in comparison to those affecting metals. The fatigue mechanism may be attributed to progressive bond degradation between coarse aggregates and the cement paste or by the development of cracks existing in the cement paste. Similarly as with metals, the process leading to fatigue failure caused by macro-crack propagation consists of three parts. The first of these is connected with crack initiation and typically takes place in the vicinity of stress concentrators in the weaker part(s) of the microstructure. The second phase is characterized by the stable growth of the initiated crack up to its critical length. The final part is associated with unstable growth of the service life of the structure, the most important is the second part, which represents up to 80% of the total life cycle. Quantification of the crack behaviour in this phase is of paramount importance.

An extensive laboratory experiment was conducted on a set of specimens of plain class C30/37 concrete. The specimens were used for determining the values of fundamental fracture characteristics and related fatigue parameters using static fracture and dynamic experiments. The problem seems to be the gradually increasing age of the concrete samples during the dynamic tests. The aim of this paper is to present results from fatigue experiments and – mainly – correction of the values of fatigue parameters using approximation curves obtained from mechanical-fracture parameter values in time (modulus of elasticity, effective fracture toughness, specific fracture energy, compressive cube strength, splitting tensile strength, maximum load force).

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#### 2. Simple approximation of the values of basic parameters of concrete

Approximation curves of the above-mentioned parameters in time were determined in the following way. In the first instance the relative values of individual parameters for all examined ages of specimens were obtained: the values of parameters obtained from measurement were divided by the appropriate mean average value for specimens at the age of 28 days. In the next step adaptation data for individual parameters were plotted in charts depending on the age of specimens. The analytical expressions for simple approximation (regression) curves were determined. Power, logarithmic and polynomial functions were used (EXCEL software). The index of dispersion ( $R^2$ ) was obtained for each type of regression curve. For illustration, approximation curves for relative maximum load force obtained from static three point bending fracture tests are depicted in Fig. 1. In the equations x indicates time in days and y indicates the dimensionless relative values of the appropriate parameter. The same is valid for Tab. 1.

Analytical expressions for simple approximation curves of all mentioned mechanical-fracture parameter values are introduced in Tab. 1, including the dimensionless index of dispersion. It stands to reason that the best approximation was obtained for the following parameters: compressive cube strength and maximum load force obtained from dynamic tests (1 cycle).



*Fig. 1: Approximation curves for relative maximum load force. Tab. 1: Coefficients of analytical expressions for simple approximation curves.* 

Regression curve	$Power$ $y = a \times x^b$			$Logarithmic$ $y = a \times \ln(x) + b$			$Polynomialy = ax^2 + bx + c$			
Relative parameter	а	b	$R^2$	а	b	$R^2$	а	b	с	$R^2$
Compressive cube strength	0.6367	0.1377	0.9396	0.1542	0.4939	0.9404	-2·10 <sup>-5</sup>	0.0060	0.8494	0.9706
Compressive strength – fractions	0.6631	0.1233	0.9227	0.1359	0.5478	0.9345	-1.10-5	0.0040	0.8974	0.9352
<i>Tensile splitting strength</i> – <i>fractions</i>	0.7373	0.0927	0.8263	0.1001	0.6713	0.8231	-2·10 <sup>-5</sup>	0.0048	0.8812	0.8877
Modulus of elasticity	0.6673	0.1113	0.2676	0.1185	0.5786	0.2906	1.10-5	-0.0010	1.0160	0.4004
Fracture toughness	0.4909	0.2007	0.5347	0.2380	0.1664	0.5255	-5·10 <sup>-5</sup>	-0.0058	1.1224	0.7849
Fracture energy	0.2877	0.3639	0.7960	0.4882	-0.6576	0.7889	2.10-5	0.0040	0.8758	0.8428
Maximum load force – static fracture tests	0.3557	0.3027	0.7128	0.3662	-0.2247	0.7496	-2·10 <sup>-5</sup>	0.0091	0.7638	0.7510
Maximum load force – dynamic tests (1 cycle)	0.6157	0.1440	0.9742	0.1596	0.4622	0.9774	-2·10 <sup>-5</sup>	0.0058	0.8447	0.9964

#### 3. Fatigue experiments

Fatigue properties were obtained from three point bending tests of beam specimens with a central edge notch. The nominal dimensions of the beams were  $100 \times 100 \times 400$  mm, span length 300 mm. The initial notch was made by a diamond blade saw. Note the depth of the notches was 10 mm.

The experimental test program was carried out at the Laboratory of the Institute of Metal and Timber Structures, Faculty of Civil Engineering, Brno University of Technology. The controlled values for temperature and relative humidity were  $22\pm2$  °C and 50%. The fatigue experiments (Wöhler curves) were carried out in a computer-controlled servo hydraulic testing machine (INOVA–U2).

Fatigue testing was conducted under load control. The stress ratio  $R = P_{min}/P_{max} = 0.1$ , where  $P_{min}$  and  $P_{max}$  refer to the minimum and maximum load of a sinusoidal wave in each cycle. The load frequency used for all repeated-load tests was approximately 10 Hz. The fatigue failure numbers of cycles were recorded.

Concrete specimens were loaded in the range of high-cycle fatigue; therefore, the upper limit to the number of cycles to be applied was selected as 2 million cycles. The test finished when the failure of the specimen occurred or the upper limit of loading cycles was reached, whichever occurred first.

#### 4. Results of the fatigue tests

The results of the fatigue tests under a varying maximum bending stress level are summarized in Fig. 2, where the maximum bending stress (S) obtained from the fatigue experiments is plotted against the logarithm of the number of cycles to failure (N). Along with data points, the analytical expressions for the curves were obtained in the following form:

$$S = a \times N^b \tag{1}$$

In an ideal case, all specimens would fail in the same cycle group and after the same number of cycles. However, the fatigue behaviour of a heterogeneous material like concrete is far from being ideal and the results are usually highly scattered; therefore, it is necessary to determine not only the analytical expression but also the index of dispersion.

The power function and the index of dispersion for the tested material are as follows:

$$S = 3.7739 \times N^{0.026}$$
 and  $R^2 = 0.4613$  (2)

The index of dispersion is relatively low; therefore, the measured data were corrected using approximation curves obtained from the values of mechanical-fracture parameters in time. The measured data were divided by a coefficient corresponding to the age of the specimens when dynamic tests were conducted. Coefficients were determined from regression curves of all observed parameters. Data obtained in this manner are standardized to a specimen age of 28 days.





In the next step, fatigue parameter adaptation data were plotted in the same way as the measured data. Fig. 2 introduces an example of a corrected S-N curve (using a logarithmic approximation curve for relative effective fracture toughness obtained from static three point bending fracture tests, see Tab. 1). The coefficients of analytical expressions (1) for S-N curves corrected by approximation curves and index of dispersion are summarized in Tab. 2.

	$S = a \times N^{o}$										
<b>Regression curve</b>		Power		Le	ogarithm	nic	Polynomial				
Relative parameter	а	b	$R^2$	а	b	$R^2$	а	b	$R^2$		
Compressive cube strength	3.3126	-0.029	0.7750	3.3096	-0.029	0.7698	3.2561	-0.029	0.8232		
Compressive strength – fractions	3.3803	-0.028	0.7475	3.3757	-0.028	0.7436	3.3246	-0.029	0.7982		
<i>Tensile splitting strength</i> <i>– fractions</i>	3.4597	-0.028	0.6807	3.4560	-0.028	0.6783	3.4505	-0.028	0.6833		
Modulus of elasticity	3.5337	-0.028	0.7224	3.5071	-0.028	0.7197	3.6758	-0.027	0.5683		
Fracture toughness	3.2923	-0.030	0.8497	3.2537	-0.030	0.8484	3.4282	-0.027	0.6706		
Fracture energy	2.8189	-0.033	0.7043	2.7722	-0.033	0.7145	2.7738	-0.032	0.6116		
Maximum load force – static fracture tests	2.9528	-0.032	0.7966	2.9102	-0.032	0.8171	2.8723	-0.032	0.7836		
Maximum load force – dynamic tests (1 cycle)	3.3356	-0.029	0.7860	3.3368	-0.029	0.7801	3.3155	-0.029	0.8083		

Tab. 2: Coefficients of S–N curves and index of dispersion.

### 5. Conclusions

In this paper results from fatigue tests carried out on plain class C30/37 concrete specimens were presented. The fatigue experiments lasted for a long time, which is problematic from the point of view of the ageing of specimen material. Because of this, data obtained from the fatigue tests were standardized to a specimen age of 28 days. Selected approximation curves obtained from mechanical-fracture parameter values in time were used for this purpose. The success of this procedure shows a substantial reduction in the values' index of dispersion from 0.46 to 0.85 [–].

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