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MODELING OF STRENGTH OF ELASTIC REINFORCED COMPOSITES BY FIBER INCLUSIONS

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Abstract: *In our contribution we will show how the Trefftz Radial Basis Functions (TRBF), i.e. RBF satisfying the governing equations can be used to increase the efficiency of modeling of such problems like composites reinforced by finite length fibres with large aspect ratio. Special attention will be given to application of the TRBF in the form of dipoles to the simulation of composite reinforced by fiber. The source functions (forces and dipoles) are continuously distributed along the fibre axis (i. e. outside of the domain, which is the domain of the matrix) and their intensities are modelled by 1D quadratic elements long the axis in order to satisfy continuity conditions between matrix and fibre. Obtained results are compared with FEM solutions obtained using own ICS FEM software.*

Keywords: ***Elastic reinforced composites, finite length fibres, short Trefftz Radial Basis Functions.***

1. Introduction

The composites of the future reinforced by stiff particles or fibres are important materials possessing excellent mechanical and also thermal and electro-magnetic properties. Understanding the behaviour of composite materials and composite structures is essential for structural design (Kormaníková, 2007). Reinforced composites contain huge number of reinforcing elements with large gradients in all fields in small parts of the matrix (in micro scale) around the reinforcing elements and accurate computational models are important for homogenization of material properties in macro scale (adjustment of local stiffness of such material) and for evaluation of strength of material. Micromechanics is essentially multiscale theory: Although a "representative volume element (RVE) can be viewed as a material point at the macro scale, it is associated with specific microstructure at the micro scale. It is well known that using volume element approximation such as FEM hundreds of elements are necessary to achieve required accuracy even for simple problem ((Filip et al., 2005) where 50 000 to 100 000 trilinear elements were used for problem containing one spherical particle in the matrix). In this paper we study the interaction of matrix-fiber-fiber for regular distributed straight in a patch inside matrix.

2. Method of Continuous Source Functions

A composite material with micro/nano structure with regularly distributed reinforcing fibres of unique dimension is to be modelled. Due to very small dimension of particles the stress and strain field can be considered to be homogeneous in the whole domain and all stresses and strains can be imagined split into a constant part, which introduces the state of the matrix without the stiffening and the local part, due to stiffening effect by the fibres.

Let's consider that the cross sectional dimensions of a fibre are much smaller than its length, the tensional stiffness of the fibre is much higher than the stiffness of matrix, the fibre is straight and ideal cohesion forces are assumed in the present model. Then the action of the fibre can be introduced by zero strains in longitudinal direction of the fibre boundary and zero difference of displacements in

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directions perpendicular to the fibre axis. Aspect ratio (length to radius) of the fibre is considered to be very large (100:1, 1000:1 in examples used here).

The classical Eshelby solution (Eshelby, 1961) was obtained for an elastic isotropic inclusion in an infinite elastic matrix. The treatment of the RVE as an infinite space implies that the inclusion concentration is dilute, and therefore, a direct application of these results to the case of finite inclusion concentration is only approximate. An improved model was suggested by (Mori & Tanaka, 1973). Their method also assumes the absence of all inhomogenities but it includes certain effect of the inhomogeneity by taking average strain in the matrix phase when all inhomogenities are present. Modification of existing homogenization methods via finite Eshelby tensors (Qu & Cherkaoui, 2006) provides significant improvement in predicting the behaviour of composites. In particular, the Hashin-Shtrikman variational bounds are modified according to the prescribed boundary condition (Hashin & Shtrikman, 1963). Recently, (Sauer, et al.) solved the elastic field of an idealized, spherical, finite RVE embedded in an infinite, homogeneous, isotropic medium using Boundary Integral Equations (BIE).

Because of large aspect ratio, continuity of strains between a matrix and a fibre can be simulated by continuously distributed source functions (forces, dipoles, dislocations, etc. (Blokh, 1964; Kachanov et al., 2003) as they are known from the potential theory) along the fibre axis (Fig. 1). The continuous source functions enable to simulate the continuity conditions with much reduced collocation points along the fibre boundary.

All displacement, strain and stress fields will be split into a homogeneous part corresponding to constant stress-strain in the matrix without the reinforcement. For simplicity an isotropic material properties are assumed in this paper.

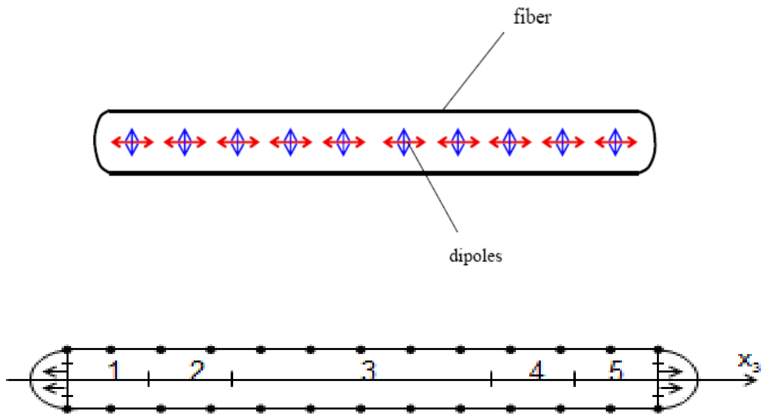

Fig. 1: Continuous force/dipole model for the fiber reinforcing element.

The field of displacements in an elastic continuum by a unit force acting in direction of the axis x_p is given by Kelvin solution

$$U_{pi}^{(F)} = \frac{1}{16\pi G(1-\nu)} \frac{1}{r} \left[(3-4\nu)\delta_{ip} + r_{,i} r_{,p} \right] \tag{1}$$

where *i* denotes the x_i coordinate of the displacement, *G* and ν are shear modulus and Poisson's ratio of the material of the matrix (isotropic material is considered here), *r* is the distance between the source point s, where the force is acting with a field point t, where the displacement is introduced, i. e.

$$r = \sqrt{r_i r_i}\ ,\ r_i = x_i(t) - x_i(s) \tag{2}$$

The summation convection over repeated indices acts and

$$r_{,i} = \partial r / \partial x_i(t) = r_i / r \tag{3}$$

is the directional derivative of radius vector *r*.

The displacement field of a dipole can be obtained from the displacement field of a force by differentiating it in the direction of the acting force, i.e.

$$U_{pi}^{(D)} = U_{pi,p}^{(F)} = -\frac{1}{16\pi G(1-v)}\frac{1}{r^2}\left[3r_{,i}r_{,p}^2 - r_{,i} + 2(1-v)r_{,p}\delta_{ip}\right] \tag{4}$$

The summation convection does not act over the repeated indices *p* here and in the following relations, too. Gradients of the displacement field are

$$U_{pi,j}^{(D)} = -\frac{1}{16\pi G(1-v)}\frac{1}{r^3}\left[-15r_{,i}r_{,j}r_{,p}^2 + 3r_{,i}r_{,j} + 2(1-2v)\delta_{ip}(\delta_{jp} - 3r_{,j}r_{,p}) + 6r_{,i}r_{,p}\delta_{jp} + \delta_{ip}(3r_{,p}^2 - 1)\right] \tag{5}$$

and corresponding strain and stress fields are

$$E_{pij}^{(D)} = \frac{1}{2}\left(U_{pi,j}^{(D)} + U_{pj,i}^{(D)}\right) = -\frac{1}{16\pi G(1-v)}\frac{1}{r^3}\left[-15r_{,i}r_{,j}r_{,p}^2 + 3r_{,i}r_{,j} + 2(1-2v)\delta_{ip}\delta_{jp} + 6v(\delta_{ip}r_{,j}r_{,p} + \delta_{jp}r_{,i}r_{,p}) + \delta_{ij}(3r_{,p}^2 - 1)\right] \tag{6}$$

$$S_{pij}^{(D)} = 2GE_{pij}^{(D)} + \frac{2Gv}{1-2v}\delta_{ij}E_{pkk}^{(D)} = -\frac{1}{8\pi(1-v)}\frac{1}{r^3}\left[(1-2v)(2\delta_{ip}\delta_{jp} + 3r_{,p}^2\delta_{ij} - \delta_{ij}) + 6vr_{,p}(r_{,i}\delta_{jp} + r_{,j}\delta_{ip}) + 3(1-5r_{,p}^2)r_{,i}r_{,j}\right] \tag{7}$$

3. Fiber-Reinforced Composites

Two different problems were simulated in order to study the interaction of fibres with matrix and also the interaction of fibres: 1) a patch of non-overlaying rows of fibres as shown in Fig. 2 on the left and 2) a patch of overlaying rows of fibres according to the Fig. 2 on the right. In the examples the modulus of elasticity of the matrix was *E = 1000* and Poisson ratio *ν = 0.3*. The matrix was reinforced by a patch of straight rigid cylindrical fibres. The length of fibres was *L = 1000* and *L = 100* and the radius *R = 1*. The distance between fibres was $\Delta_1 = \Delta_2 = \Delta_3 = 16$ and for longer fibres also $\Delta_3 = 200$ in the fibre direction. The fibres in the patch contain approximately 1% of the volume of the composite material.

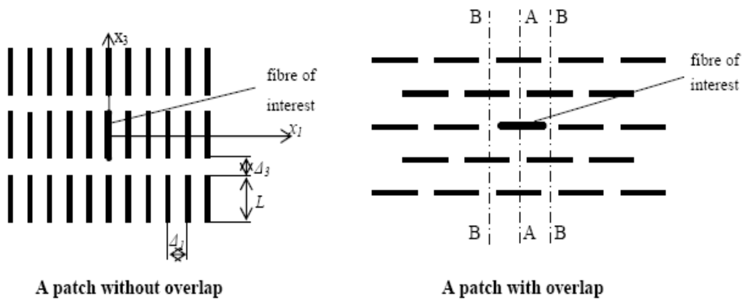

Fig. 2: 3D patches of regularly distributed fibres.

The patches of fibres consisted of 5 x 5 x 7 fibres in presented examples and "the fibre of interest" (FOI) were chosen in the middle to study the interaction of the fibre with matrix and with the other fibres as well. The domain is supposed to be loaded by far field stress $\sigma_{33\infty} = 10$ in the direction (x_3), which is also parallel to fibres' axes. The model of the fibre used in these examples contained less than 100 unknown parameters (intensities of the source functions) and about 200 collocation points. The problem is solved by LS method.

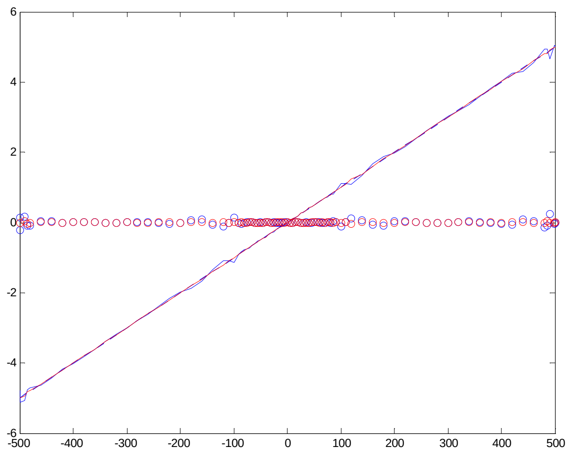

Fig. 3: Local displacements along a fibre.

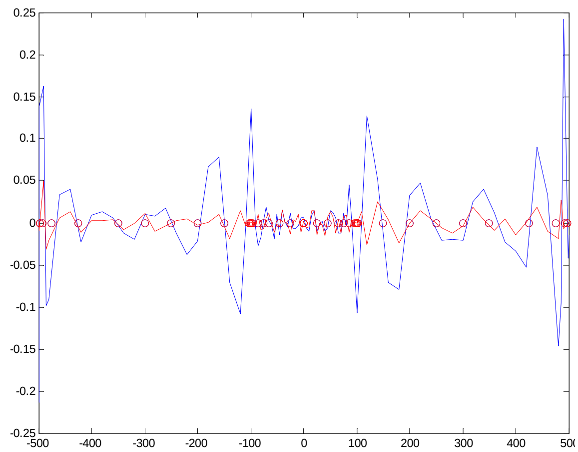

Fig. 4: Errors in local displacements along fibre.

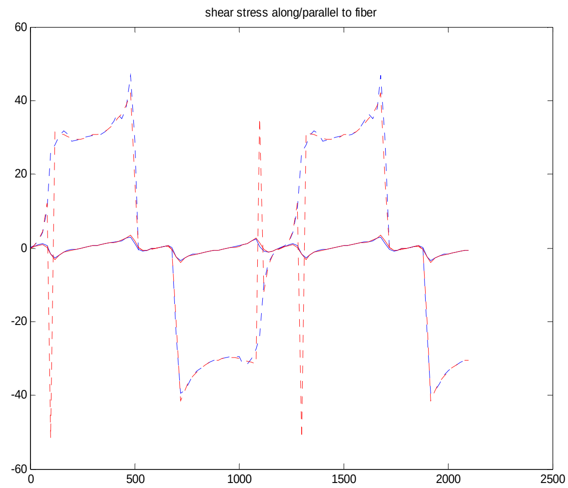

Fig. 5: Shear stress parallel to fibre axis.

Figs. 3 to 5 contain the local fields in the vicinity of the fibre of interest (coordinates origin is in the middle of the fibre) for length fiber (L = 1000 R) with overlay and the distance Δ_3 =200 R. Displacement differences of the points on the fibre boundary were linear along the fibre (Fig. 3). As the LS method was used in the procedure the errors were examined as shown in Fig. 4. The circles denote nodal points in distributed source functions (fictive forces of the Kelvin functions and dipoles) along the fibre axis. Two different models were used: with discontinuities (A - red) by the ends of neighbour fibres where the fields have large gradients and with continuous distribution of source functions (B - blue) along whole fibre axis.

4. Conclusions

The MCSF enables to simulate both the interaction of matrix with stiff reinforcing fibres and the fibre with other fibres very effectively. Computational experiment have shown that very large gradients in all fields occur not only in the end parts of fibres, but also in the points close to the ends of neighbouring fibres, more precisely in points on the line perpendicular to the axis of the neighbouring fibre. The large gradients in the end parts of fibres influence also the numerical models. If polynomial interpolation of source functions is chosen in the models then finer division of the continuous function is to be defined in these parts.

Acknowledgement

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References

Blokh, V.I. (1964) Theory of Elasticity. University Press, Kharkov.

Eshelby, J. D. (1961) Elastic inclusions and inhomogenities, in N. I. Sneddon and R. Hill, eds., Progress in Solid Mechanic, Vol. 2, North-Holland.

Filip, C., Garnier, B. & Danes, F. (2005) Prediction of the effective thermal conductivity of composites with spherical particles of higher thermal conductivity than the one of the polymer matrix, in Proc. Of COMSOL Multiphysics User's Conference, Paris.

Hashin, Z. & Shtrikman, S. (1963) A variational approach to the theory of the elastic behaviour of multiphase materials, J. Mech. Phys. Solids, 11, pp. 127-140.

Kachanov, M., Shafiro, B. & Tsukrov, I. (2003) Handbook of Elasticity Solutions. Kluver Academic Publishers, Dordrecht

Kormaníková, E. (2007) Optimization of laminated circular cylindrical shell, in Proc.Computational Modeling and Experiments of the Composite materials with Micro- and Nano-Structure. Liptovský Mikuláš, Slovak Republic, CD ROM.

Mori, T. & Tanaka, K. (1973) Average stressing matrix and average elastic energy of materials with misfitting inclusions, Acta Metall., 21, pp. 571-574.

Qu, J. & Cherkaoui, M. (2006) Fundamentals of Micromechanics of Solids, John Wiley &Sons, Hoboken, New Jersey.

Sauer, R. A., Wang, G. & Li, S. The composite Eshelby Tensors and their application to homogenization, Acta mechanica, 197, pp. 63-97.