18th International Conference
ENGINEERING MECHANICS 2012
Svratka, Czech Republic, May 14 – 17, 2012

pp. 1581–1588
Paper #29

MATHEMATICAL MODELLING OF A DAMPING ELEMENT WORKING ON THE PRINCIPLE OF SQUEEZING TWO LAYERS OF NORMAL AND MAGNETORHEOLOGICAL OILS ARRANGED IN SERIES AND ITS APPLICATION FOR VIBRATION ATTENUATION OF A RIGID ROTOR

J. Zapoměl*, P. Ferfecki*, J. Kozánek*

Abstract: *The oscillations amplitude and the forces transmitted between the rotor and the stationary part can be considerably reduced if damping devices are added to the coupling elements placed between the rotor and its casing. To achieve their optimum performance the damping effect must be controllable. In this paper there is proposed a new concept of a damping element working on the principle of squeezing two lubricating layers that are formed by normal and magnetorheological oils and that are mutually separated by a movable thin ring. Unlike to previous solutions, the magnetorheological layer always decreases the amount of damping. This decrease can be controlled and is required by certain operating conditions. In the developed mathematical model the normal and magnetorheological oils are represented by Newtonian and Bingham materials respectively and the pressure distribution in the lubricating layers is governed by the Reynolds' equations. The examination of the properties and efficiency of the proposed damping element was carried out by means of its application for vibration attenuation of a rigid unbalanced rotor. Results of the performed computational simulations proved that a suitable control of the damping effect made it possible to reach an optimum compromise between reduction of the oscillation amplitude and minimizing magnitude of the force transmitted into the rotor casing. A big advantage of the proposed damping element is that it does not need a complicated and expensive control system for its operation.*

Keywords: ***Controlled damping, new semiactive damping element, magnetorheological oil layer, Bingham fluid, modified Reynolds equation.***

1. Introduction

Unbalance of the rotating parts is one of the main sources of time variable forces that are transmitted between the rotor and its stationary part. Their magnitude can be significantly reduced if the rotor is flexibly supported and if the damping devices are added to the coupling elements. The theoretical analyses, confirmed by practical experience, show that to achieve the optimum performance of the rotating machines during their steady state running, the damping devices must be controllable.

One of the first reported works dealing with controllable dampers was published by Burrows et al., (1984). The damper was of a squeeze film kind and the authors examined the effect of controlling the oil-supply pressure on the change of the system damping coefficients. Another design of a controllable squeeze film damper was reported by Mu et al., (1991). In their solution, the gap between the damper inner and outer rings is conical filled with classical oil. Moving of the outer ring in the axial direction changes the damper radial clearance and land length, which changes the damping force. A new concept of controlling the damping devices is represented by magnetorheological dampers. Much work has been already done in this field. Wang et al., (2003) studied experimentally the vibration characteristics and the control method of a flexible rotor equipped with a magnetorheological squeeze

* Prof. Ing. Jaroslav Zapoměl, DrSc.: Institute of Thermomechanics, Dolejškova 1402/5; 182 00, Prague 8; CZ, e-mail: jaroslav.zapomel@vsb.cz

* Ing. Petr Ferfecki, PhD.: Institute of Thermomechanics, Dolejškova 1402/5; 182 00, Prague 8; CZ, e-mail: petr.ferfecki@vsb.cz

* Ing. Jan Kozánek, CSc.: Institute of Thermomechanics, Dolejškova 1402/5; 182 00, Prague 8; CZ, e-mail: kozanek@it.cas.cz

film damper. Zapoměl & Ferfecki (2009a) developed a mathematical model of a short magnetorheological damper of a squeeze film kind and utilized it for computational simulations of a transient response of a rigid rotor passing the critical speeds (Zapoměl & Ferfecki, 2009b). The same authors (Zapoměl & Ferfecki, 2011) proposed a combined damper formed by two lubricating layers of normal and magnetorheological oils arranged in parallel. Their investigations done by means of computational simulations were focused on its contribution to attenuation of the steady state vibration of a flexibly supported rigid rotor and on its stability.

In this paper there is investigated a new semiactive damping element generating the damping force by squeezing two mutually separated layers of normal and magnetorheological oils. The damping effect is controlled by changing the magnetic flux passing through the magnetorheological film. Results of the performed simulations show that a suitable setting of magnitude of the damping effect enables to reach the optimum compromise between attenuation of the rotor vibration amplitude and reduction of the forces transmitted into the rotor casing and its foundation plate.

2. Mathematical model of the new damping element

The principal parts of the proposed damping element (Figure 1) are the inner and outer rings between which there are two mutually separated layers of lubricating liquids. The inner ring is movable. It is coupled with the damper's body by a flexible element (squirrel spring) and supports the rolling element bearing, in which the rotor journal is mounted. The outer ring is stationary, fixed to the damper housing. The lubricating films are formed by normal (inner) and magnetorheological (outer) oils and their mutual separation is accomplished by a thin ring flexibly coupled with the damper's body. The damping device is equipped with an electric coil generating magnetic field. The magnetic flux passes through the layer of magnetorheological liquid and because its resistance against the flow depends on magnetic induction the change of the electric current can be used to control the damping effect.

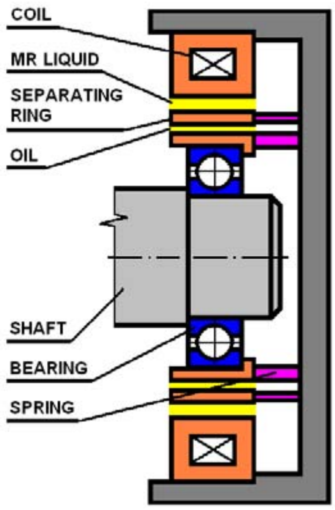

Fig. 1: Scheme of the proposed damping element

The developed mathematical model of the proposed constraint element is based on assumptions of the classical theory of lubrication but with some modifications. The normal and magnetorheological oils are represented by Newtonian and Bingham materials with the yield shear stress depending on magnetic induction. A further attention is focused only on the dampers whose geometric and design parameters make it possible to treat them as short (Krämer, 1993).

The thickness of the oil films depends on positions of the centres of the damper rings relative to the damper body (Krämer, 1993)

$$h_{CO} = c_{CO} - e_H \cos(\varphi - \gamma), \tag{1}$$

$$h_{MR} = c_{MR} - e_H \cos(\varphi - \gamma). \tag{2}$$

h_{CO}, h_{MR} denote the thickness of the films of classical and magnetorheological oils, c_{CO}, c_{MR} are the widths of the gaps between the rings filled with classical and magnetorheological oils, e_H denotes the rotor journal eccentricity, φ is the circumferential coordinate and γ denotes the position angle of the line of centres.

The pressure distribution in the layer of classical oil is governed by a Reynolds equation modified for short squeeze film dampers (Krämer, 1993)

$$\frac{\partial^2 p_{CO}}{\partial Z^2} = \frac{12\eta}{h_{CO}^3}\dot{h}_{CO} \,. \tag{3}$$

p_{CO} denotes the pressure in the layer of classical oil, η is the normal oil dynamic viscosity, Z is the axial coordinate and $(\dot{})$ denotes the first derivative with respect to time.

To describe the pressure field in the layer of the magnetorheological fluid, the Reynolds equation has been modified for Bingham material (Zapomĕl & Ferfecki, 2009a)

$$h_{MR}^3 p_{MR}'^3 + 3\left(h_{MR}^2\,\tau_y - 4\,\eta_B\,\dot{h}_{MR}\,Z\right)p_{MR}'^2 - 4\tau_y^3 = 0\,, \qquad \text{for} \qquad p_{MR}' < 0\,, \tag{4}$$

$$h_{MR}^3 p_{MR}'^3 - 3\left(h_{MR}^2\,\tau_y + 4\,\eta_B\,\dot{h}_{MR}\,Z\right)p_{MR}'^2 + 4\tau_y^3 = 0 \qquad \text{for} \qquad p_{MR}' > 0\,. \tag{5}$$

Both these equations are valid for $Z > 0$. p_{MR}, p_{MR}' denote the pressure and the pressure gradient in the axial direction in the layer of magnetorheological liquid, η_B is the Bingham viscosity and τ_y represents the yield shear stress.

Equations (3), (4) and (5) are solved for the boundary conditions expressing that the pressure at the damper's ends is equal to the pressure in the ambient space. Relationships (4) and (5) represent polynomial algebraic equations of the third order. The sought root must fulfil the conditions that the pressure gradient p_{MR}' is real (not complex), is negative (for equation (4)) or positive (for equation (5)) and satisfies the relation

$$\left|p_{MR}'\right| > \frac{2\,\tau_y}{h_{MR}} \;. \tag{6}$$

After calculation of the pressure gradient from equations (4) or (5) the pressure distribution in the axial direction is obtained by the integration

$$p_{MR} = \int p_{MR}'\,\mathrm{d}Z\,. \tag{7}$$

In the damper simplest design arrangement the rings, between which there is a layer of magnetorheological liquid, can be considered as a divided core of an electromagnet. Then the dependence of the yielding shear stress on magnetic induction can be approximately expressed

$$\tau_y = k_y\left(\frac{N_C\,I}{2\,h_{MR}}\right)^{n_y} . \tag{8}$$

k_y and n_y are material constants of the magnetorheological liquid, N_C is the number of the coil turns and I is the electric current.

In the areas where the thickness of the lubricating films rises with time ($\dot{h}_{CO} > 0\,, \dot{h}_{MR} > 0$) a cavitation is assumed. Pressure of the medium in these regions remains constant and equal to the pressure in the ambient space. In noncavitated areas the magnitude of the pressure is governed by solutions of the Reynolds equation (3) and integral (7).

Components of the damping forces are calculated by integration of the pressure distributions around the circumference and along the length of the damping element taking into account a cavitation in the oil films.

$$F_{MRy} = -2\,R_{MR}\int_0^{2\pi}\int_0^{\frac{L}{2}} p_{DMR}\cos\varphi\,\mathrm{d}Z\,\mathrm{d}\varphi\,, \tag{9}$$

$$F_{MRz} = -2\,R_{MR} \int_0^{2\pi} \int_0^{\frac{L}{2}} p_{DMR} \sin\varphi \,\mathrm{d}Z\,\mathrm{d}\varphi\,, \tag{10}$$

$$F_{COy} = -2\,R_{CO} \int_0^{2\pi} \int_0^{\frac{L}{2}} p_{DCO} \cos\varphi \,\mathrm{d}Z\,\mathrm{d}\varphi\,, \tag{11}$$

$$F_{COz} = -2\,R_{CO} \int_0^{2\pi} \int_0^{\frac{L}{2}} p_{DCO} \sin\varphi \,\mathrm{d}Z\,\mathrm{d}\varphi\,. \tag{12}$$

F_{COy}, F_{COz}, F_{MRy}, F_{MRz} are the y and z components of the hydraulic forces produced by the layers of normal and magnetorheological oils respectively, R_{CO}, R_{MR} are the mean radii of the layers of normal and magnetorheological oils, L is the axial length of the damping element and p_{DCO}, p_{DMR} denote the pressure distributions (taking into account different pressures in cavitated and noncavitated regions) in the layers of normal and magnetorheological oils.

3. The equation of motion of a rigid rotor damped by the new damping elements

The investigated rotor (Figure 2) consists of a shaft and of one disc and with the stationary part it is flexibly coupled by the proposed damping elements at both its ends. The rotor is considered to be absolutely rigid, it is unbalanced, turns at constant angular speed and is loaded by its weight. The system is symmetric relative to the plane perpendicular to the shaft axis. The squirrel springs are prestressed to be eliminated their deflection caused by the rotor weight.

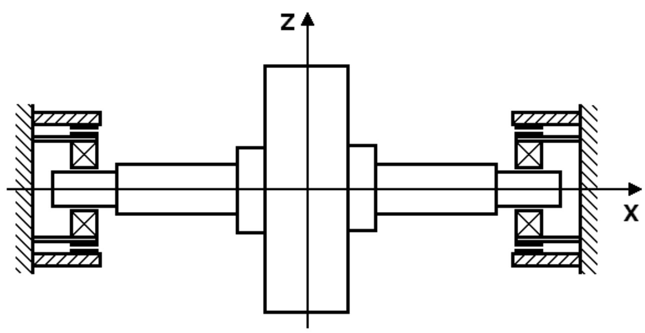

Fig. 2: Scheme of the investigated rotor system

The rotor vibration is governed by a set of four equations of motion

$$m_R \ddot{y}_1 + b_P \dot{y}_1 + 2\,k_{DA} y_1 = m_R e_T \omega^2 \cos(\omega t + \psi_o) + 2\,F_{COy}(y_1, z_1, y_2, z_2, \dot{y}_1, \dot{z}_1, \dot{y}_2, \dot{z}_2)\,, \tag{13}$$

$$m_R \ddot{z}_1 + b_P \dot{z}_1 + 2\,k_{DA} z_1 = m_R e_T \omega^2 \sin(\omega t + \psi_o) + 2\,F_{COz}(y_1, z_1, y_2, z_2, \dot{y}_1, \dot{z}_1, \dot{y}_2, \dot{z}_2)\,, \tag{14}$$

$$m_{SR} \ddot{y}_2 + 2\,k_{SR} y_2 = -2\,F_{COy}(y_1, z_1, y_2, z_2, \dot{y}_1, \dot{z}_1, \dot{y}_2, \dot{z}_2) + 2\,F_{MRy}(y_2, z_2, \dot{y}_2, \dot{z}_2)\,, \tag{15}$$

$$m_{SR} \ddot{z}_2 + 2\,k_{SR} z_2 = -2\,F_{COz}(y_1, z_1, y_2, z_2, \dot{y}_1, \dot{z}_1, \dot{y}_2, \dot{z}_2) + 2\,F_{MRz}(y_2, z_2, \dot{y}_2, \dot{z}_2)\,. \tag{16}$$

m_R is the rotor mass, b_P is the coefficient of the rotor external damping, k_{DA} is stiffness of the squirrel spring, $m_{SR,}$ k_{SR} are the mass of the ring separating the lubricating layers and the stiffness of its support respectively, ω is angular speed of the rotor rotation, e_T is eccentricity of the rotor unbalance, t is the time, ψ_o is the phase shift, y_1, z_1, y_2, z_2 are displacements of the rotor centre (centre of the rotor journal) and of the centre of the ring separating the lubricating layers and $(\dot{})$, $(\ddot{})$ denote the first and second derivatives with respect to time.

The steady state solution of the equations of motion is obtained by a trigonometric collocation method. As the system is symmetric and the squirrel springs are prestressed, it can be assumed that trajectories of the rotor and the rings centres are circular. Then it holds for the displacements

$$y_1 = r_{C1} \cos \omega t - r_{S1} \sin \omega t \, , \tag{17}$$

$$z_1 = r_{C1} \sin \omega t + r_{S1} \cos \omega t \, , \tag{18}$$

$$y_2 = r_{C2} \cos \omega t - r_{S2} \sin \omega t \, , \tag{19}$$

$$z_2 = r_{C2} \sin \omega t + r_{S2} \cos \omega t \, . \tag{20}$$

Substitution of (17) - (20) and of their first and second derivatives with respect to time in (13) - (16) gives a set of four nonlinear algebraic equations. As the number of resulting equations is equal to the number of unknown parameters (r_{C1}, r_{S1}, r_{C2}, r_{S2}), only one collocation point of time is needed for their determination. For the moment of time $t = 0$ they read

$$\left(2 k_{DA} - \omega^2 m_R\right) r_{C1} - \omega b_P r_{S1} = m_R e_T \omega^2 \cos \psi_o + 2 F_{COy} \, , \tag{21}$$

$$\omega b_P \, r_{C1} + \left(2 k_{DA} - \omega^2 m_R\right) r_{S1} = m_R e_T \omega^2 \sin \psi_o + 2 F_{COz} \, , \tag{22}$$

$$\left(2 k_{SR} - \omega^2 m_{SR}\right) r_{C2} = -2 F_{COy} + 2 F_{MRy} \, , \tag{23}$$

$$\left(2 k_{SR} - \omega^2 m_{SR}\right) r_{S2} = -2 F_{COz} + 2 F_{MRz} \, . \tag{24}$$

Solution of the set of equations (21) - (24) makes it possible to determine amplitudes of the rotor steady state vibration and of the forces transmitted into the stationary part for the specified angular speeds.

4. Results of the computational simulations

The basic data of the investigated system are : mass of the rotor 450 kg, stiffness of the squirrel spring 2 MN/m, linear coefficient of the rotor external damping 1000 Ns/m, mass of the separating ring 2 kg, eccentricity of the rotor centre of gravity 100 μm, viscosity of the normal oil 0.002 Pas, Bingham viscosity of the magnetorheological liquid 0.3 Pas, length of the damping element 60 mm, mean diameters of the lubricating layers 110 mm (normal oil), 150 mm (magnetorheological oil) and the damper gap width 0.2 mm (normal oil), 1 mm (magnetorheological oil). The task was to analyse dependences of amplitude of the rotor vibration and of the force transmitted through the coupling elements into the stationary part on the rotor angular speed and magnitude of the applied current and to compare them with the allowed values (maximum vibration amplitude 140 μm, maximum transmitted force 800 N).

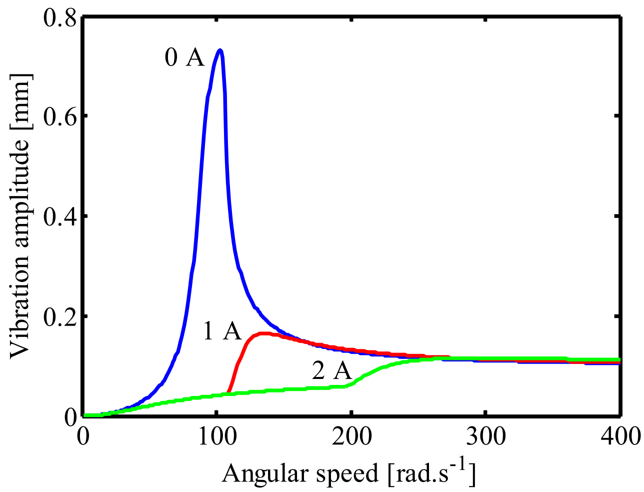

Fig. 3: Frequency response characteristic (centre of the rotor)

Figures 3 and 4 depict the frequency response characteristics of the centres of the rotor (rotor journal) and of the separating ring for several magnitudes of the applied electric current. It is evident that the increasing current has only little influence on amplitude of the rotor vibration in the region of high velocities. But in the area of low angular speeds the system behaves in a different way. Only large applied current makes it possible to attenuate the vibration sufficiently. The vibration amplitude of the centre of the ring separating the layers of normal and magnetorheological oils approaches to zero if magnitude of the current rises and angular velocity of the rotor decreases. If magnitude of the current is sufficiently high (depending on the speed of rotation), the layer of magnetorheological liquid becomes almost absolutely rigid and the rotor behaves as it would be damped only by normal squeeze film dampers.

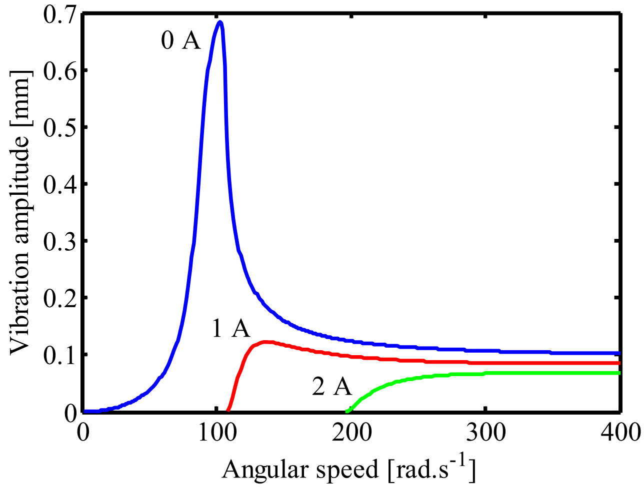

Fig. 4: Frequency response characteristic (centre of the separating ring)

The force is transmitted between the rotor and its casing through the squirrel springs and the damping layers. Figure 5 shows that to minimize magnitude of the total transmitted force in the area of low angular speeds the applied current must be large, in the area of higher angular velocities its magnitude must be zero or very low.

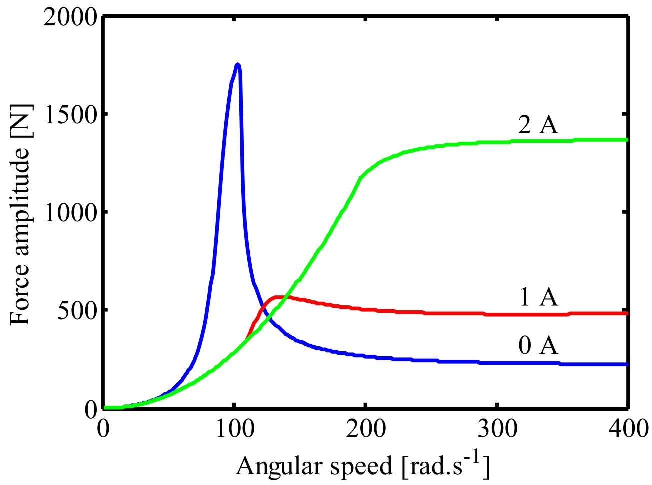

Fig. 5: Transmitted force amplitude – angular velocity relationship

Figures 6 and 7 show the admissible magnitudes of the electric current, for which the requirements put on the maximum allowable vibration amplitude and magnitude of the force transmitted into the stationary part are satisfied, in dependence on the speed of the rotor rotation. The combination of these results is depicted in Figure 8. It is evident that the condition for allowed values of parameters of the rotor steady state vibration can be satisfied in the whole extent of the operating speeds except for the ones from a small velocity range approximately between 150 and 180 rad/s. In this speed interval amplitude of the vibration cannot be reduced without exceeding the allowed value of the transmitted force.

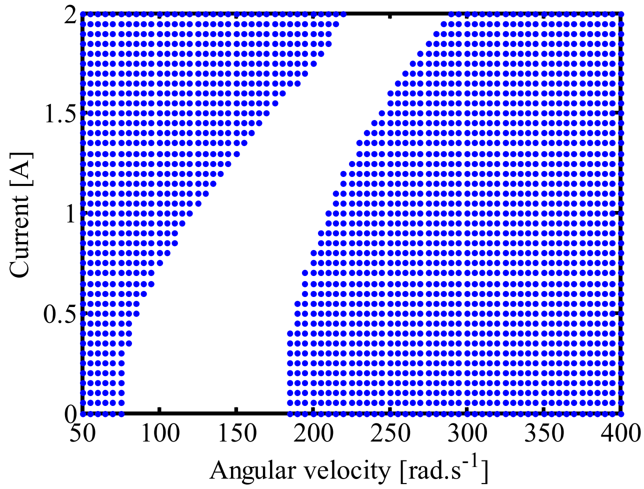

Fig. 6: Current - angular velocity relationship from the point of view of allowed vibration

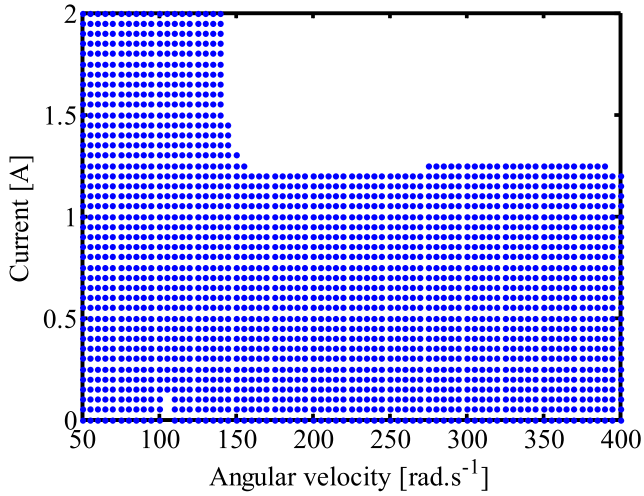

Fig. 7: Current - angular velocity relationship from the point of view of allowed force

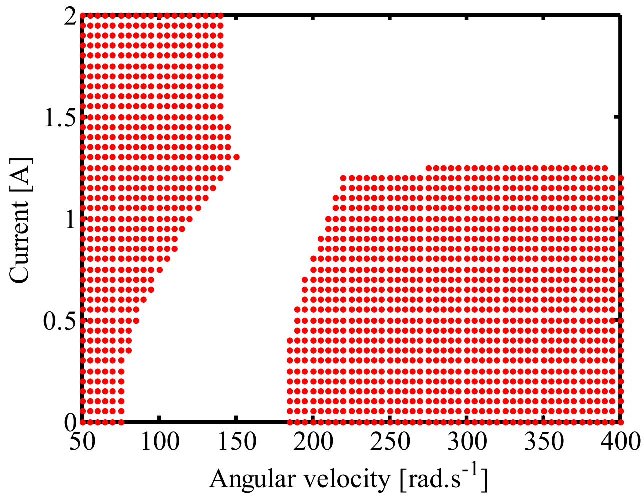

Fig. 8: Current - angular velocity relationship for allowed vibration and force

A detailed analysis of the frequency response characteristics given in Figures 3, 4 and 5 shows that to achieve the optimum performance of the damping device in the velocity interval above approximately 200 rad/s the electric current should be switched off. The decrease of the damping arrives at considerable reduction of the force transmitted into the rotor casing and increase of the rotor vibration amplitude is only negligible.

5. Conclusions

The damping effect of the new proposed damping element is produced by squeezing two layers of lubricating liquids, which are mutually separated by a ring. The layers are formed by normal and magnetorheological oils. The control of the damping force is accomplished by means of the change of magnetic flux passing through the film of magnetorheological liquid. Results of the carried out computational simulations show that a suitable setting of magnitude of the applied current makes it possible to reach the optimum compromise between attenuation of the rotor vibration and reduction of the force transmitted into the stationary part and into the foundation plate. It is evident that at lower angular velocities a higher damping effect is needed to suppress both the vibration and the transmitted force. In the interval of higher velocities large damping arrives at a strong increase of the transmitted force but only at little attenuation of the rotor oscillations. This implies that for these operating conditions it is desirable to switch off the auxiliary magnetorheological damping or to apply only a very small electric current.

Acknowledgement

The work reported here has been supported by research projects AVO Z20760514 and P101/10/0209. This help is gratefully acknowledged.

References

Burrows, C.R., Sahinkaya, M.N. & Turkay, O.S. (1984) An adaptive squeeze-film bearing. *ASME Journal of Tribology*, 106, 1, pp. 145-151.

Krämer, E. (1993) *Dynamics of Rotors and Foundations*. Springer-Verlag, Berlin-Heidelberg-New York.

Mu, C., Darling, J. & Burrows, C.R. (1991) An appraisal of a proposed active squeeze film damper. *ASME Journal of Tribology*, 113, 4, pp. 750-754.

Wang, J., Meng, G. & Hahn E.-J. (2003) Experimental study on vibration properties and control of squeeze mode MR fluid damper-flexible rotor system, in: *Proceedings of the 2003 ASME Design Engineering Technical Conference & Computers and Information in Engineering Conference*, ASME, Chicago, pp. 955-959.

Zapoměl, J. & Ferfecki, P. (2009a) Mathematical modelling of a short magnetorheological damper. *Transactions of the VŠB – Technical University of Ostrava, Mechanical Series*, LV, 1, pp. 289-294.

Zapoměl, J. & Ferfecki, P. (2009b) A computational investigation of vibration attenuation of a rigid rotor turning at a variable speed by means of short magnetorheological dampers. *Applied and Computational Mechanics*, 3, 2, pp. 411-422.

Zapoměl, J. & Ferfecki, P. (2011) Stability investigation of the steady state response of flexibly supported rigid rotors, in: *Vibration Problems ICOVP 2011: The 10th International Conference on Vibration Problems* (J. Náprstek et al. eds), Springer Proceedings in Physics 139, Prague, pp. 521-527.