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MODELING OF FIBER BRIDGING IN MULTIPLY-CRACKING MORTAR

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Abstract: *The use of composite materials is one of current trends in civil engineering. Proper description of their behavior is one of prerequisites for correct and appropriate application of these materials. The subject of our research are fiber reinforced composite materials that exhibit tensile pseudo-ductile and strain hardening behavior. Response of a single fiber is one of many factors that affect the overall response of the composite. In the literature analytical relations describing the behavior of a fiber during its pull-out from the surrounding matrix can be found. Up to now, these models did not take into account the possibility that a fiber bridges more than one crack. In the present paper, we refine the model for one fiber by considering that it may cross several parallel cracks. A numerical study is performed to investigate the effect of this consideration on the relation between force acting in the fiber and its pull-out displacement.*

Keywords: ***Fiber-reinforced composite, fiber bridging, pseudo-ductility, strain hardening, multiple cracking***

1. Introduction

Historical monuments are at the present time subjected to various effects, which their builders couldn't take into account at the time of construction and which contribute to their deterioration. Excessive loads caused by temperature fluctuations or technical seismicity due to traffic and technological processes cause degradation and cracking of historical masonry. The cracks pave the way for penetration of water and contaminants into the masonry, which leads to further degradation. These cracks often formed in the masonry joints, because mortar is usually the weaker element. To alleviate the degradation we are developing a fiber reinforced mortar that under tensile stress undergoes multiple cracking as opposed to failing by a single brittle crack. During the multiple cracking process, a large number of fine cracks with controlled width forms, while the mortar retains macroscopic integrity. Keeping the small crack width may prevent penetration of contaminants. For these cases a methodology for systematic design of materials with brittle matrix reinforced with short fibers was developed (Li, 2003). This methodology employs micromechanics and fracture mechanics based models of the damage phenomena taking place at the level of the composite microstructure, such as fiber debonding and pullout and matrix cracking. It was successfully used, for example, for design of Engineered Cementitious Composites – ECC (Li, 2003). Our intention is to use this approach to develop a new lime mortar reinforced with short random fibers, which could be applied to restoration works on historic buildings. As part of this effort we further refine the existing micromechanical models to take into account previously neglected phenomena.

2. Single fiber response

Bridging effect of fibers crossing a crack in a brittle-matrix composite has a dominant influence on whether the material eventually exhibits multiple cracking (Marshall et al., 1988). When a fiber-bridged crack forms and opens, the fibers are being extracted from the surrounding matrix. This process can be divided into two main stages, which can be described by the relationship between force P on the pulled-out end of the fiber and displacement u at the same point. In the first stage, the fiber

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gradually debonds from the matrix and as the force P increases. Assuming that debonding is resisted by fiber-matrix chemical bond strength G_d, that the debonded portion of the fiber elastically deforms, and that constant friction τ_0 acts on the debonded fiber-matrix interface, this stage can be described by Eq. (1). This stage is completed when the embedded end of the fiber becomes fully debonded from the matrix. The corresponding displacement of u then reaches the value of δ_c given in Eq. (2). A pull-out phase follows, during which the fiber slips out from the matrix while the contact area with the matrix diminishes. The pull-out phase is described by Eq. (3). The whole $P-u$ relation is shown in Fig. 1.

$$P_{deb} = \sqrt{\frac{\pi^2 \tau_0 E_f d_f^3}{2} u + \frac{\pi^2 G_d E_f d_f^3}{2}} \tag{1}$$

$$\delta_c = \frac{2\tau_0 L_e^2}{E_f d_f} + \sqrt{\frac{8 G_d L_e^2}{E_f d_f}} \tag{2}$$

$$P_{pull} = \pi d_f \tau_0 \left(1 + \frac{\beta(u-\delta_c)}{d_f}\right)(L_e - u + \delta_c) \tag{3}$$

In equations is E_f fiber elastic modulus, L_e is fiber embedment length, d_f is fiber diameter, τ_0 is frictional stress on debonded interface, G_d is fiber-matrix chemical bond strength and β is fiber-matrix interface slip-hardening parameter.

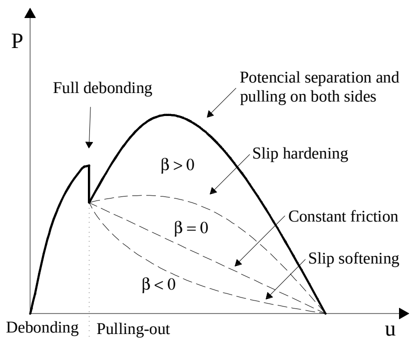

Fig. 1: Single fiber pull-out response

3. Probability of fiber bridging several cracks

The model described above adopts the assumption, that a fiber bridges only one crack. However, the length of fibers that are typically used in short-fiber reinforced mortars is in the order of 10 mm, while the crack to crack distance during multiple cracking can be as low as few mm. A question naturally arises, whether the $P-u$ relation described above is realistic in the multiply-cracking composites. Thus, the aim of this paper is to describe the influence of a state when a fiber bridges more than one crack.

First of all, let us investigate how many fibers may bridge more than one crack when the composite undergoes multiple cracking. To this end, we assume that fibers of length L_f are randomly distributed and oriented in the composite. Furthermore, we assume that matrix cracks, bridged by these fibers, are perfectly planar and parallel. The analytical relation derived below describes the probability with which a fiber passing through a fixed point P on a one crack crosses another crack at a distance d_c.

All possible cases of the position of the fiber end points fill the space corresponding to the sphere with radius L_e and center at point P. For one side of the crack it is a hemisphere. End points of fibers, which intersect the second crack, fill the space corresponding to the spherical cap of the hemisphere with a height L_e-d_c. (see Fig. 2). The ratio of the volume of the hemisphere and the spherical cap (Eq. 4) describes the probability with which a fiber intersects two parallel cracks with the given distance.

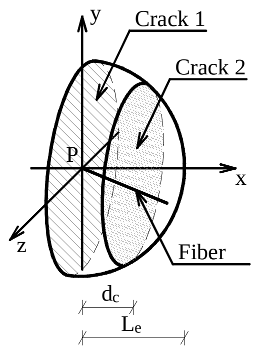

Fig. 2: Randomly oriented fiber in space with two parallel cracks

$$P(d_c) = \frac{(L_e - d_c)(2L_e^2 - L_e d_c - d_c^2)}{2L_e^3} \tag{4}$$

To verify the validity of the analytical relationship in Eq. (4), a numerical simulation was performed. We considered prismatic specimens of fiber reinforced composite with the same length of 200 mm but different square cross-sections with widths a) 50 mm b) 100 mm c) 200 mm.. Within these volumes, random fibers were generated keeping fiber volume fraction constant and equal to 2 %. Pairs of cracks (perpendicular to the specimen axis) with different mutual distances were inserted into each specimen and the number of fibers bridging both cracks was counted. The results for each specimen were averaged and the probability P was calculated. Figure 3 shows a very good agreement between the results of the analytical solution (Eq. 4) and numerical simulations.

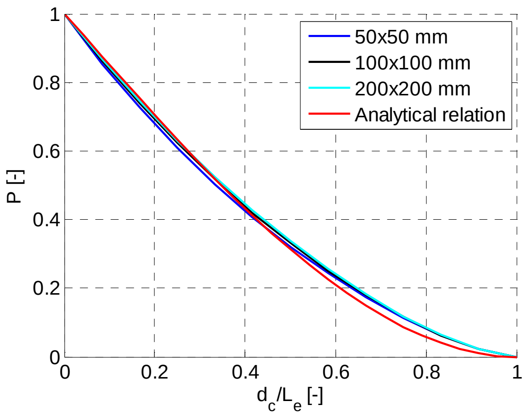

Fig. 3: Dependence of the number of fibers intersecting two parallel cracks on their distance

4. Crack spacing

At a crack plane, each fiber carries its bridging force. The fiber force decreases along its length with increasing distance from the crack plane due to transfer of the load to the surrounding matrix through the friction at the interface. At the end of debonded fiber-matrix interface (at distance d_c from the crack) the force is completely transferred to the matrix. Assuming constant frictional stress τ_0 along the interface, this force can be expressed as:

$$P(d_c) = \sqrt{\frac{\pi^2 G_d E_f d_f^3}{2}} + \pi d_f \tau_0 d_c \tag{5}$$

If we consider spatial randomness of fiber orientation, the number of fibers bridging a crack of unit area N_s is (Naaman 1972 referenced in Naaman, 2008):

$$N_s = \frac{2V_f}{\pi d_f^2} \tag{6}$$

And the area of crack corresponds to single fiber is:

$$A_m = \frac{\pi d_f^2}{2V_f} \tag{7}$$

The stress in matrix at distance d_c can be expressed as:

$$\sigma_m = P(d_c)/A_m \tag{8}$$

Assuming that a new crack forms when stress σ_m reaches the matrix tensile strength f_t, we can express the crack to crack distance for single fiber perpendicular to crack with sufficient embedment length as:

$$d_c = \frac{1}{\tau_0}\left(\frac{f_t d_f}{2V_f} - \sqrt{\frac{d_f G_d E_f (1+\eta)}{2}}\right) \tag{9}$$

Where $\eta = (E_f V_f)/(E_m V_m)$ expresses deformation of the matrix, V_m is matrix volume fraction, E_m is matrix elastic modulus, V_f is fiber volume fraction. For material and geometric parameters of typical ECC with PVA fibers d_f = 0,04 mm, L_e = 12 mm, E_f = 21 800 MPa, τ_0 = 2,21 MPa, G_d = 0,004 N/mm, E_m = 15900 MPa, f_t = 4,3 MPa and V_f = 0,02 we get d_c = 1,34 mm. From Eq. (4) we get that approximately 84 % of fibers crossing two cracks in this distance, which shows, that the possibility of fibers bridging multiple cracks should be taken into account.

5. Response of fiber bridging several cracks

For a description of behavior of single fiber bridging several cracks a numerical model was created and implemented in software MATLAB. This model utilizes the analytical relations of Eq. (1) – (3). We consider that a fiber bridges one main and one or two adjacent cracks. The distances between the main and adjacent cracks were being changed and we monitored the P_f - w relationship of the fiber at the main crack, where w crack opening and P_f is force in the fiber.

Cracks divide the fiber into several parts. Computation was controlled by displacement u_s on shorter of edge parts, because this part determines maximum force in the fiber P_f. During the debonding stage, separation occurs on every side of each crack. There is tunnel crack propagation along the fiber and debonded length L_{deb} is:

$$L_{deb} = \frac{P_f - \sqrt{\frac{\pi^2 G_d E_f d_f^3}{2}}}{\pi d_f \tau_0} \tag{10}$$

When the debonded length L_{deb} reaches the embedment length L_e on shorter edge part, pull-out phase occurs and force in fiber decreases in case $\beta = 0$. The crack beside shorter edge part is opening and others are closing due to fiber stiffness. Another case occurs when tunnels propagating from two nearby cracks meet. Debonding stops and when force P_f increases, only elastic deformation of fiber continues, which is restrained by frictional stress τ_0 at the interface. Displacement of fiber at any point can be described by equation:

$$u(x) = \frac{1}{E_f A_f}\int P_f - \pi d_f \tau_0 x \, dx \tag{11}$$

Considering, that center of full debonded part don't change its position on fiber and embedment length on this part corresponds to half distance of cracks d_c, we get contribution to the crack opening as elongation of fiber from half of this part:

$$u = \frac{2P_f d_c}{\pi E_f d_f^2} - \frac{\tau_0 d_c^2}{2E_f d_f} \tag{12}$$

Finally, we obtain crack opening w as the sum of pulled length on both sides of the crack:

$$w = \sum_{i=1}^{2} u_i \tag{13}$$

Where u_i is prescribed u_s for debonding or u for elastic deformation of the fiber from Eq. (12). Responses of fiber in the main crack are shown in Figure 4 and schematic drawings (possible states) at the end of debonding stage are shown in Figure 5.

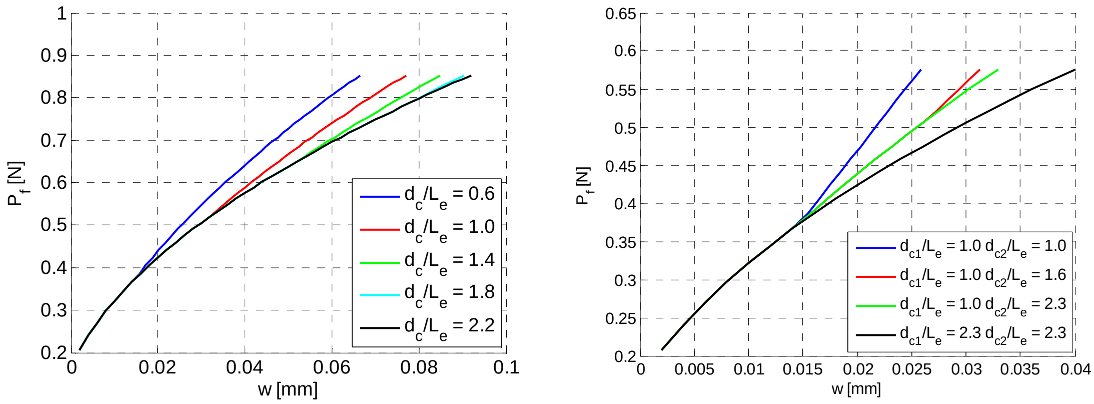

Fig. 4: Single fiber response for one nearby crack (left) and two nearby cracks (right)

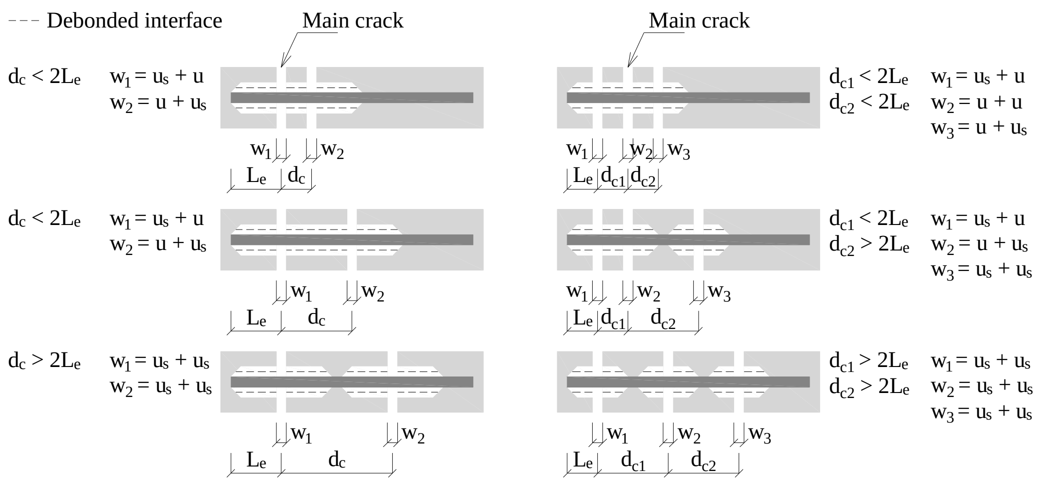

Fig. 5: Schematic drawings for different d_c – one nearby crack (left) and two nearby cracks (right)

6. Conclusions

According to the results, the single fiber response is affected by the interaction of cracks. This interaction occurs when $d_c < 2L_e$. It causes that for the same force P_f we get smaller crack opening w. Future work on this topic will be focused on examination how the interaction of cracks affects the behavior of cracks themselves.

Acknowledgement

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