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ACTIVE VIBRATION CONTROL OF A CANTILEVER BEAM

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Abstract: *The paper deals with analysis of a cantilever beam equipped by the active vibration control system. The cantilever beam as a continuum is approximated by a lumped-parameter system. The lumped-parameter model enables to derive the transfer functions relating forces to displacements, to determine the appropriate controller type and to determine under what conditions the system will work.*

Keywords: ***cantilever beam, lumped-parameter model, active vibration control, controller type.***

1. Introduction

Control theory is based on the models with one (single) input and one output (SISO) or models with multiple inputs and multiple outputs (MIMO). These models are described by systems of ordinary differential equations. The description of a continuum on the contrary uses partial differential equations. For the analysis of the continuum, suitable approximation of systems with a lumped parameter model must be used (Preumont, & Seto, 2008).

The computed transfer functions of the lumped-parameter system allow to determine the appropriate controller type and to determine under what conditions the system will work (Genta, 2009).

2. Lumped parameter model

A cantilever beam of the length L as a continuum can be divided into discrete elements of the same length ΔL that are modeled using rigid-body dynamics. How to create the lumped parameter model of the cantilever beam of the rectangular cross section and to associate this multibody system with the Cartesians coordinates x, y, z is shown in Fig. 1. The cantilever beam is clamped at the xy-plane and its centerline is parallel to the z-axis. It is assumed only planar motion of the cantilever beam in the yz-plane. Let N be the number of flexible links in the model. Linking of a pair of adjacent beam elements is considered in the mentioned plane as free with a torsion spring. The coordinates of the multibody systems are usually associated with the gravity centers. Such coordinate system requires the additional set of constrains for linking of the individual beam elements in one point. Additional equations are not needed if the coordinate system is chosen in such a way that describes motion of the meeting points of two adjacent elementary beams. The vertical coordinates of these points are marked by y_1, y_2,…, y_N. The angle of rotation with respect to the horizontal axis can be marked by δ_1, δ_2, …, δ_N.

The deflection Δy of the beam may be expressed as a function of the beam length and the difference $\Delta\delta$ of the adjacent elements $\Delta y = \Delta\delta\,\Delta L$. The bending stiffness K_δ of the elementary cantilever beam relates the applied bending moment M to the resulting relative rotation $\Delta\delta$ of the elementary beam

$$K_\delta = \frac{M}{\Delta\delta} = \frac{3EI_x}{\Delta L} \tag{1}$$

where E is Young's modulus of the beam material [N/m^2], $I_x = bh^3/12$ is the area moment of inertia of the beam cross-section [m^4] about the horizontal x-axis, b is the beam width and h is the beam height.

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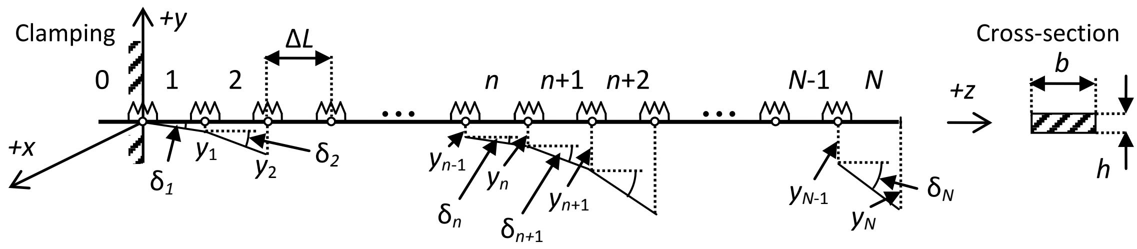

Fig. 1: Coordinates and elements of a cantilever beam.

The coordinates of the beam equidistant points in the Cartesian coordinates and the independent generalized coordinates for Lagrangian equations of motion are identical. For further derivation it makes sense only motion in the direction of the *y*-axis. Because they are assumed small deformations, the shifts of the meeting points in the direction of the *z*-axis are neglected. If all angles are small enough, then their measure in radians is given by the formula

$$\begin{aligned} \delta_n &= (y_n - y_{n-1})/\Delta L, \\ \Delta\delta_n &= \delta_{n+1} - \delta_n. \end{aligned} \tag{2}$$

The coordinates of the gravity centre of the elementary beams are as follows

$$Y_1 = y_1/2, \quad Y_n = (y_n + y_{n-1})/2 \tag{3}$$

The potential V and kinetic T energy of the cantilever beam in the horizontal position as a continuum replaced by its lumped parameter model is as follows

$$\begin{aligned} V &= \sum_{n=0}^{N-1} \frac{1}{2} K_\delta (\Delta\delta_n)^2 + \sum_{n=1}^{N} \Delta m g Y_n, \\ T &= \sum_{n=1}^{N} \left(\frac{1}{2} \Delta m \left(\frac{\mathrm{d} Y_n}{\mathrm{d} t} \right)^2 + \frac{1}{2} J_x \left(\frac{\mathrm{d}\,\delta_n}{\mathrm{d} t} \right)^2 \right), \end{aligned} \tag{4}$$

where J_x is the moment of inertia [kgm^2] about the horizontal x-axis and perpendicular to the centerline of the elementary beam. For a solid cuboid of height h, and length ΔL it is $J_x = \Delta m \left(\Delta L^2 + h^2 \right)/12$. The cantilever beam is considered as a conservative system. Lagrange's equations of motion of such a system are as follows

$$\frac{\mathrm{d}}{\mathrm{d} t} \left(\frac{\partial T}{\partial \dot{y}_n} \right) - \frac{\partial T}{\partial y_n} + \frac{\partial V}{\partial y_n} = 0, \quad n = 1, 2, \ldots, N \tag{5}$$

After introduction symbols **M** for a mass square matrix and **K** for a stiffness square matrix and **G** for a gravity force column vector and **y** for a coordinate column vector into the matrix equation of motion we obtain

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{K}\mathbf{y} + \mathbf{G} = \mathbf{0} \tag{6}$$

where

$$\mathbf{M} = \begin{bmatrix} B & A & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ & A & B & A & \\ & \vdots & \vdots & \vdots & \vdots \\ & & & A & B/2 \end{bmatrix}, \quad \mathbf{K} = \frac{K_\delta}{\Delta L^2} \begin{bmatrix} 6 & \cdots & & & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & 1 & -4 & 6 & -4 & 1 & \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & & & \cdots & 1 \end{bmatrix}, \quad \mathbf{G} = mg \begin{bmatrix} 1 \\ \vdots \\ 1 \\ \vdots \\ \frac{1}{2} \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \\ \vdots \\ y_N \end{bmatrix}, \tag{7}$$

$$A = \frac{\Delta m}{4} \left(1 - \frac{1}{3} \left(1 + \left(\frac{h}{\Delta L} \right)^2 \right) \right), B = \frac{\Delta m}{2} \left(1 + \frac{1}{3} \left(1 + \left(\frac{h}{\Delta L} \right)^2 \right) \right) \tag{8}$$

2.1 Steady state deformation shape of the cantilever beam

Steady state deformation of the beam in the horizontal position due to the gravity force is resulting from solution of the equation $\mathbf{Ky} + \mathbf{G} = \mathbf{0}$. If the cantilever beam is in vertical position then $\mathbf{y}_0 = \mathbf{0}$. For testing a beam with the following parameters is prepared: L = 0.5 [m], b = 0.04 [m], h = 0.005 [m]. The beam is divided into N = 10 elements. Deflection of the beam's own self-weight is shown in Fig. 2A.

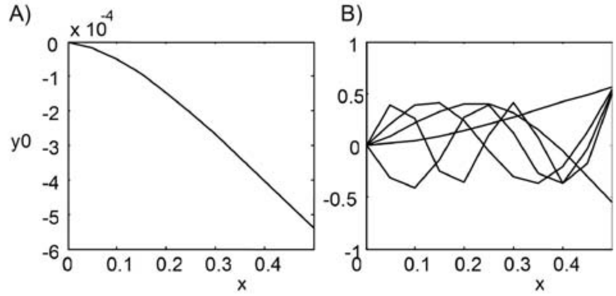

Fig. 2: A) Deflection of the cantilever beam's own self-weight, B) The first 5 of 10 modal shapes of the cantilever beam.

2.2 Free vibration

For vertical position of the cantilever beam the governing equation of free vibration is as follows

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{Ky} = \mathbf{0} \tag{9}$$

The solution of this equation of the homogenous type is assumed to be in the form of $\mathbf{y} = \mathbf{u}\exp(j\omega t)$, where **u** is an n-dimensional vector of the oscillation amplitudes and ω is an angular frequency. After substitution into (9) we obtain

$$\left(\mathbf{K} - \omega^2\mathbf{M}\right)\mathbf{u} = \mathbf{0} \tag{10}$$

Given that the mass matrix is symmetric and positive definite then substitution $\lambda = \omega^2$ and multiplication of the previous equation by the inverse mass matrix on the left side results in

$$(\mathbf{A} - \lambda\mathbf{I})\mathbf{u} = \mathbf{0}, \quad \mathbf{A} = \mathbf{M}^{-1}\mathbf{K} \tag{11}$$

where **I** is a unit matrix. Given that the mass and stiffness matrices are symmetric then the matrix **A** is symmetric as well. For the nonzero vector u the determinant of the matrix $(\mathbf{A} - \lambda\mathbf{I})$ has to be zero. Because the determinant $\det(\mathbf{A} - \lambda\mathbf{I})$ is an n-degree characteristic polynomial of λ, the number of roots λ_n, called the eigenvalues, is equal to the degree of polynomial. The corresponding nonzero solution of the homogenous equation is called an eigenvector. It is proved in the linear algebra theory that all the eigenvalues of the symmetric matrix are real and an arbitrary pair of the eigenvectors, corresponding to different eigenvalues, is orthogonal. We form a spectral matrix $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_N)$ and an eigenvector matrix $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_N]$.

Because the beam is divided into 10 elements, it is possible to calculate 10 modal frequencies and 10 modal shapes of vibration. Only 5 out of the modal shapes, identifiable by the number of nodes, are shown in Fig. 2B. The modal frequencies are summarized in Tab. 1.

Tab. 1: Modal frequencies for N = 10.

Mode	1	2	3	4	5	6	7	8	9	10
Freq in Hz	26.6	167	470	922	1522	2256	3093	3965	4755	5309

2.3 Excited vibration

The excited vibration of the cantilever beam in the vertical position describes the equation of motion with the external forces p_1, p_2, …, p_N assembled into a vector **p** on the right side and acting at the gravity centers of the beam elements

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{p} \tag{12}$$

The presence of viscous damping, such as a dissipative force, extends the left side of the equation of motion by an additional term which is proportional to velocity

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{p}, \quad \mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}, \tag{13}$$

where the matrix of proportionality **C** for Rayleigh damping is a linear combination of the mass and stiffness matrices, and α, β are constants of proportionality. The relationship to the damping ratio ξ can be seen using the formula $\xi = \pi(\alpha/f_0 + \beta f)$, where f_0 is the frequency in hertz

3. Transfer Function

Vibration of mechanical structures is dampened very slightly. It's only a few percent of critical damping. The purpose of active vibration control is increase ability of structures to absorb vibration by adding an artificial electronic feedback. To analyze the effect of active vibration damping it is firstly assumed that the system is not damped at all.

It is assumed that **y** and **p** are complex harmonic functions of time ($\exp(\omega t)$) and **Y** and **P** are complex amplitudes, respectively. The transfer function in the form of a squared matrix **H**, relating the displacement $y_r, r = 1,\ldots,N$ of the lumped masses to the force $p_q, q = 1,\ldots,N$ acting at these masses, is defined by the following formula (Hi & Fu, 2001)

$$\mathbf{Y} = (\mathbf{K} - \lambda\mathbf{M})^{-1}\mathbf{P} = \mathbf{H}\,\mathbf{P} \tag{13}$$

A modal transform $\mathbf{y} = \mathbf{V}\,\mathbf{q}$ is the basis for the derivation of the transfer function. The coordinates y are transformed into generalized coordinates q by using the matrix **V**. The relationship of the transfer function to the modal properties of the structure can be defined if the modal transformation matrix V has the following property $\mathbf{V}^T\mathbf{M}\,\mathbf{V} = \mathbf{I}$. It can be proved that the orthonormal eigenvectors vn arranged in the matrix V are given by

$$\mathbf{v}_n = \mathbf{u}_n \Big/ \sqrt{\mathbf{u}_n^T\mathbf{M}\mathbf{u}_n}, \quad n = 1,\ldots,N \tag{14}$$

The transfer function matrix **H**, called the receptance, as a function of $\lambda = \omega^2$ depends on the eigenvectors and eigenvalues according to the formulas

$$\begin{aligned}&\mathbf{H} = (\mathbf{K} - \lambda\mathbf{M})^{-1} = \mathbf{V}\mathbf{D}\mathbf{V}^T,\\ &\mathbf{D} = \mathrm{diag}\left(1/(\lambda_1 - \lambda), 1/(\lambda_2 - \lambda),\ldots,1/(\lambda_N - \lambda)\right),\end{aligned} \tag{15}$$

where **D** is a diagonal matrix. The matrix **H** relates the force acting at the *q*-th lumped mass, to the displacement y_r of the *r*-th lumped mass, where is measured. The Laplace transform of the individual elements of the matrix **H** is as follows

$$H_{r,q}(s) = \sum_{n=1}^{N} \frac{v_{nr}v_{qn}}{\omega_n^2 + s^2}, \quad r,q = 1,2,\ldots,N, \tag{16}$$

where $v_{q,r}$ q, r =1,...,N is the *r*-th element of the *q*-th normalized eigenvector. The poles of the transfer function lie on the imaginary axis of the complex plane. The system is on the stability margin, not stable and simultaneously not unstable.

Tab. 2: Values of the product $v_{nN}v_{1n}$ for N=10.

n	1	2	3	4	5	6	7	8	9	10
$v_{nN}v_{1n}$	0.146	0.669	1.339	-1.775	1.866	1.628	1.176	0.670	-0.264	0.049

The product $v_{nN}v_{1n}$ of the elements of the matrix **V** is given in Tab. 2.

The assumption (16) about the stability margin of the cantilever beam as a dynamic system is the worst one, because the amplitude of vibration always decays after some time.

4. Active vibration control

The purpose of the system for the active vibration control (AVC) is to compensate the effect of a disturbing external force on the vibration of the beam. It is desirable to relocate the poles of the transfer function of the controlled system from the imaginary axis in the left half-plane of the complex plane. The cantilever beam is considered as an MIMO system composed of the lumped masses whose count is equal to N. Vibration of all these masses can be controlled by forces acting at all of them as it is shown in Fig. 3.

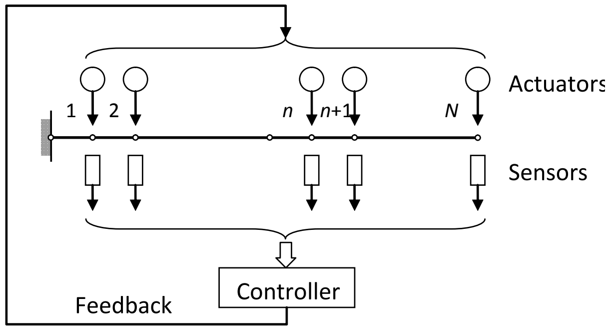

Fig. 3: Active vibration control.

We assume that the system is of the SISO type with a controller with a transfer function $R(s)$. There are two possible solutions, the collocated and non-collocated active vibration control. For the collocated system the correcting force p_n acts and the response Y_n is measured in the same centre gravity of the beam element. For the non-collocated system it is assumed that the correcting force p_q acts at the lumped mass indexed by q and the vibrations are sensed at the lumped mass indexed by r. An example of the non-collocated system is shown in Fig. 4. Vibration of the free end element of this cantilever beam is sensed at $r = N$ and the correcting force acts at the element just next to the clamped end, therefore $q = 1$. A block diagram of the closed loop system is shown in Fig. 5.

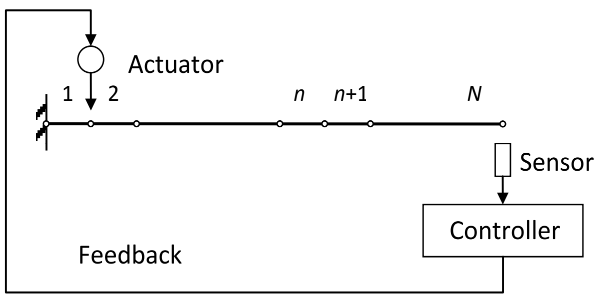

Fig. 4: Non-collocated system of active vibration control.

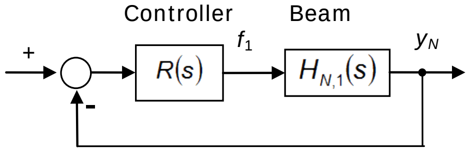

Fig. 5: Closed loop system of AVC

There is a transfer function of the closed-loop system $\tilde{H}_{r,SP}$, relating the displacement y_r of r-th lumped mass to the set point x_{SP}, and the function $H_{r,q}$, relating the displacement y_r to the feedback force p_q acting at q-th lumped mass

$$\tilde{H}_{r,SP}(s)=\frac{X_r(s)}{X_{SP}(s)}=\frac{R(s)H_{r,q}(s)}{1+R(s)H_{r,q}(s)}=\frac{R(s)\sum_{n=1}^{N}\frac{v_{nr}v_{qn}}{\omega_n^2+s^2}}{1+R(s)\sum_{n=1}^{N}\frac{v_{nr}v_{qn}}{\omega_n^2+s^2}}=\frac{R(s)\sum_{n=1}^{N}v_{nr}v_{qn}\prod_{\substack{k=1\\k\neq n}}^{N}\left(\omega_k^2+s^2\right)}{\prod_{n=1}^{N}\left(\omega_n^2+s^2\right)+R(s)\sum_{n=1}^{N}v_{nr}v_{qn}\prod_{\substack{k=1\\k\neq n}}^{N}\left(\omega_k^2+s^2\right)} \tag{17}$$

The term in the denominator of (2) is a polynomial of the squared variable *s*, the coefficients of the odd power are equal to zero. For a stable system, the variable *s* raised to odd powers must be added

$$R(s)\sum_{n=1}^{N}v_{nr}v_{qn}\prod_{\substack{k=1\\k\neq n}}^{N}\left(\omega_k^2+s^2\right)=\sum_{n=1}^{N}T_{D,n}s^{2n-1} \quad\Rightarrow\quad R(s)=\frac{\sum_{n=1}^{N}T_{D,n}s^{2n-1}}{\sum_{n=1}^{N}v_{nr}v_{qn}\prod_{\substack{k=1\\k\neq n}}^{N}\left(\omega_k^2+s^2\right)} \tag{18}$$

where $T_{D,n}$, n = 1,...,N are the mentioned positive coefficients. The degree of the polynomial in the numerator of the transfer function $R(s)$ is greater than the degree of the polynomial in the denominator. The feedback controller of the derivative type is on the stability margin by itself due to lack of terms with variable *s* raised to an odd power. The additional proportional part of the controller ensures stability (Tůma, 2012).

$$R(s)=F+Ts \tag{19}$$

where F is a gain and T is a time constant.

The disturbance force can be of the broad or narrow frequency spectrum. Suppose that the frequency spectrum of disturbance affects only the *d*-th mode of vibration.

$$H_{r,q}(s)=\frac{v_{dr}v_{qd}}{\omega_d^2+s^2}=\frac{K_d}{\omega_d^2+s^2} \tag{20}$$

The transfer function of the close loop with the controller described by the transfer function (19) is as follows

$$\tilde{H}_{r,SP}(s)=\frac{X_r(s)}{X_{SP}(s)}=\frac{(F+Ts)\frac{K_d}{\omega_d^2+s^2}}{1+(F+Ts)\frac{K_d}{\omega_d^2+s^2}}=\frac{K_d(F+T\,s)}{s^2+K_dT\,s+\left(\omega_d^2+K_dF\right)} \tag{21}$$

where a gain factor $K_d=v_{dr}v_{qd}$ is called a residuum. As it is shown in Tab. 2 some of the gain factors are negative. The damping ratio ξ and the decay constant σ of the system described by (21) is equal to

$$\xi=\frac{1}{2}\frac{K_dT}{\sqrt{\omega_d^2+K_dF}},\quad \sigma=\xi\sqrt{\omega_d^2+K_dF}=\frac{K_dT}{2} \tag{22}$$

The decay constant σ for $0<\xi<1$ and $\omega_d^2+K_dF>0$ determines an envelope of decaying vibration $\exp(-\sigma t)$. Strictly speaking if the decay constant is negative since $K_d<0$, then the vibration of the corresponding frequency does not decay and the system is unstable. Only natural damping of the cantilever beam can compensate this instability caused by active vibration control. Design of the controller parameters requires taking into account a number of external influences and mainly the natural material damping of the beam vibration.

The effect of active vibration control is often demonstrates on the vibration decay of the beam which is bended into a deflected position and suddenly released. In this case, only the lowest modes of vibration are excited.

Both the controller parameters can be calculated using the partial pole placement (Mottershead & Tehrani & Ram, 2009) as well. The pair of the complex conjugate poles $j\omega_k$, and $-j\omega_k$ of the transfer function $H_{r,q}(s)$ will be placed in the complex conjugate poles $\mu_k = -\sigma_k + j\Omega_k$, and $\mu_k = -\sigma_k - j\Omega_k$ of the closed loop transfer function $\tilde{H}_{r,SP}(s)$. For both the replaced poles the denominator of $\tilde{H}_{r,SP}(s)$ have to be zero. We obtain two equations with unknown parameters T and F

$$\begin{aligned} &1 + (F + T\mu_k) H_{r,q}(\mu_k) = 0, \\ &1 + (F + T\mu_k^*) H_{r,q}(\mu_k^*) = 0. \end{aligned} \tag{23}$$

After rearranging the equations (19) we get

$$\begin{aligned} &H_{r,q}(\mu_k) F + H_{r,q}(\mu_k)\, \mu_k T = -1, \\ &H_{r,q}(\mu_k^*) F + H_{r,q}(\mu_k^*)\, \mu_k^* T = -1. \end{aligned} \tag{24}$$

The solution of these two equations is as follows

$$\begin{aligned} F &= -\frac{H_{r,q}(\mu_k^*)\,\mu_k^* - H_{r,q}(\mu_k)\,\mu_k}{H_{r,q}(\mu_k) H_{r,q}(\mu_k^*)\,\mu_k^* - H_{r,q}(\mu_k^*) H_{r,q}(\mu_k)\,\mu_k} \\ T &= +\frac{H_{r,q}(\mu_k) - H_{r,q}(\mu_k^*)}{H_{r,q}(\mu_k) H_{r,q}(\mu_k^*)\,\mu_k^* - H_{r,q}(\mu_k^*) H_{r,q}(\mu_k)\,\mu_k} \end{aligned} \tag{25}$$

Using the controller of the proportional-derivative type only one pair of poles can be placed into the stable part of the complex plane.

5. Simulation of active vibration control

Equations of motion of a mechanical system with an electronic feedback can be written in the form

$$\left(\mathbf{M}s^2 + \mathbf{C}s + \mathbf{K}\right)\mathbf{y} = \mathbf{b}u + \mathbf{p}, \quad u = -(\mathbf{F} + \mathbf{T}s)^T \mathbf{y}, \tag{26}$$

where **b, F** and **T** are column vectors defined as follows

$$b_i = \begin{cases} 1, & i = q \\ 0, & i \neq q \end{cases}, \quad F_i = \begin{cases} F, & i = r \\ 0, & i \neq r \end{cases}, \quad T_i = \begin{cases} T, & i = r \\ 0, & i \neq r \end{cases}. \tag{27}$$

Then, by combining equations (25), we obtain the corrected damping and stiffness matrices

$$\left(\mathbf{M}s^2 + \left(\mathbf{C} + \mathbf{b}\mathbf{T}^T\right)s + \left(\mathbf{K} + \mathbf{b}\mathbf{F}^T\right)\right)\mathbf{y} = \mathbf{p}. \tag{28}$$

For the cantilever beam described above we assume now that N = 5. Vibration of the free end element of this cantilever beam is sensed at r = 5 and the correcting force acts at the element just next to the clamped end, therefore q = 1. The damping **C** and stiffness **K** matrices are influenced by the feedback in the following way

$$\mathbf{K} + \mathbf{b}\mathbf{F}^T = \mathbf{K} + \begin{bmatrix} 0 & \cdots & F \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \quad \mathbf{C} + \mathbf{b}\mathbf{T}^T = \mathbf{C} + \begin{bmatrix} 0 & \cdots & T \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \tag{29}$$

Both the matrices **C** and **K** become asymmetric.

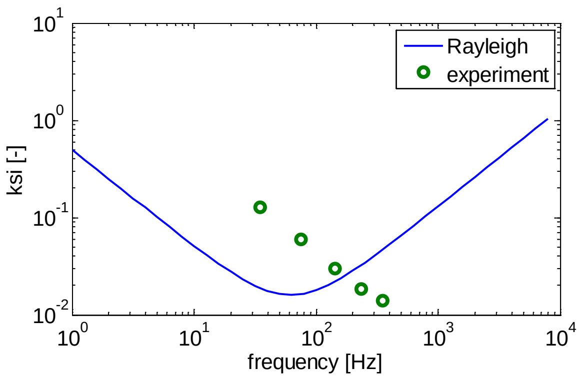

Fig. 6: The dependence of the damping ration ξ on frequency

It is assumed the viscous damping force (13), defined by the matrix $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$, where $\alpha = 0.159$, $\beta = 0.0000411$. The damping ratio “ksi” (ξ) for the first two modes of vibration is about 0.004 and the corresponding damping constant (σ) is about 0.6. The dependence of the damping ration ξ on frequency is shown in Fig. 6. Small circles indicate the damping ratio from the experimental modal analysis.

A force of 9.81 [N] is acting at the free end of the cantilever beam which is on the opposite side of the clamped end. The force is suddenly released at t = 0. A weak damping is added to avoid instability of the undamped response. The result of simulation of free vibration is shown in the left panel of Fig. 7 (AVC OFF).

The effect of the active vibration control in operation is shown in the right panel of Fig. 7 (AVC ON) as well. The gain F = 66.8 and the time constant T = 20.7 [s] were designed for the undamped cantilever beam. The damping constant of the first vibration mode was increased 10 times by using a simulated feedback. All the responses were calculated using Newmark’s method for integrating differential equations.

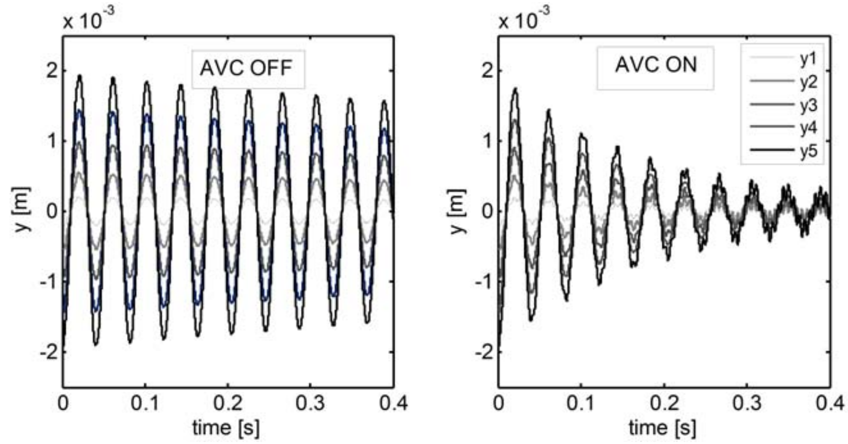

Fig. 7: Free vibration of the cantilever beam with AVC OFF and ON

6. Conclusions

The lumped-parameters model of the cantilever beam was designed using the method based on the modal analysis. It was proved that the cantilever beam can be actively damped only by a force which is controlled by the PD controller. The feedback of the D type is not sufficient for damping undamped systems. This paper is focused on the computation of the PD controller parameters using the pole placement method. The formulas for calculation the controller gain and time constant are derived.

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