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SINGULAR CASES OF PLANAR AND SPATIAL PARALLEL MANIPULATOR

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Abstract: *Singularities are places in the workspace of the robot where kinematic equations have no solution. It is of course desired to minimize number of such places in the workspace. The analysis of these places is then important from the construction point of view where singularities might be suppressed by suitable changes in design of construction. The paper deals with description of singular cases types of a parallel manipulator and their analysis via analysis of the determinants of the system Jacobians. This is demonstrated for two examples – planar and spatial parallel manipulator.*

Keywords: ***Parallel manipulator, singular cases, Jacobian matrices*.**

1. Introduction

Parallel mechanisms are generally based on closed – loop kinematic chain which leads to very high stiffness of such a mechanism and related properties as high positioning accuracy and repeability, possibility of mounting of the actuators to the base thus achieving of high accelerations. On the other hand such a construction has also its drawbacks. It is among others smaller workspace which is restricted by singularity areas. These are defined as places in the workspace where direct or inverse kinematics has no solution. The appearance of such a singularity may cause fast changes in accelerations and force effects which may lead to loosing of the controllability or destruction of the device or its parts.

The methods for the singularity detection are either numerical or graphical. The numerical methods are often based on the analysis of determinants of Jacobian matrices (Sefrioui & Gosselin, 1992 or Belda & Stejskal, 2003), i.e. on the analysis of the presence of the solution for the kinematics. There are also sometimes used geometry methods, i.e. based on the Grassmann geometry (Merlet, 1989). The proposed article presents a singularity analysis for a spatial and planar manipulator via the method analyzing determinants of Jacobian matrices.

2. Analysis method

The method is based on the analysis of determinants of Jacobian matrices as was already mentioned. The singularity is detected for the singular value of the determinant. The behavior of the manipulator is also rapidly getting worse for the determinants which are "close to zero" – how close to zero must the determinant be to observe the unacceptable behavior have to be tested individually for each manipulator. The Jacobians are typically obtained in the following way.

The relation between joint and cartesian coordinates may be expressed as

$$f\left(q,z\right)=0, \tag{1}$$

where $q=\left[q_1,q_2,...,q_n\right]^T$ is the vector of the joint coordinates and $z=\left[z_1,z_2,...,z_n\right]^T$ is the vector of the Cartesian coordinates.

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Jacobians of the systems are then defined as

$$\Phi_q = \begin{bmatrix} \dfrac{\partial f_1}{\partial q_1} & \cdots & \dfrac{\partial f_1}{\partial q_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_n}{\partial q_1} & \cdots & \dfrac{\partial f_n}{\partial q_n} \end{bmatrix}, \Phi_z = \begin{bmatrix} \dfrac{\partial f_1}{\partial z_1} & \cdots & \dfrac{\partial f_1}{\partial z_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_n}{\partial z_1} & \cdots & \dfrac{\partial f_n}{\partial z_n} \end{bmatrix} \tag{2}$$

There are then three possible situations (the determinants are consequently analyzed for the whole workspace:

a) $\det\Phi_q = 0$ and $\det\Phi_z \neq 0$ for the singularities of the first type,

b) $\det\Phi_q \neq 0$ and $\det\Phi_z = 0$ for the singularities of the second type,

c) $\det\Phi_q = 0$ and $\det\Phi_z = 0$ for the combined singularities.

3. Spatial mechanism analysis

The spatial mechanism for the analysis is consisting of six extendible links and a platform (Fig. 1). The effector has six degrees of freedom.

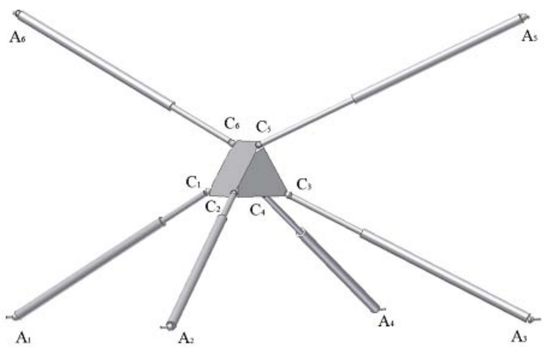

Fig. 1 Analysed spatial 6 DOF parallel manipulator

The relation between the joint and Cartesian coordinates is

$$\begin{aligned} f_i = &\left(n_x C_{ix} + o_x C_{iy} + w_x C_{iz} + x_{c1} - A_{ix}\right)^2 + \left(n_y C_{ix} + o_y C_{iy} + w_y C_{iz} + y_{c1} - A_{iy}\right)^2 + \\ &+\left(n_z C_{ix} + o_z C_{iy} + w_z C_{iz} + z_{c1} - A_{iz}\right)^2 - S_i^2 = 0 \end{aligned} \tag{4}$$

for $i = 1, ..., 6$, where

$$R = \begin{bmatrix} \cos\psi\cos\nu & \text{-}\cos\psi\sin\varphi & \sin\psi \\ \sin\varphi\sin\psi\cos\nu{+}\cos\varphi\sin\nu & \text{-}\sin\varphi\sin\psi\sin\nu{+}\cos\varphi\cos\nu & \text{-}\sin\varphi\cos\psi \\ \text{-}\cos\varphi\sin\psi\cos\nu{+}\sin\varphi\sin\nu & \cos\varphi\sin\psi\sin\nu{+}\sin\varphi\cos\nu & \cos\varphi\cos\psi \end{bmatrix} = \begin{bmatrix} n_x & o_x & w_x \\ n_y & o_y & w_y \\ n_z & o_z & w_z \end{bmatrix}$$

and $C_{ix,y,z}$ are (cartesian) coordinates of the platform points, $A_{ix,y,z}$ are coordinates of the base points, x_{c1}, y_{c1}, z_{c1} are describing translation of the platform point C_1, ψ, ν, φ are Euler angles and S_i are link lengths (joint coordinates).

The equation (4) may be used for the derivation of the system Jacobians according to (2). Consequently are analyzed singularities (Fig. 2, 3).

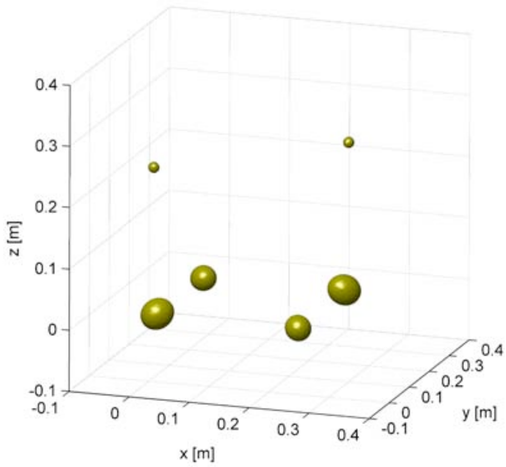

Fig. 2 Singular areas of the first type for $\psi, \nu, \varphi = 0°$

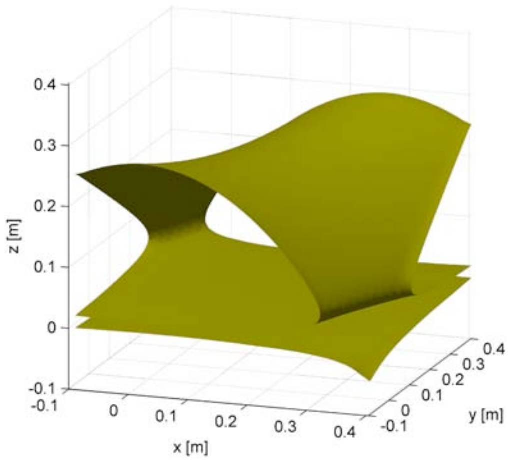

Fig. 3 Singular areas of the second type for $\psi, \nu, \varphi = 0°$

The following examples are mapping the workspace of the manipulator for the different orientations of the platform (Fig. 4- 7).

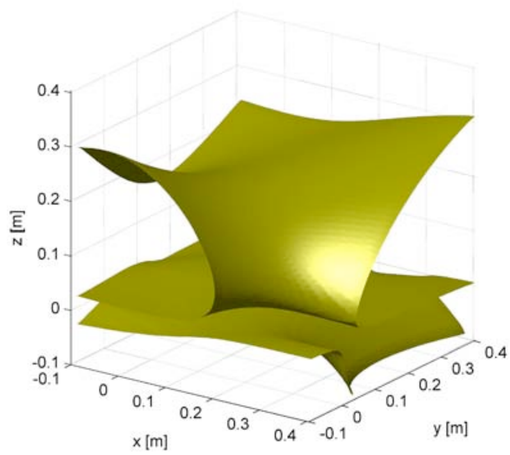

Fig. 4 $\psi = 5°, \nu = 0°, \varphi = 0°$

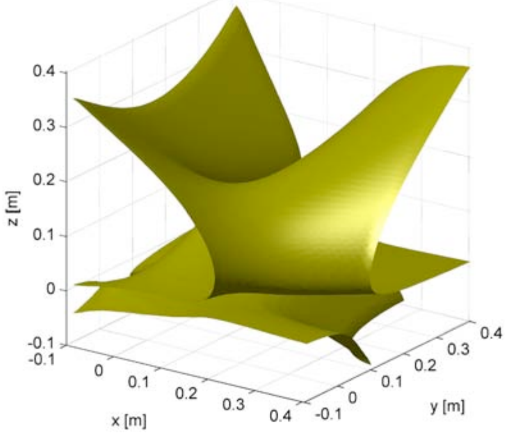

Fig. 5 $\psi = 5°, \nu = 5°, \varphi = 0°$

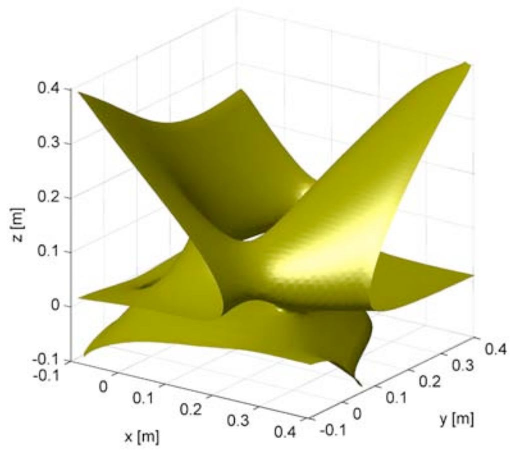

Fig. 6 $\psi = 5°, \nu = 5°, \varphi = 5°$

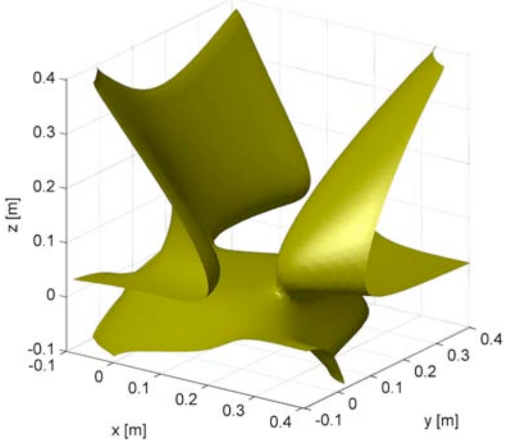

Fig. 7 $\psi = 10°, \nu = 5°, \varphi = 5°$

4. Planar manipulator

The planar manipulator is classical 3RPR 3 DOF parallel mechanism according to Fig. 8. It consists of the three extendable links and triangular effector.

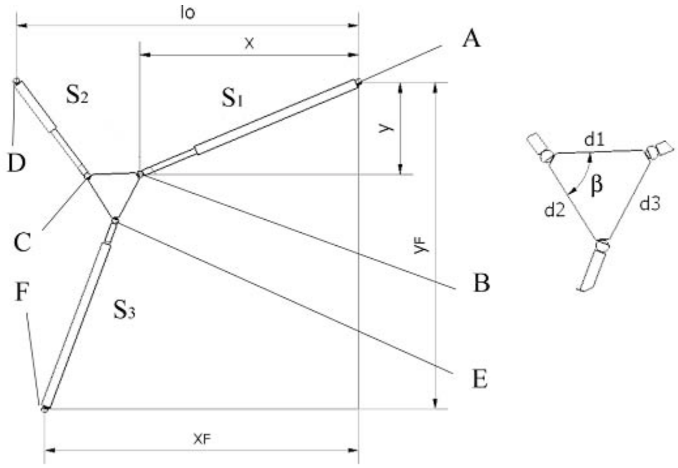

Fig. 8 3RPR parallel manipulator

The singularities (Fig. 9, 10) are analyzed in the same manner as for the spatial example.

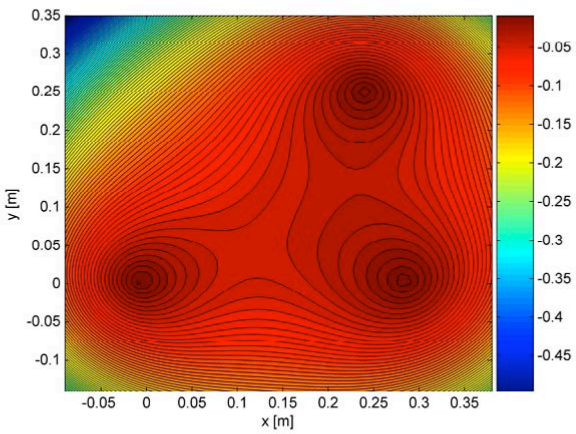

Fig. 9 Singularities of the first type $\varphi = 0°$

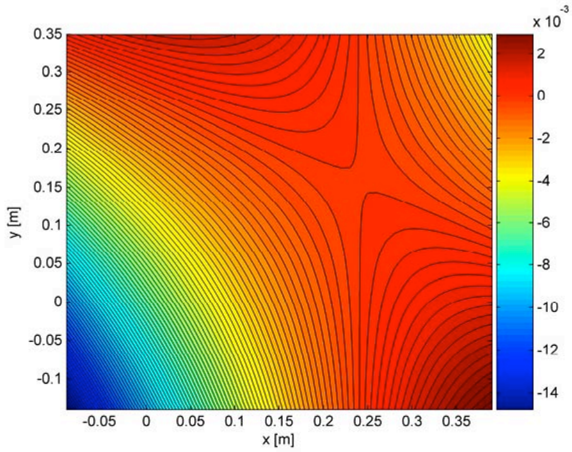

Fig. 10 Singularities of the second type $\varphi = 0°$

5. Conclusions

The article describes possible areas of singularities in a parallel manipulator workspace. There were analyzed singularities for the spatial and planar manipulator via the method analyzing determinants of the system Jacobians. The study is useful especially for the trajectory planning purposes where the avoiding of singularities is important because of the mechanism damage prevention.

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References

Belda, K. & Stejskal, V. (2003) Singular cases of the planar parallel robot. *Proceedings, Engineering mechanics*.

Merlet, J. P. (1989) Singular configurations of parallel manipulators and Grassmann geometry. *The international journal of robotics research*, 8, 5, pp. 1099 – 1113.

Sefrioui, J. & Gosselin, C. M. (1992) Singularity analysis and representation of planar parallel manipulators. *Robotics and autonomous systems*, 10 pp. 209 – 224.