DETERMINATION OF BURST PRESSURE OF THIN-WALLED PRESSURE VESSELS

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Abstract: *A simple fracture-mechanics based method is described for assessing a part-through crack in the wall of a pipe subjected to internal pressure of liquid and/or gas. The method utilizes simple approximate expressions for determining the fracture parameters K, J, and employs these parameters to determine critical dimensions of a crack on the basis of equality between the J-integral and the J-based fracture toughness of the pipe steel. The crack tip constraint is taken into account by the so-called plastic constraint factor C, by which the uniaxial yield stress in the J-integral equation is multiplied. The results of the prediction of fracture conditions are verified by burst tests on test pipes.*

Keywords: *Fracture mechanics, J integral, pressure vessels, burst tests*

1. Introduction

In thin-walled gas pipelines, similarly as in other (especially welded) structures, we should expect defects to occur. Under certain conditions, the defects can grow and they will gradually shorten the residual life of gas pipelines. Using fracture mechanics we can assess the threat that such defects can pose to the pipeline wall taking into account whether a brittle, quasi-brittle or ductile material is involved. A model description of crack-containing systems, based on the stress intensity factor (SIF), K, can be used for brittle and quasi-brittle fracture, and in addition for subcritical fatigue growth, corrosion fatigue and stress corrosion [1], [2]. In these cases, the surface crack is usually located in the field of one of the membrane tensile stress components or in the field of bending stress, or in a combination of these two stresses. In comparison with the dimensions of the crack and the cross section of the pipeline the extent of the plastic zone at the crack tip is small. If the gas pipeline is made of a high toughness material, the plastic strains become extensive before the crack reaches instability. Hence, some elasto-plastic fracture mechanics methods, such as J-integral, crack opening displacement, the two-criterion method or some other procedure, should be employed to assess the fracture condition of the pipeline [3], [4]. Because the method of determination of burst pressure of thin-walled pressure vessels utilizes simple approximate expressions for determining the fracture parameters K, J, a brief background of some fracture-mechanics formulae will be made first.

2. A brief background of some fracture-mechanics formulae

SIF for an axial through crack
The stress intensity factor can be determined by equation (1)

$$K_I = M_T \sigma_\varphi \sqrt{\pi c} \tag{1}$$

Where $\sigma_\varphi = pD/2t$ is the hoop stress, and

M_T is the Folias correction factor, taking account of curvature of a pipe

One of the most widely used expressions to determine the Folias factor is the following [5]:

$$M_T = \sqrt{1 + 1.255 \frac{c^2}{Rt} - 0.0135 \frac{c^4}{R^2 t^2}} \tag{2}$$

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where R is the mean radius of the pipe, and
               t is the pipe wall thickness

SIF for an axial part-through crack

Various methods are used for analyzing the problem of axial semi-elliptical surface cracks in the wall of a cylindrical shell (Fig. 1).
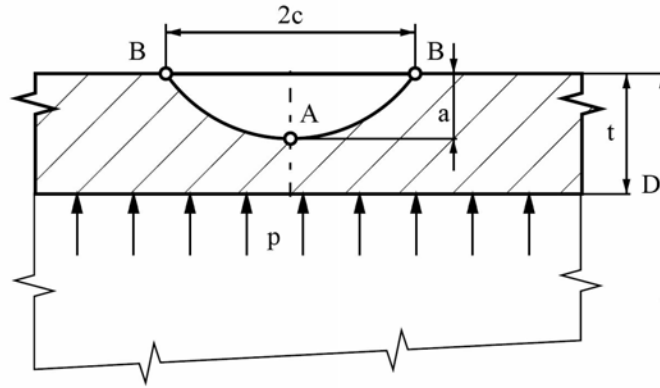

Fig. 1 An external longitudinal semi-elliptical crack in the wall of a cylindrical shell

A very good estimate of the stress intensity factor for such a crack is given by expression (3).

$$K_I = \left[M_F + \left(E_{(k)}\sqrt{c/a} - M_F \right)\left(\frac{a}{t}\right)^p \right] \frac{\sigma_\varphi \sqrt{\pi a}}{E_{(k)}} M_{TM} \qquad (3)$$

This is an adjusted form of the Newman solution [6] for a thin-walled shell. Here

M_F - a function depending on the crack geometry (on the ratio a/c),

$$E_k = \int_0^{\pi/2} \sqrt{1 - \frac{c^2 - a^2}{c^2}\sin^2\theta}\, d\theta \quad \text{- an elliptical integral of the second kind}$$

p - the function depending on the crack geometry (on the ratio a/c) and on the relative crack depth
        (on the ratio   a/t)

$$M_{TM} = \frac{\left(1 - \dfrac{a/t}{M_T}\right)}{1 - a/t} \quad \text{- the correction factor for curvature of a cylindrical shell and for an increase in}$$

        stress owing to radial strains in the vicinity of the crack tip

Functions M_F and p differ in form for the lowest point of the crack tip (point A in Fig. 1) and for the crack mouth on the surface of the cylindrical shell (point B in Fig. 1).

The next fracture-mechanics parameter we need for determination of the burst pressure of a pipeline is J integral. The actual magnitude of this quantity will be compared to its critical value – the fracture toughness. From this comparison a magnitude of the burst pressure will result.

3. Engineering methods for determination of J integral

3.1 FC method

This method was proposed in Addendum A16 of the French nuclear code [7] as the J_s method. It stems from the second option of describing the transition state between ideally elastic and fully plastic

behaviour of material, that is to say from the function $f_2(L_r)$ of the R6 method [8]. This function takes the form:

$$f_2(L_r) = \left(\frac{E\varepsilon_{ref}}{L_r R_e} + \frac{L_r{}^3 R_e}{2E\varepsilon_{ref}} \right)^{-1/2} \tag{4}$$

where

$L_r = \sigma/\sigma_L$ (σ – applied stress, σ_L – stress at the limit load)
R_e is the yield stress
E is Young´s modulus
ε_{ref} is the reference strain corresponding to the reference (nominal) stress σ_{ref}

If we identify function $f_2(L_r)$ with function $f_3(L_r) = \left(\frac{J}{J_e} \right)^{-\frac{1}{2}}$ and express L_r as σ_{ref} / R_e and elastic J integral J_e as $K^2/ E´$, where E´ =E for plane stress state and E´ =E / (1−v²) for plane strain state, we have:

$$J = \frac{K^2}{E'} \left(\frac{E.\varepsilon_{ref}}{\sigma_{ref}} + \frac{\sigma_{ref}{}^3}{2E.R_e{}^2.\varepsilon_{ref}} \right) \tag{5}$$

The stress σ_{ref} in the above equation is a nominal stress – i.e. a stress acting in the plane where the crack occurs. Taking into consideration the description of the stress-strain dependence by the Ramberg-Osgood relation (6) and adjusting Eq. (5) we obtain the *J*-integral in the form (7).

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0} \right)^n \tag{6}$$

$$J = \frac{K^2}{E'} \left[A + \frac{0.5(\sigma/\sigma_0)^2}{A} \right] \tag{7}$$

where

$$A = 1 + \alpha \left(\frac{\sigma}{\sigma_0} \right)^{n-1} \tag{8}$$

As the pipeline is a body of finite dimensions, stress σ in Eqs. (7) and (8) is a nominal stress – i.e. a stress acting in the plane where the crack occurs. Referring to the R6 method [8] this stress for the pipe containing longitudinal part-through thickness crack may be written as:

$$\sigma = \frac{\sigma_\phi}{1 - \dfrac{\pi a c}{2t(t+2c)}} \tag{9}$$

In eq. (9) $\quad \sigma_\phi = \dfrac{pD}{2t}\quad$ is the hoop stress and the meaning of the symbols *a, c,* and *t* is clear from Fig.2.

3.2 GS method

The GS method was derived by Gajdoš and Srnec [9] on the basis of the limit transition of *J*-integral, formally expressed for a semi-circular notch, to a crack, with the variation of the strain energy density along the notch circumference being approximated by the third power of the cosine function of the polar angle. If the stress-strain dependence is further expressed by the Ramberg-Osgood relation (6) with $\varepsilon_0 = \sigma_0/E$, (α, n – material constants) it can be arrived at Eq. (10)

$$J = \frac{K^2}{E'} \left[1 + \frac{2\alpha n}{(n+1)} \left(\frac{\sigma}{\sigma_0} \right)^{n-1} \right] \tag{10}$$

where σ is the nominal stress in the reduced cross-section of a body. For a pipe containing longitudinal part-through thickness crack it may be determined by the relation (9).

4. Accounting the constraint

The crack tip constraint is accounted here by a simple procedure based on the so-called plastic constraint factor on yielding, C. This factor is given by the ratio of the stress needed to obtain plastic macrostrains under constraint conditions to the yield stress at a homogeneous uniaxial state of stress [10]. The C factor can be expressed by the relation (11)

$$C = \frac{\sigma_1}{\sigma_{HMH}}$$
(11)

where $\sigma_{HMH,}$ the Huber-Mises-Hencky stress, is put equal to the yield stress.

Let us now consider the state of stress at the crack tip in a thick-walled body, where the stress perpendicular to the crack plane, σ_1, and the stress in the direction of the crack, σ_2, are equal, and the stress in the direction of the thickness of the body, σ_3, is governed by the expression $\sigma_3 = \nu(\sigma_1 + \sigma_2)$. Then, based on the HMH criterion and assumed elastic conditions ($\nu \cong 0.33$), the plastic constraint factor $C \approx 3$. If the stress in the thickness direction, σ_3, falls within $2\nu\sigma_1$ and zero (thin-walled body), the value of the plastic constraint factor will range between $C = 1$ and $C = 3$. These data can be used to assess fracture conditions in gas pipelines with surface part-through cracks, employing a C-factor which has to be experimentally determined. After determination of the C factor the value of $C\sigma_0$ would be used instead of the yield stress σ_0 in relations for the calculation of *J*-integral.

The C factor was experimentally investigated at the Institute of Theoretical and Applied Mechanics of the Academy of Science of the Czech Republic in the framework of a broader project concerned with research of the reliability and operational safety of high pressure gas pipelines. Fracture conditions were investigated on five pipe bodies, made of steels X52, X65 a X70, with cycling-induced cracks. Data on the pipe bodies used, cracks in the walls, and mechanical and fracture-mechanical material properties of the bodies are given in Table 1.

Tab. 1. Summary of data concerning the assessment of the fracture behaviour of pipe bodies

Material	X 52	X 65	X 65	X 70	X 70
D (mm)	820	820	820	1018	1018
t (mm)	10.2	10.7	10.6	11.7	11.7
c (mm)	50	100	100	127	115
a (mm)	7.0	7.7	7.0	6.7	7.1
a/t	0.686	0.720	0.660	0.573	0.607
a/c	0.14	0.077	0.07	0.053	0.062
p (MPa)	8.05	9.71	9.86	9.86	9.55
$p/p_{0.2}$	1.034	0.750	0.769	0.800	0.775
σ_0 (MPa)	313	496	496	536	536
α	2.40	5.34	5.34	5.92	5.92
n	6.25	8.45	8.45	9.62	9.62
C	2.1	2.4	2.4	2.0	2.07
J_{cr} (N/mm)	487	432	432	439	439
$- T/\sigma_0$	0.672	0.575	0.544	0.606	0.611
$- Q$	0.667	0.591	0.546	0.648	0.651

The individual rows in the table show the following data (top to bottom): body diameter D, body wall thickness t, half-length of longitudinal part-through crack c, crack depth a, relative crack depth a/t, aspect ratio a/c of a semi-elliptical crack, fracture pressure p, ratio of fracture pressure p and pressure $p_{0.2}$ corresponding to the hoop stress at the yield stress, yield stress in the circumferential direction of the body σ_0, Ramberg-Osgood constant α, Ramberg-Osgood exponent n, plastic constraint factor C, *J*-integral critical value J_{cr}, determined as J_m (corresponding to attaining the maximum force at the "force – force point displacement" curve), T-stress to yield stress ratio T/σ_0, and Q parameter. Values of σ_0, α and n were derived from tensile tests and the values of J_{cr} from fracture tests run on

CT specimens. Values of the fracture pressure *p* were read at the moment the ligament under the crack in the pipe body ruptured. Values of the plastic constraint factor on yielding, *C*, were determined on the basis of *J*-integral in such a way that agreement was reached between the predicted and experimentally established fracture parameters for the given crack and fracture toughness of the material. The *J*-integral value was calculated from the GS method [9] on the one hand and the French nuclear code [7] on the other hand.

It should be noted that in determining the C factor the critical value of J-integral established on CT specimens was considered– namely J_{cr} = 439 N/mm for steel X70, J_{cr} = 432 N/mm for steel X65 and J_{cr} = 487 N/mm for steel X52. It was found out by a computational analysis of CT specimens, employed to construct the R curve, that the Q parameter for these specimens had been Q = 0.267. A comparison of this with the Q parameter for pipe bodies (Q ≈ -0.55 ÷ -0.65) reveals that the constraint in the CT specimens was much higher. This implies that the real fracture toughness – i.e. the critical value of J-integral, J_{cr} – was higher in the pipe bodies. The real C factor for a cracked pipe body is lower, so that the *J-a* curve for a pipe body is steeper than that for CT specimens with a greater C factor [11]. Due to this, the J integral for the axial part-through crack reaches the corresponding higher fracture toughness (for a lower constraint) for the same crack depth as the J integral with a higher C factor reaches lower fracture toughness (determined on CT specimens). The situation is illustrated in Fig.2.
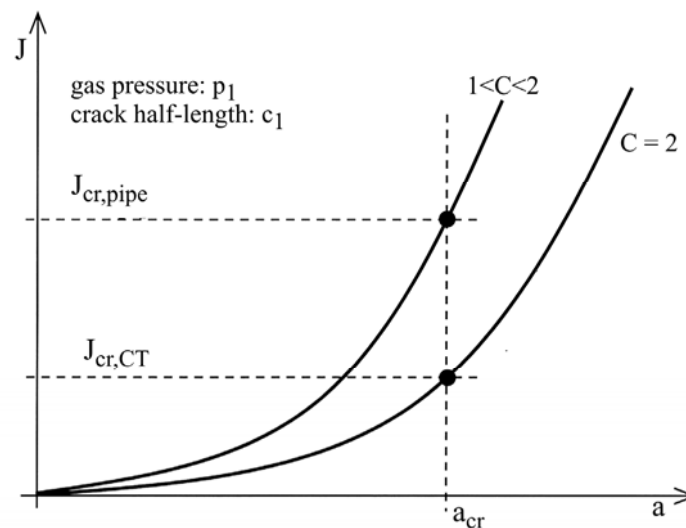

Fig. 2 Schematic J-a dependence for (i) a CT specimen, and (ii) a pipe with an axial part-through crack

Normalized T–stress values in Table 1 were obtained by the use of the plane solution – i.e. solution for an infinite length of a crack oriented longitudinally along the pipe. The problem was solved in the Institute of Physics of Materials, Brno, by the finite elements method. The solution included two steps: (i) a corresponding FEM network was established and corresponding boundary conditions were formulated for each crack depth, (ii) magnitudes of the stress intensity factor and the T-stress were calculated for each FEM network by means of the FEM system CRACK2D with hybrid crack elements. Values of the Q parameter were derived from $Q – T/\sigma_0$ curves, obtained by O´Dowd and Shih [12] by modified boundary layer analysis for different values of the strain coefficient (Ramberg-Osgood exponent, *n*). Strictly speaking, the *Q* parameter values from Table 1 do not correspond accurately to values for the examined cracks, because T-stresses were not computed for real semi-elliptical cracks, but for cracks spreading along the entire length of pipe body (*a/c ≈ 0*). Nevertheless, with regard to the fact that the ratio of the depth to the surface half-length of examined cracks (*a/c*) was close to zero (*a/c=0.053÷0.14*), we can assume that the differences between real values of the *Q* parameter and the values listed in Table 1 will be small. Relations between the C and the Q parameter will be discussed later.

5. Burst tests

For burst tests of pipe bodies it is usually sufficient that the length of the pipe between the welds of dished bottoms is at least 3.5 D. Such a length permits placing a number of starting cuts axially along the length of the body. The cuts are made to initiate crack growth when the body is subsequently pressurized by a fluctuating pressure of water. They can be made in several ways, of which one uses a thin grinding wheel. The smallest real functional thickness of such a wheel is about 1.2 mm and the corresponding width of the cuts made with it is approximately 1.5 mm. Depending on the type of pipes of which gas pipelines are built (seamless, spirally welded, longitudinally welded) the starting cuts can be provided in the base material, transition region or the weld metal, their orientation being axial, circumferential or along the spiral weld. The depth of an initiated fatigue crack must be at least 0.5 mm along the whole perimeter of the cut tip so that the cut with the initiated crack at its tip can be considered as a crack after the pipe body has been subjected to cycling. This value follows from work of Smith and Miller [13].

As it was seen in the preceding section, the results of burst tests are presented in Table 1. The cracks were made in such a way that the test pipes were first provided with working slits and the check slits. The latter slits were of the same surface length as the working slits but their depth was greater. These check slits functioned as a safety measure to prevent cracks that developed at the working slits from penetrating through the pipe wall. For illustration, a DN1000 test pipe body of the working length 3.5 m is shown in Fig. 3.
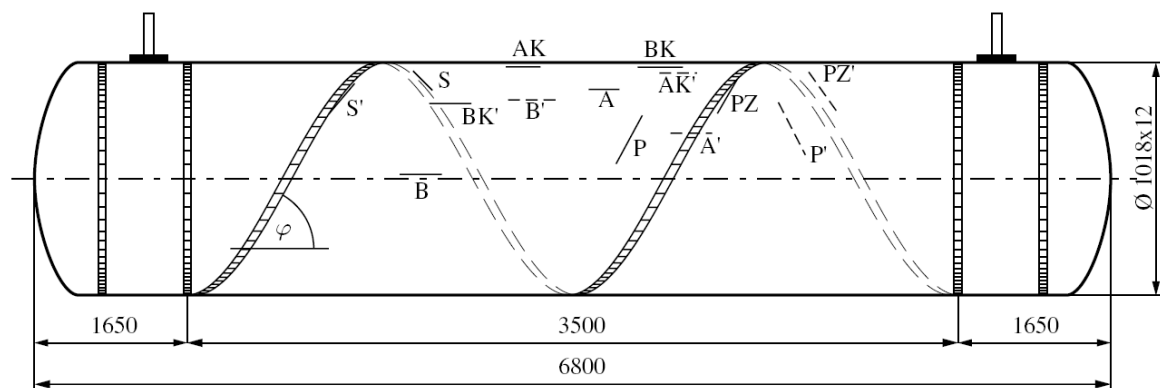

Fig.3 Test pipe body DN1000 with marking the starting cuts

The check slits are denoted in Fig. 3 by a supplementary letter K. The material of the test pipe body is a thermo-mechanically treated steel X70 according to API specification. The pipe is spirally welded, the weld being inclined at an angle of $\varphi = 62°$ to the pipe axis. It is provided with starting cuts oriented either axially or in the direction of the strip axis (i.e. in the direction of the spiral) and then along or inside the spiral weld. The cuts differ in length ($2c = 115$ mm or 230mm) and depth ($a = 5$, 6.5, 7, and 7.5 mm). We are particularly interested in axial (longitudinal) slits situated aside welds because these are sites where axial cracks will be formed in the basic material of the pipe.

Efforts were made in the fracture tests to keep the circumferential fracture stress below the yield stress, because the operating stress in gas pipelines is virtually around one half of the yield stress (and does not exceed two thirds of the yield stress even in intrastate high-pressure gas transmission pipelines). Calculations reveal that in order to comply with this, the depth of the axial semi-elliptical cracks should be greater than one half of the wall thickness. Oblique cracks should be even deeper, as the normal stress component opening these cracks is smaller. If the crack depth is to have a certain magnitude before the fracture test is begun, the depth of the starting slit should be smaller than this magnitude by the fatigue extension of the crack along the perimeter of the slit tip. At the same time, we should bear in mind that the higher the fatigue extension of the crack, the better the agreement with a real crack.

5.1 Procedure of the tests

After the starting slits were made, the test pipes were subjected to water pressure cycling to produce fatigue cracks in the tips of the starting slits. The cycling was carried out in a pressurizing system, which included a high-pressure water pump, a collecting tank, a regulator designed to control the amount of water that was supplied and, consequently, the rate at which the pressure is increased in the pipe section. This was effected by opening by-pass valves. A scheme of the pressurizing system is shown in Fig. 4.
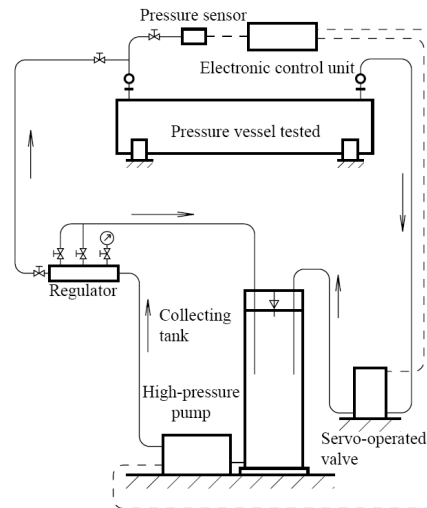

Fig.4 Pressurizing system used for cyclic pressure tests of a test pipe body

In cycling the cracks, the water pressure fluctuated between $p_{min} = 1.5$ MPa and $p_{max} = 5.3$ MPa, and the number of pressure cycles was between 3 000 and 4 000. The period of a cycle was approximately 150 seconds. The cycling went on until a crack, initiated in one of the check slits, became a through crack. This moment was easy to detect, because it was accompanied by a water leak. By choosing an appropriate difference between the depths of the working slits and the check slits it was possible to obtain a working crack depth (= starting slit depth + fatigue crack extension) approximately of the required size. To run a test for a fracture, however, it was necessary to remove the check slit which had penetrated through the wall of the test pipe from the body shell and to repair the shell, e.g. by welding a patch in it.

After removing the check slit with a crack which penetrated through the wall, and repairing the shell of the test pipe, the pipe was loaded by increasing water pressure to burst. The test procedure, which was common to all test pipes, will now be briefly described for the DN1000 pipe shown in Fig. 3. As the figure suggests, slits A, A´, B and B´ were oriented along the axis of the pipe. The nominal length of notches B, B´ had twice the length of notches A, A´, but they were shallower. As mentioned above, cracks at the slit tips were extended by fluctuating water pressure, and this proceeded until the cracks from the check slits (BK, BK´) grew through the wall and a water leak developed. Then the damaged parts of the shell were cut out, patches were welded in their place, and the test pipe was monotonically loaded to fracture at the location of crack B or B´. The burst of the test pipe at crack B is shown in Figs. 5 and 6 (in detail).
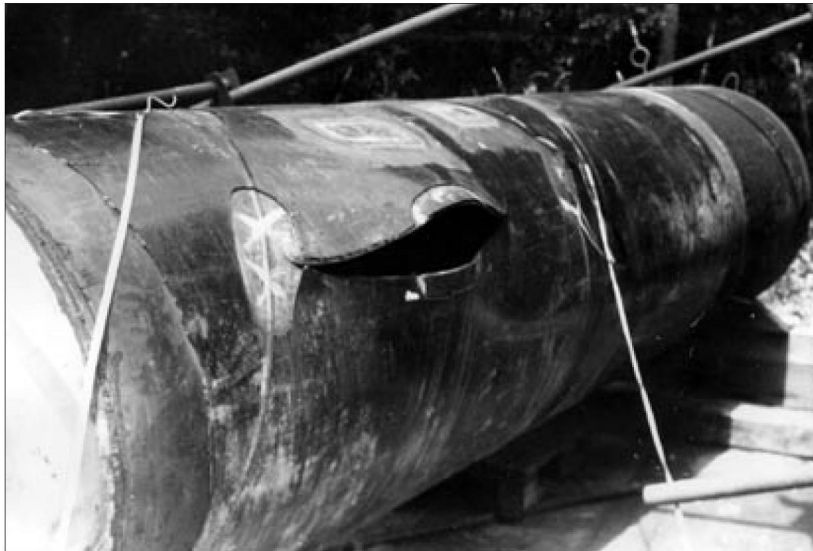
Fig.5 Burst initiated on the slit B with a fatigue crack

Fig.6 Burst initiated on the slit B - detail

Evidently, at the instant of fracture the crack spread not only through the remaining ligament, but also lengthwise. It means the LBB criterion was not observed. After removing the part of the pipe shell with crack B, a patch was welded in and the second burst test followed. In the Table 2 there are extracted from Table 1 the numerical values of the geometrical parameters, the J-integral fracture values, the Ramberg-Osgood constants, the fracture pressure and the fracture depth for cracks B and B´, respectively.

5.2 Prediction of fracture parameters

Fracture parameters for a cracked pipe, i.e. the crack size for a given pressure and/or the pressure for a given crack size, can be predicted by solving Eq. 12 (GS method) or Eq. 13 (FC method) for the unknown fracture parameter (e.g. crack depth *a* and/or pressure *p*).

$$\frac{K^2}{E'}\left[1+\frac{2\alpha n}{(n+1)}\left(\frac{\sigma}{C\sigma_0}\right)^{n-1}\right]=J_{cr} \tag{12}$$

$$\frac{K^2}{E'}\left[A+\frac{0.5\left(\sigma/C\sigma_0\right)^2}{A}\right]=J_{cr} \tag{13}$$

In the above equations *K* is given by Eq. 3, σ is given by Eq. 9 and A is $A=1+\alpha\left(\frac{\sigma}{C\sigma_0}\right)^{n-1}$.

J_{cr} in Eq. 12 and 13 is the critical magnitude of the J integral – the fracture toughness – taken here as the value J_m and $α$, n, $σ_0$ are the Ramberg-Osgood parameters of the steel in the hoop direction.

As it can be deduced from Table 1 the crack B fractured first, namely when the water pressure reached the value p = 9.55 MPa. The damaged part of the pipe shell was then cut out, the fracture surfaces were released to enable their fractographic examination. The missing part of the shell was replaced by welding a patch on it. In the second burst test the crack B´ fractured at the pressure p = 9.86 MPa. After cutting out the damaged part of the pipe shell the fracture surfaces were released and were then subjected to fractographic examination. The results are digestedly presented in Table 2.

Tab 2. Some characteristics referring to crack B and crack B´

Characteristics	Crack B	Crack B´
CRACK DIMENSIONS		
half-length, c (mm)	115	127
depth in fracture, a_f (mm)	7.1	6.7
RAMBERG-OSGOOD PARAMETERS		
$α$ / n /$σ_0$ (MPa)	5.92 / 9.62 /536	5.92 / 9.62 /536
FRACTURE TOUGHNESS		
$J_{cr} = J_m$ (N/mm)	439	439
FRACTURE PRESSURE		
p_f (MPa)	9.55	9.86

It is seen from here that the crack depth at fracture was 7.1 mm for crack B and 6.7 mm for crack B´. These values are also shown in the last two columns of Table 1.

Now let us predict the fracture conditions according to engineering approaches, and compare the prediction results with real fracture parameter values (pressure, crack depth). As it was already stated the procedure for verifying the predictive engineering methods involves determining either the fracture stress for a given (fracture) crack depth or the fracture crack depth for a given (fracture) pressure. To illustrate this, we select the latter case – i.e. determining the fracture depth of a crack for a given (fracture) pressure. We will not use directly Eqs. 12 and 13 but general dependences of the J integral (given by the left-hand side of Eqs. 12 and 13) on the crack depth a. Fig. 7 shows the J-integral vs. crack B depth dependences, as determined by the FC and GS predictions for the fracture hoop stress corresponding to the measured fracture pressure. When using appropriate equations to determine J integrals, the following parameters were used: D = 1018 mm; t = 11.7 mm ; $p = p_f$ = 9.55 MPa; c = 115 mm; $α$ = 5.92; n = 9.62; $σ_0$ = 2.07×536 = 1110 MPa (i.e. C = 2.07).

Similarly, Fig. 8 shows J - a dependences for crack B´.
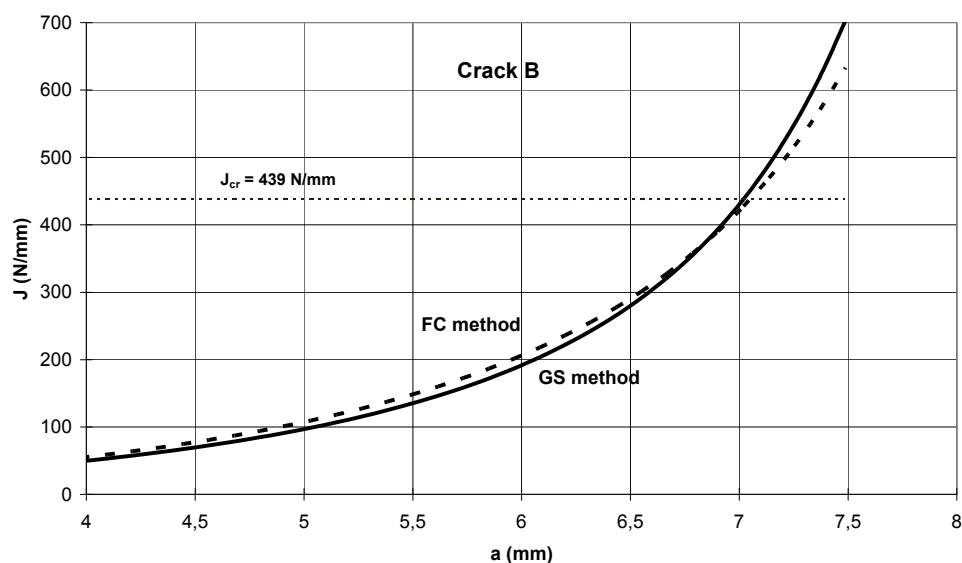

Fig.7 Prediction of the fracture depth for the crack B ($p = p_f$ = 9.55 MPa and C = 2.07)
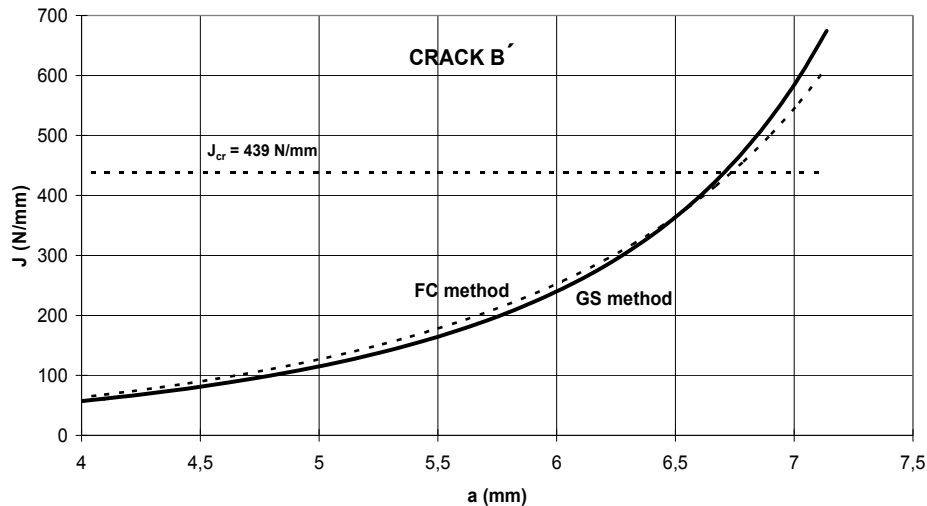
Fig.8 Prediction of the fracture depth for the crack B´ (p = p$_f$ = 9.86 MPa and C = 2.0)

6. Conclusions

On the basis of both experimental work and a fracture-mechanical evaluation of experimental results, an engineering method has been worked out for assessing the geometrical parameters of critical axial crack-like defects in a high-pressure gas pipeline wall for a given internal pressure of a gas.

The method makes use of simple approximate expressions for determining fracture parameters K, J, and it accommodates the crack tip constraint effects by means of the so-called plastic constraint factor on yielding, C. Involving this idea in the fracture analysis leads to multiplication of the uniaxial yield stress by the C factor in the expression for determining the J-integral.

Two independent approximate equations for determining the J-integral provided very close assessments of the critical geometrical dimensions of part-through axial cracks.

With the use of the crack assessment method proposed, the critical gas pressure in a pipeline can also be determined for a given crack geometry.

Acknowledgment

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