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LIFETIME PREDICTION OF WIND LOADED MAST AND TOWERS WITH RESPECT TO LATERAL AND LONGITUDINAL WIND SPECTRUM

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Abstract: *The paper deals with a theoretical lifetime prediction of telecommunication towers, guyed masts and antenna's cantilevers and comparison with the long-term measurements. A simple and practical calculation method is presented. The wind load is described taking into account the probability distribution function of the mean velocity and corresponding wind pressures. The dynamic response of structure caused by the turbulence uses wind models for both longitudinal and lateral direction. Structural response takes into account the contribution of more vibration modes. Based upon this knowledge, the number of cycles for certain time period together with the residual time life prediction of antennas cantilevers and cables was determined.*

Keywords: ***Lifetime prediction, Masts, Towers, Fatigue, Wind, Spectrum.***

1. Introduction

Slender structures like masts and towers are repeatedly exposed to dynamic wind loading. The consequent vibration may cause the cumulating damage, which sometimes leads to the collapse of structure. To prevent the breakdown by timely replacement or reconstruction, or to predict the expenses for further tower operation, one should determine the remaining lifetime. A method for the determination of remaining life of guyed masts, antennas and guy ropes is presented in this article. It is based on the theoretical work described in Pospíšil et al. (1997) and it deals with the vibration caused not only by the longitudinal but also by the lateral wind turbulence monitored during a long-term measurement, the latter being in some cases of specific structural characteristics more important, see e.g. Repetto & Solari (2002). The age and regular inspection revealed that these components could be in danger with regards to the remaining life. Based on the long-term monitoring stress cycles by strain gauges the remaining life of the elements was determined. For practical as well as analytical reasons, it was compared with theoretical calculation and prediction using just the knowledge of dynamic characteristics of the structure and the wind.

2. Damage and fatigue model

There exists number of methods to determine the fatigue damage of a specimen, see for example Degrieck & Paepegem (2001), Benasciutti & Tovo (2005), Epaaratchchci & Clausen (2005), Wahl et al. (2001). For the sake of practical engineering analysis, the linear theory of damage accumulation is used, given in the mathematical form by the so-called Palmgren-Miner law:

$$D = \sum \frac{n_i}{N_{i,f}} \tag{1}$$

where n_i is the number of cycles with certain amplitude and $N_{i,f}$ is the total number of cycles to the damage, i.e. cycles corresponding to the *i*-th level of the load. Linear theory is based upon the assumption of the constant energy accumulated by one cycle and the characteristic amount of the energy in time of damage. The energy accumulation leads therefore to the summation of partial

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damage. For the total collapse state holds $D = 1$. The relation between the damage and the number of cycles is not dependent on the level of the stresses. Some studies show, that in the case of cyclic loading with alternation of low and high amplitude cycles, the value $D = 1$ is very overrated and lead to the erroneous conclusion. Several modifications of the Palmgren-Miner criterion with the usage of non-linear equation for D involving the *i*-th level of the stresses can be found in Degrieck & Paepegem (2001). Before the analysis of the damage is performed according to the equation (1), there is a need to identify the amplitudes of individual cycles. The authors have chosen the rain-flow method for the separation of cycles into the stress ranges, see e.g. Amzallag et al. (1994).

3. Theoretical determination of number of cycles

There are several causes of the dynamic response of slender structures and consequently the sources of cyclic stresses. In the observed structures four sources were identified: a) the first one gives the number of load cycles caused by the fluctuation component of the wind acting in the direction of mean wind speed; b) the second part gives the number of cycles caused by lateral turbulent wind loads in the direction perpendicular; c) number of cycles caused by the direction changes in mean wind speed; d) number of cycles of the mast (or of the guy rope) caused by the shed vortices (Strouhal's effect).

The case c) needs some comments. For the determination of the stress range spectra, caused by changes of the mean wind speed and the wind direction changes, following simplifications are used.

It is assumed that the change of the mean velocity or direction occurs every 10 minutes, i.e. the period during which is the mean component of the wind velocity is considered as a constant. The stress range (hereafter labeled $\Delta_{\sigma, fl+mean}$, which occurs after this time is equal to the difference between the maximum value of the stress increase in the corresponding ten-minute interval and the minimum value of the stress increase in the previous interval.

$$\Delta_{\sigma,fl+\mathrm{mean}} = \sigma_{\mathrm{max}} - \sigma_{\mathrm{min}} \tag{2}$$

The maximum and minimum values of the stress increase $\sigma_{\max}$ and $\sigma_{\min}$ are defined as:

$$\sigma_{\mathrm{max}} = \sigma_{\mathrm{m}} + 3,7 \cdot \sigma_{Fl}; \ \sigma_{\mathrm{min}} = \sigma_{\mathrm{m}} - 3,7 \cdot \sigma_{Fl} \tag{3}$$

where σ_m denotes the stress caused by the mean wind load and σ_{Fl} the standard deviation of the fluctuation component response in the corresponding ten-minute interval. Another simplification is introduced taking into account that the minima of the stress increases $\sigma_{\min}$ are approximately the same low values (near zero) for all mean wind velocities.

Number of cycles of the stress range by the stress range $\lambda_{\sigma, fl+mean}$ is defined as:

$$n_i = P_1 \cdot P_3 \cdot d_{10,\mathrm{min}} \tag{4}$$

where $d_{10,\min}$ is the number of the ten-minute intervals in the total lifetime of the structure.

3.1. Probability P1

We call P_1 the probability of occurrence of the mean wind speed, i.e. that the mean wind speed will lie in certain interval. This probability is based upon the knowledge of distributive function. We use Weibull distribution with parameters $k = 2$ and $c = 0.35 \cdot v_{\max}$ applicable for most locations in the Czech Republic. It may be written as:

$$f(\mu) = \frac{k}{c} \cdot \left(\frac{\mu}{c}\right)^{k-1} \exp\left[\left(-\frac{\mu}{c}\right)^{k}\right] \tag{5}$$

where μ is the ten-minutes wind speed according to ČSN EN Eurocode 1 (2007). Dividing the whole band into several intervals and integrating the curve in these intervals, we obtain the probability that the mean wind speed will lie in that interval.

3.2. Probability P2

The probability of the fluctuating stress component at certain mean wind speed is P_2. We use the assumption of Gaussian distribution around the mean stress. According to the standard ČSN EN Eurocode 3 (2008) the stress probability density function (PDF) at certain cross-section of the structure may be expressed as follows:

$$f(\tau) = \frac{1}{\sigma_\tau} \exp\left(\frac{-(\tau - \overline{\tau})^2}{2\sigma_\tau^2}\right) \tag{6}$$

where $\overline{\tau}$ is the mean stress value in the structural element and σ_τ is a stress standard deviation. The mean stress is calculated using mean wind speed from each interval. For the reference wind speed in order to calculate stresses we selected the upper limit of each interval. The stress standard deviation can be stated for example according to the standard ENV as $\sigma_\tau = \tau \cdot G/4$, where G is the gust factor, see ČSN P ENV 1993-3-1 (2000), Simiu & Scanlan (1996) . Let us assume that the stress response is composed from the harmonic components with the amplitudes $\tau_{ampl,fl,i}$. For each of these amplitudes we calculate the probability density function P_{2i} and consequently also the probability P_2 as the sum of all mean wind speed contributions; see Fig. 1.

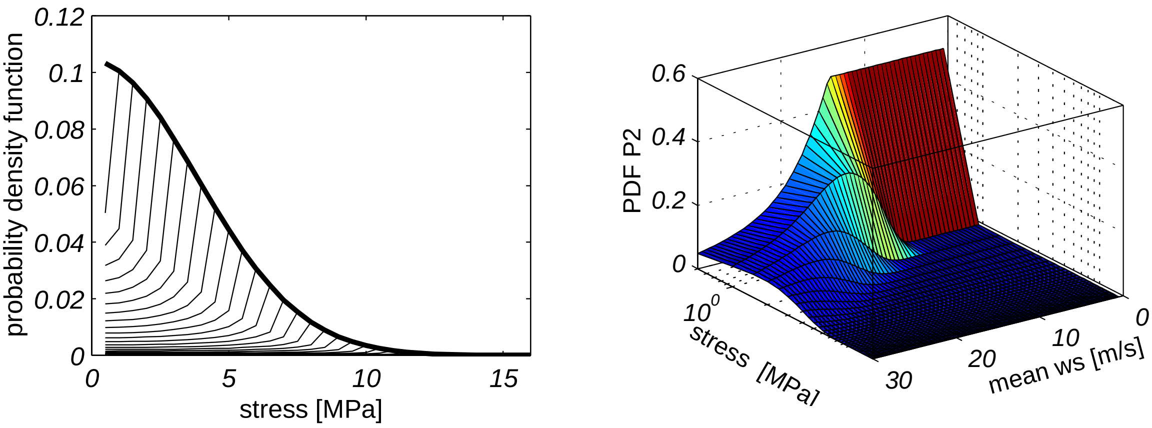

Fig. 1: Left: Probability density function of mechanical stress P_{2i} and partial probability density functions associated with the harmonic components. Right: Probability P_2 as the function of mean wind speed.

3.3 Probability P3

Alternatively we should consider also the occurrence frequency of the mean wind speed blowing in one direction. This we may call P_3. It is given by the fraction of the area bellow histogram (usually taken from the meteorological observations; wind rosette) and the area below the whole curve.

3.4 Probability P4

The probability P_4 expresses the fact that the structure vibrates in certain shape with certain frequency, close to one of the eigen-frequencies. It is the ratio between root mean square acquired by the integration of the response spectrum close to certain eigen-frequency respectively the background component of the response and the total integral of the response power spectrum over the whole frequency range.

3.5 Total number of the cycles due to wind fluctuation

The number of cycles with the amplitude of fluctuating component of the stresses $\tau_{ampl,fl,i}$ can be finally formed according to the practical formula, expressing the cumulative probability of mutually independent phenomena. We may write:

$$n_i = P_1 \cdot P_2 \cdot P_3 \cdot d_t \cdot \sum_{k=1}^{N} P_{4,f_k} \cdot f_k \tag{7}$$

where f_k is the *k*-th resonant frequency and d_t is the desired (projected) lifetime of the structure.

4. Damage assessment of existing structures-examples

In the period 2005-2011, the authors collected several measurements on several broadcast towers. The suitability of the procedure is documented on the analysis of towers and guyed masts. Two examples are presented here: The tower Veselský kopec (Example-1a), Vraní Vrch (Example-1b) and guyed mast Domamil with two sets of ropes (Example-2).

4.1. Data measurement

4.1.1. Wind characteristics

The ultrasonic anemometers in several elevations have measured the wind speed up to 60 m/s and the direction of the flow. The measured spectral density corresponded to EN spectra; see ČSN EN Eurocode 1 (2007). The characteristic profile is prevalently of category II as can be seen from Fig. 2.

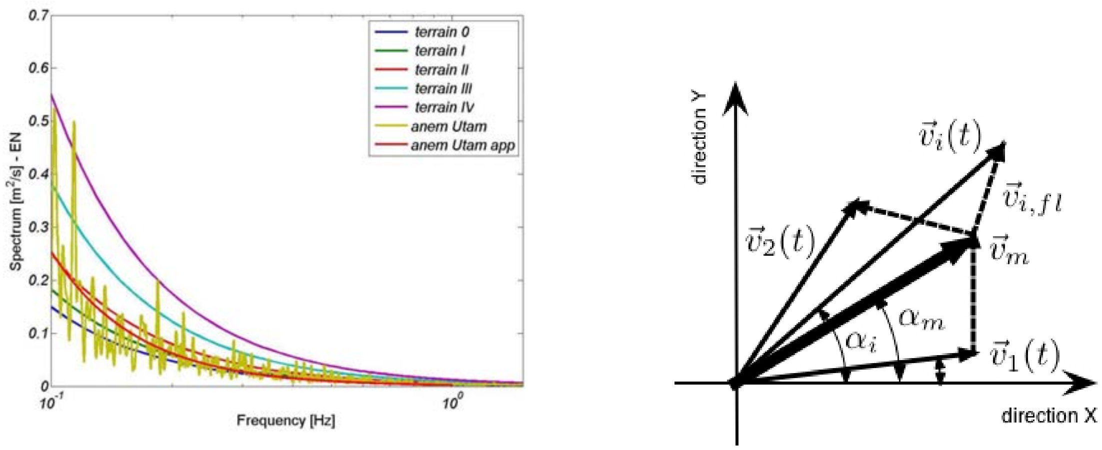

Fig. 2: Left: In order to analyze if the assumption of the load is based on the real values, the measured spectral densities were compared to theoretical ones given by the Eurocode. Right: Decomposition of the wind speed into its lateral and longitudinal components.

Decomposition of the velocity into the longitudinal and the lateral components was carried out as shown in Fig. 2. Wind speed is considered as a complex values and distribution is based on trigonometric relationship:

$$v_{\mathrm{fl,long}} = |\vec{v}| \cdot \cos(\alpha_{\mathrm{fl}}); \; v_{\mathrm{fl,lat}} = |\vec{v}| \cdot \sin(\alpha_{\mathrm{fl}}) \tag{8}$$

4.1.2. Response measurement

Dynamic measurements consisted in determining of resonant frequency of the tower based on the evaluation of the ambient vibration of the structure. The dynamic measurement was taken from the acceleration time histories (in case of mast measurement) along the mast shaft or tower or from the strain gauges (in case of the rope measurement). Frequencies were evaluated directly from the assessed spectra by simple peak picking method and the shapes of vibration were identified. Also the logarithmic damping δ was evaluated from the measurements.

Strain measurements were carried out using strain gauges mounted on the tubular antenna or directly on the guy-rope tensioning rods, see Fig. 5. These strain gauges were connected to a bus and the numbers of cycles and the stress range was calculated using the Rainflow method. In the case of rope measurement, the stress range obtained from long-term measurements for calculation of damage accumulation were multiplied by the factor $\gamma = 1.10$ in order to include the influence of bending moments at the tailpiece of the rope and aeroelastic effects, see Fischer (1983). The measured number of cycles is extrapolated to the projected lifespan, usually 45 or 50 years.

4.2. Example 1-prediction for the towers (fiber reinforced cantilevers)

The fiber-glass cantilevers were about 20 m tall. They were made with the diameter of 1.9 m and with the wall thickness $12 \div 16$ mm. The determination of the fatigue can be based on literature references. This however may lead to uncertainties with regards to the lack of knowledge of the material parameters and past fabrication conditions. In this study we have used the properties of the fiber-glass elements taken from the experiments made at older antenna extensions, see Pirner & Fischer (1999). Their characteristics were closest to the characteristics of the laminates under examination and they were made in the same time period and with similar fabrication procedure describing the limit stress values for certain number of cycles.

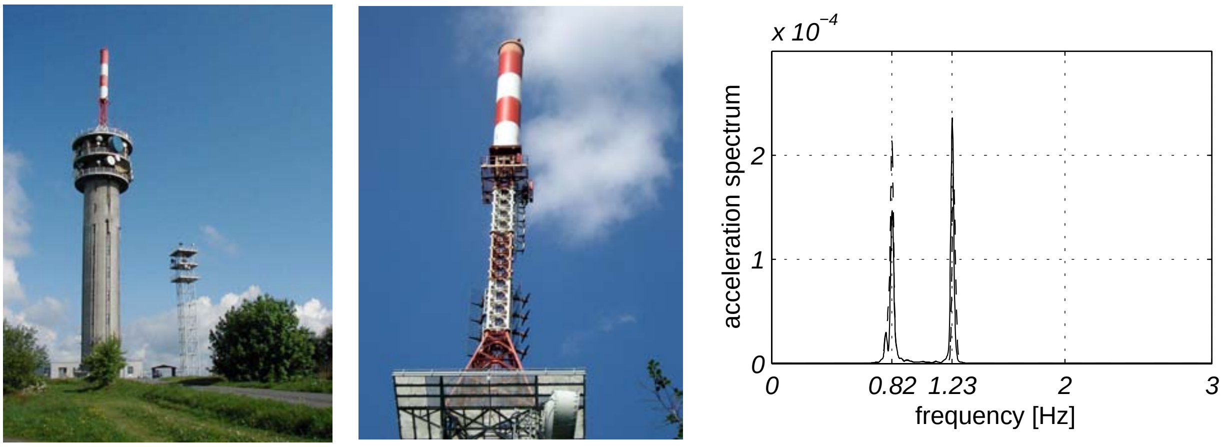

Fig. 3: Left: Photograph of the TV tower Veselský kopec (Example 1a), Middle: Vraní Vrch (Example 1b) with GRP antenna cantilevers. Right: Example of the spectral densities of the acceleration of a tower Veselský kopec to determine the resonance frequencies.

The forced vibration using two men swinging at the top was used to excite the towers. In case of strong wind, also the ambient wind excitation was analyzed and the resonant frequencies were determined. Also the wind speed, see Fig. 2, together with the spectrum have been measured. The spectral peaks of the response indicate usually the values of eigen-frequencies. It should be analyzed with care, however, because the influence of some mode shapes could be incorrectly concluded from the acceleration spectra of the whole tower and not just of the antenna cantilever. Fig. 3 (left) shows such an example; the structure as a whole vibrates predominantly with the first frequency 0.82 Hz; however, the dominant antenna's frequency was the second one with higher frequency equal to 1.23 Hz.

In case we know the measured values, we may set the number of cycles in every stress range according to Eq. 7. For the illustration, the probability, that the cantilever (depicted in Fig. 3. left) is vibrating in the first eigen-form is $P_{4,fl} = 0.06$, in the second $P_{4,f2} = 0.93$ and in the third and the fourth shape we may expect probability $P_{4,f3} = 0.00002$ and $P_{4,f4} = 0.00005$ respectively. Finally we may assess if it complies with the Palmgren-Miner criterion and/or determine the residual life of the structure.

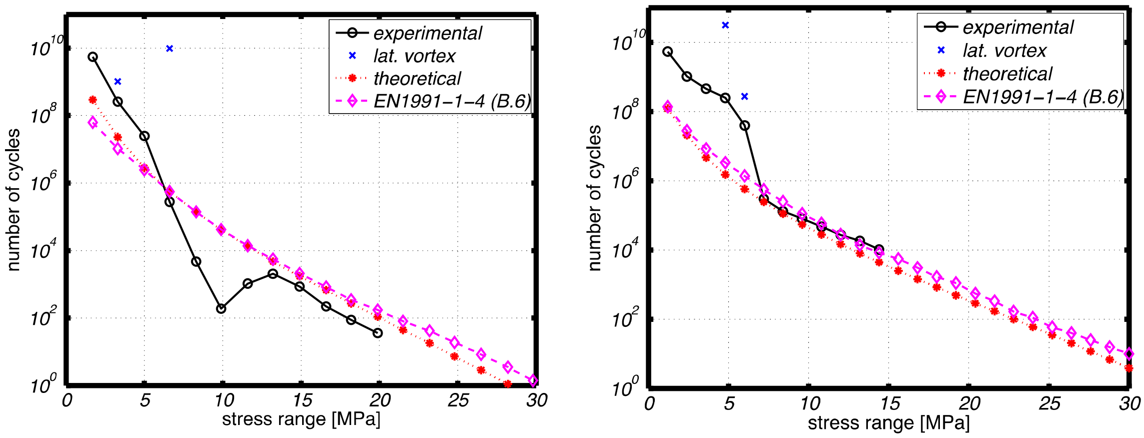

Fig. 4: Stress spectra ascertained from the strain-gauges measurement (circles) and number of cycles determined theoretically (red asterisks, magenta diamonds). Two blue crosses stand for the lateral vibration. Left: Example 1a (Veselský kopec), Right: Example b1 (Vraní vrch).

The suitability of the proposed method is demonstrated in Fig. 4, where the comparison between measured and calculated number of cycles is presented for the observed time interval. It is obvious, that there exists good agreement between proposed method and the measured values.

4.3. Example 2-prediction for the guy ropes

4.3.1. Response in the lateral direction

The calculation of the number of the cycles and the range of the stress is performed in the same manner as for the longitudinal direction of the wind except for the determination of the standard deviations of the fluctuation component, of the response and probability P_3.

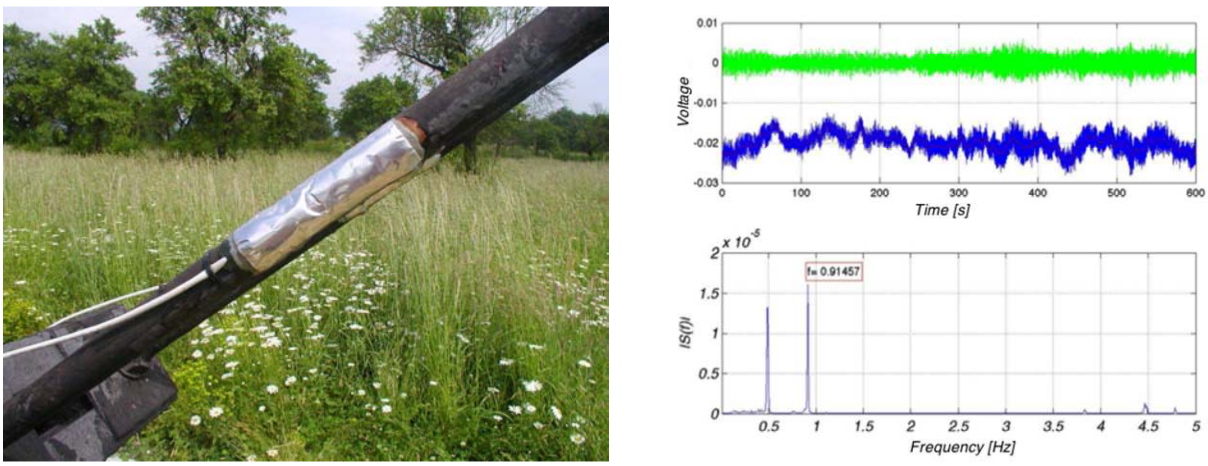

Fig. 5: Left-Strain-gauges glued at the guy rope. Right-stress time history and the spectral density of the response of lower rope.

Standard deviations of the resonant responses were determined approximately as $\sqrt{1/3}\cdot\sqrt{0.5}=0.82$ times the standard deviations of the resonant response in the direction of the wind medium, see Madugula et al. (2002). The term $\sqrt{(1/3)}$ expresses the ratio of the approximate size of the fluctuation in the resonant frequency and in the perpendicular direction. It is considered that the size of aerodynamic damping for perpendicular direction is about 50 % compared to the aerodynamic damping for the direction of mean wind speed.

The probability that the wind will blow in the perpendicular direction is P_{3p} . It is derived from the probability P_3 by following simple approach considering the direction perpendicular to the most frequent direction of the wind; with a possible deviation 45°. The probability value P_{3p} is equal to $1-P_3$.

4.3.2 Resonant components, net stress spectra

The probability P_4 expresses the influence of the background and the resonant responses respectively. It is given as the ratio of standard deviation of resonant responses and the background response respectively and the net dynamic response. This ratio is expressed in accordance with the definition of λ_3 (see Madugula et al. (2002)) as $1/\lambda_R$ where λ_R coefficient is considered to have a conservative value of 1.2 in our case).

The frequency of the background response as well as the resonant frequencies for the guys in the presented example can be seen on the Table 1, together with the individual probabilities P_4.

Tab. 1: The background and resonant frequencies of the lower cables 1 (left) and upper cables 2 (right) together with the individual values of the probability P_4.

	Lower cables		**Upper cables**	
	Freq. [Hz]	Probability P_4	Freq. [Hz]	Probability P_4
backgr. resp.	0,03	0,83	0,03	0,82
1. shape	0,43	0,03	0,29	0,02
2. shape	0,49	0,02	0,43	0,06
3. shape	0,76	0,05	0,49	0,07
4. shape	1,50	0,05	0,91	0,02
5. shape	1,80	0,02	1,10	0,01

Calculation of the tension in the guy rope was carried out for 3 different wind velocities. Based on the measurement of the wind spectra, the terrain category II was chosen and the static calculation used the patch load method according to ČSN EN Eurocode 1 (2007) was carried out. The coefficient of the maximum value (peak factor) was determined by the more accurate calculation according Sparling et al. (1993). Partial factor for fatigue loading was considered to have the value of $\gamma_{Ff} = 1.00$ as recommended in ČSN EN Eurocode 3 (2008), article 9.5.

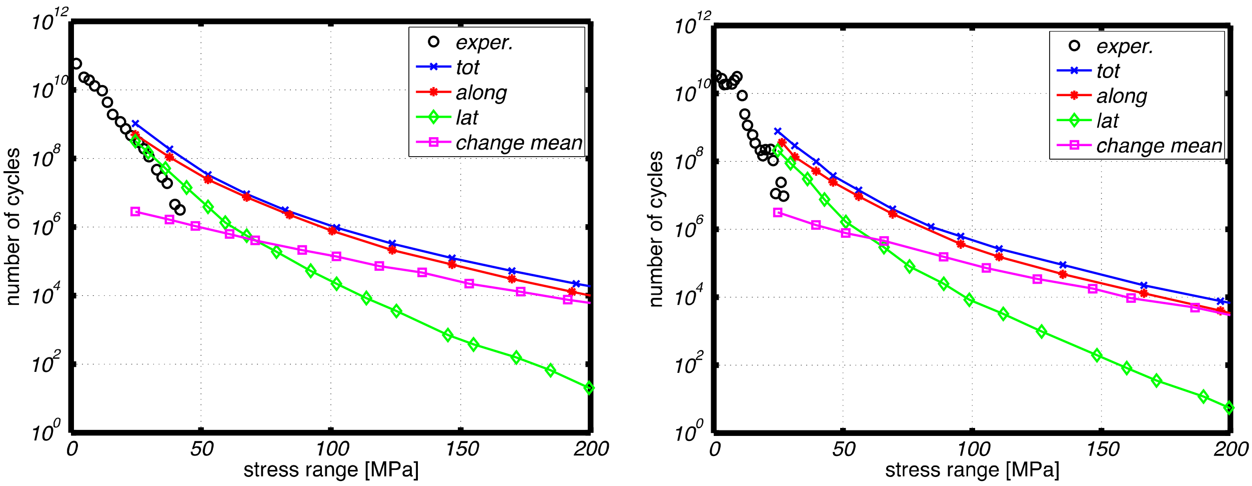

Fig. 6: Stress spectra ascertained from the strain-gauges measurement (blue circles) and total number of cycles determined theoretically with the use of known structure characteristics (green crosses). Red circles stand for stress spectra due to change of mean wind speed. Squares are the stress spectrum due to fluctuating lateral wind. Diamonds are due the along wind fluctuations. Left: Lower cables, Right: Upper cables.

5. Conclusions

The paper presented the analysis focused on the prediction of theoretical lifetime of antenna cantilever and mast guys and the comparison with long/short time measurements on the real structures. Examples of slender towers and masts subjected to the turbulent wind were presented. The wind load has been described by relatively simple formulas and the number of cycles during certain period was determined. The load description agreed very well with the measurements on the structure. Though there are differences in some of the stress ranges especially higher one which cannot be measured in the relatively short interval, there exists general agreement between theoretical predictions and onsite measurements, when correct assumption of the wind and the structure are employed. The work offers practical method for the clients to better estimate the service life of towers and their parts and to better organizes the maintenance work.

From the comparison obtained by theoretical calculation (for the measured wind speed) it is evident that the theoretical number of cycles is higher than measured ones, especially in the high stress range, which are significant from the lifetime point of view. This deviation may be caused by some conservative assumptions especially for lower wind speeds, which includes all kinds of excitation.

There are some open problems, behind the scope and limitation of this paper, like the correlation of the load with respect to the eigen-modes, superposition of lateral and longitudinal direction. However, in case there is attention paid to the short-term measurements of the dynamic characteristics of a structure, the method can give good results with regards to the lifetime prediction. This is useful when the structure is hardly accessible for the long-term monitoring.

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