

## ASSESSMENT OF MODEL UNCERTAINTIES IN THE ANALYSIS OF REINFORCED CONCRETE STRUCTURES

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**Abstract:** *Numerical methods of structural analysis enable consideration of material and geometrical non-linearity of reinforced concrete structures. While the effect of variability of materials and geometry can be relatively well described, the model uncertainty is not yet well understood. The present contribution is, therefore, focused on resistance model uncertainties in the analysis of reinforced concrete structures. Available definitions of the model uncertainties are critically reviewed. Statistical characteristics of the model uncertainties are obtained from previous studies. Simple engineering formulas (beam models, section-oriented approaches) as well as complex numerical solutions are considered. To facilitate practical applications the partial factors for the model uncertainties related to various types of the analysis are derived using the design value method.*

**Keywords:** *Model uncertainties, reinforced concrete structures, partial factors.*

### 1. Introduction

Recent development in structural design of concrete structures reflects advances in the fields of material engineering, reliability theory, structural mechanics and numerical methods of structural analysis. These advances provide exact tools for the reliability assessment of structural resistance in engineering practice.

Safety formats for the analysis of reinforced concrete structures were investigated in previous studies by Bertagnoli et al. (2004), Červenka (2008), Schlune et al. (2011) and Sýkora & Holický (2011). Some of these studies were used as background materials for the new Model Code of fib (2010). Different approaches for resistance modelling and safety were compared and suggestions for design applications proposed. These studies covered resistance models based on both the finite element technique as well as section-oriented engineering models. Safety formats were based on probabilistic analyses and estimates of resistance variability.

It has been indicated that structural resistances can be predicted by appropriate modelling of material properties, geometry variables and uncertainties associated with an applied model. The effect of variability of materials and geometry is relatively well understood and has been extensively addressed by the aforementioned studies. However, better description of model uncertainties seems to be desired as concluded by Vrouwenvelder (2010).

The submitted study is, therefore, aimed at the model uncertainties with a particular focus on the analysis of reinforced concrete structures. Statistical characteristics of the model uncertainties are summarized from data available in scientific literature considering simple engineering formulas (beam models, section-oriented approaches) as well as complex numerical solutions based on FE methods. To facilitate practical applications based on the partial factor method, the partial factors for model uncertainties related to various types of the analysis are derived using the design value method.

The study is an initial step of the research project of the authors' team in which the model uncertainties related to resistance models of reinforced concrete structures are to be systematically

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investigated. Outcomes of the project are also foreseen to provide background materials for the Joint Committee on Structural Safety (JCSS) that is currently updating the models for uncertainties provided in the Probabilistic Model Code, JCSS (2006).

## 2. Definitions of the model uncertainties

According to JCSS (2006) the model uncertainty is generally a random variable accounting for effects neglected in the models and simplifications in the mathematical relations. The model uncertainties can be related to:

- Resistance models (based on structural mechanics, constitutive laws),
- Models for action effects (assessment of load effects and their combinations).

This study is fully focused on the uncertainties related to resistance models of reinforced concrete structures. It is assumed that the uncertainty of actions can be treated separately.

The uncertainty of a resistance model should cover the following aspects (if relevant):

- Simplifications of known physical principles in an applied model,
- Approximations inherent to numerical methods,
- Influence of different interpretations of complex software tools and related errors by users.

Commonly variability of material properties and possibly related statistical uncertainty are included in relevant models for material properties.

In general the model uncertainty can be obtained from comparisons of physical tests and model results. A great care should be taken to define correctly test conditions and evaluate test results. It should be always assured that a specimen fails in an investigated failure mode. For instance when the model uncertainty in shear is investigated, beams failed in bending should be excluded from the assessment. Accuracy of tests (related to the test method and execution of an individual test) should be considered in the assessment of model uncertainties.

JCSS (2006) proposed the following definitions of the model uncertainty  $\theta$  based on different relationships between the response of a structure (actual resistance)  $R$  and a model resistance  $R_{\text{model}}$  (estimate of the resistance based on a numerical model or analytical expression) as follows:

$$R = \theta R_{\text{model}}(\mathbf{X}) \quad (1)$$

or

$$R = \theta + R_{\text{model}}(\mathbf{X}) \quad (2)$$

or a combination of both;  $\mathbf{X}^T = (X_1, \dots, X_m)$  is the vector of basic variables  $X_i$ . In this paper the model uncertainty is assumed to be a random variable  $\theta$ . However, in more advanced analyses it may be represented by functions of several auxiliary random variables  $\boldsymbol{\theta}$  and variables  $\mathbf{X}$  involved in the model resistance.

It is difficult to specify general conditions under which Eq. (1) or (2) becomes more preferable than the other since the choice always depends on task-specific conditions. Current practice indicates that the multiplicative definition in Eq. (1) is widely applied to the model uncertainties while the additive relationship in Eq. (2) is used to account for systematic measurement errors.

From a purely statistical point of view the multiplicative relationship is more appropriate when the structural resistance  $R$  and the model resistance  $R_{\text{model}}(\cdot)$  are described by lognormal distributions since the model uncertainty  $\theta$  is likewise lognormally distributed and its statistical characteristics can be readily derived. Similarly, the additive formula becomes preferable when normal distributions are relevant.

It is worth noting that Eq. (1) can be transformed to Eq. (2) using the logarithmic transformation:

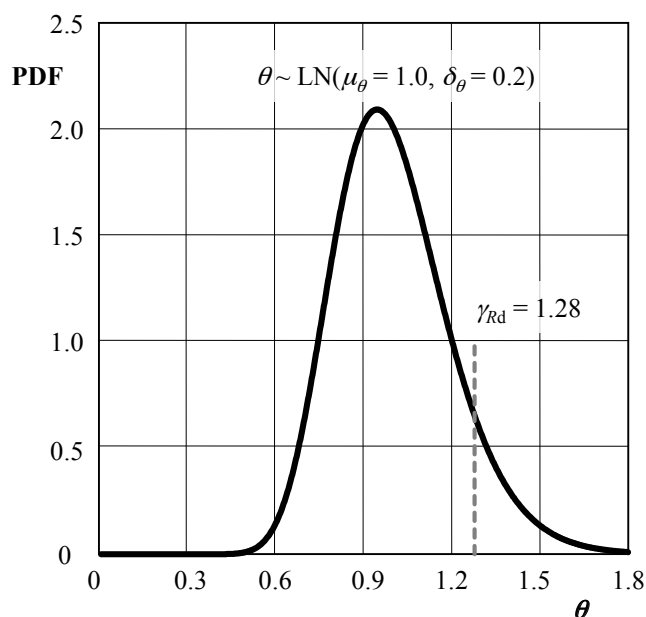


Fig. 1: Probability density function of  $\theta$  and the model uncertainty factor  $\gamma_{Rd}$

$$\ln R = \ln \theta + \ln[R_{\text{model}}(X_1, \dots, X_m)] \quad (3)$$

The model uncertainty  $\theta$  in general depends on basic variables  $(X_1, \dots, X_m)$ . Influence of individual variables on  $\theta$  can be assessed by a regression analysis as described e.g. by Ditlevsen & Madsen (1996). It is also indicated that the model describes well the essential dependency between  $R$  and  $(X_1, \dots, X_m)$  only if the model uncertainty:

- Has either a suitably small coefficient of variation (how small is the question of the practical importance of the accuracy of the model) or
- Is statistically independent of the basic variables  $(X_1, \dots, X_m)$ .

For deterministic reliability verifications EN 1990 (2002) introduces the partial factor  $\gamma_{Rd}$  to describe the uncertainty associated with the resistance model (“design value of the model uncertainty”). Fig. 2 illustrates the relationship between the probabilistic distribution of  $\theta$  and factor  $\gamma_{Rd}$ . As an example the lognormal distribution (mean  $\mu_\theta = 1$  and coefficient of variation  $\delta_\theta = 0.2$ ) and the relevant model uncertainty factor  $\gamma_{Rd} = 1.28$  are shown (more details are provided in Section 5).

### 3. Uncertainties related to the models provided in EN 1992

The model uncertainty should be always clearly associated with an assumed resistance model. In this section uncertainties related to basic resistance models provided in EN 1992-1-1 (2004) are considered. Model resistances of structural members exposed to compression, bending, shear without and with stirrups are assumed to be given as follows, respectively:

$$R_{\text{model}}(\mathbf{X}) = b^2(\alpha_{cc}f_c + \rho_l f_y) \quad (4a)$$

$$R_{\text{model}}(\mathbf{X}) = \rho_l b d f_y [d - 0.5 \rho_l d f_y / (\alpha_{cc} f_c)] \quad (4b)$$

$$R_{\text{model}}(\mathbf{X}) = \max[0.18k \min(\rho_l; 0.02)(100\rho_l f_c)^{1/3} b_w d; 0.035k^{3/2} f_c^{1/2} b_w d] \quad (4c)$$

$$R_{\text{model}}(\mathbf{X}) = \max_{1 \leq \cot\theta \leq 2.5} \{\min[\rho_w b_w z f_{yw} \cot\theta, \alpha_{cw} b_w z v f_c / (\cot\theta + \tan\theta)]\} \quad (4d)$$

where:

$b$  denotes width

$b_w = b$  minimum width between tension and compression chords

$d$	effective depth
$f_c$	concrete compressive strength
$f_y, f_{yw}$	yield strength of reinforcement (longitudinal, shear)
$k = \min[1 + \sqrt{(200 \text{ mm} / d)}; 2.0]$	
$z \approx 0.9d$	inner lever arm
$\alpha_{cc} = 1$	coefficient accounting for long-term effects on concrete strength
$\alpha_{cw} = 1$	coefficient taking account of the state of the stress in the compression chord
$\theta$	angle between concrete compression struts and the main tension chord
$\nu = 0.6(1 - f_{ck} / 250 \text{ MPa})$	strength reduction factor for concrete cracked in shear
$\rho_l, \rho_w$	longitudinal/shear reinforcement ratio.

Note that coefficients  $\alpha_c$  are assumed to be one and thus not included in Eqs. (4). For shear without stirrups no axial compressive force is considered and the partial factor  $\gamma_c$  is not applied in the assessment of the coefficient  $C_{Rd,c}$  and thus  $C_{Rd,c} = 0.18$ .

Statistical characteristics of the uncertainties related to the resistance models in Eqs. (4) obtained in previous studies are provided in Tab. 1. These characteristics were derived using the definition of the model uncertainty given in Eq. (1). The reported values seem to be in a broad agreement for the failure modes where several results are available. It should be noted that some of the studies are based on overlapping experimental results and thus are partly dependent.

The previous studies also provided a valuable insight into factors influencing the model uncertainties for bending and shear. As a background information for the paper by Holický et al. (2007), the analysis of experimental data revealed no significant statistical correlation between the model uncertainty and any of the basic variables influencing flexural resistance (including reinforcement ratio  $0.25 \% < \rho_l < 3.5 \%$  and concrete compressive strength  $17 \text{ MPa} < f_c < 45 \text{ MPa}$ ).

For shear without stirrups, researchers at the University of Stellenbosch concluded that the model uncertainty is significantly decreasing with the ratio of the shear span to the effective depth  $a/d$  and the effective depth  $d$  while it is not subject to trends with the other shear parameters, Retief (2007). The results were limited to the range of  $2.9 < a/d$  as deep beam and shear bond failures are expected for lower values of  $a/d$  in accordance with O'Brien & Dixon (1995). For lower  $a/d$  the shear resistance is likely to be underestimated since the positive contribution of an arch action is neglected in the Eurocode model. Cladera & Mari (2004a) reported decrease of the model uncertainty with  $d$  and also with  $\rho_l$ . The discrepancy with results by Retief (2007) should be further investigated. It is important to emphasise that the failure of members without shear reinforcement is sudden and brittle and adequate reliability should be assured.

For shear with stirrups, Cladera & Mari (2007) and Mensah (2012) indicated that the model uncertainty significantly decreases with an increasing strength of shear reinforcement  $\rho_w f_y$ . Influences of the other basic variables are much lower. In addition Cladera & Mari (2004b) noted that influence of the amount of shear reinforcement is not linearly proportional to the shear strength and the truss model in EN 1992-1-1 (2004) may be unconservative for highly reinforced concrete members ( $\rho_w f_y > 2 \text{ MPa}$ ).

Based on the results given in Tab. 1 and neglecting trends with the basic variables, the following stochastic characteristics of  $\theta$  may be applied as a first approximation:

- Axial compression without effects of buckling:  $\mu_\theta \approx 1$ ;  $V_\theta \approx 0.05$ ,
- Bending:  $\mu_\theta \approx 1.1$ ;  $V_\theta \approx 0.1$ ,
- Shear of the members without shear reinforcement:  $\mu_\theta \approx 1$ ;  $V_\theta \approx 0.2$ ,
- Shear of the members with shear reinforcement:  $\mu_\theta \approx 1.7$ ;  $V_\theta \approx 0.35$ .

When different failure modes are combined, less favourable model uncertainty may be considered.

Tab. 1: Statistical characteristics of the model uncertainties related to the resistance models provided in EN 1992-1-1

Failure type	Note	$\mu_s$	$V_s$	Source
Axial compression	Validated to comply with the model uncertainty factors used in ENs	1	0.05	Working draft of the bulletin on semi-probabilistic methods for verifications of reinforced concrete structures by <i>fib</i> SAG7
Bending	Number of tests $n = 109$	1.08	0.093	Holický et al. (2007)
Shear without stirrups	$n = 718$	1.75**	0.33	Hawkins et al. (2005)
	$n = 193$	1.02	0.22	Cladera & Marí (2004a)
	$n = 184, 2.9 < a/d < 8.03^*$	0.94	0.13	Retief (2007)
Shear with stirrups	$n = 160$	1.7	0.37	Hawkins et al. (2005)
	$n = 123$	1.83	0.40	Cladera & Marí (2004b)
	$n = 122$	1.64	0.32	Cladera & Marí (2007)

\*  $a/d$  denotes the ratio of the shear span to the effective depth; \*\* discrepancy with the other sources needs to be further clarified.

#### 4. Uncertainties related to FE models

In general well validated advanced FE models should perform much better than the standardised formulas, with mean model uncertainty approaching unity. However, Vrouwenvelder (2010) indicated that variability of the uncertainty of FE models might be greater than that of the standardised formulas since:

- There may be lower experience with applications of FE models,
- Additional input data (material properties) and decisions in set-up of a FE model (discretization) may significantly influence predicted results.

JCSS (2006) recommends probabilistic models for the model uncertainties associated with “a more or less standard structural FE model”, see Tab. 2.

Schlune et al. (2011) provided an overview of the model uncertainties related to FE analyses of different failure modes. The study was based on an extensive review of round robin exercises and modelling competitions published in scientific literature as well as on engineering judgement. The proposed models (accepted here in Tab. 2) were intended to be used as a first approximation of quantification of the model uncertainty.

Schlune et al. (2011) noted that the round robin exercises and modelling competitions dealt mainly with statically determinate structures for which the resistance in one critical section is decisive. However, for statically indeterminate structures the deformation capacity allowing for redistribution often becomes important. Since modelling of the deformation capacity is usually more difficult than of the ultimate strength, the coefficients of variation given in Tab. 2 may need to be increased. It was also emphasised that values in Tab. 2 are not affected by gross human errors.

For shear authors' experience from previous round robin exercises and competitions (see e.g. Jaeger & Marti (2009)) indicates the mean value of the model uncertainty of about 1.15 and the coefficient of variation ranging from 0.05 to 0.3 (exceptionally up to 0.6).

Tab. 2: Indicative probabilistic models for the model uncertainties associated with FE models.

Failure type	Note	$\mu_e$	$V_e$	Source
Compression	Normal strength concrete	0.9 - 1.0	0.1 – 0.2	Schlune et al. (2011)
	High strength concrete	1.0	0.2 – 0.3	Schlune et al. (2011)
Bending	Including the effects of normal and shear forces, standard structural FE models	1.2	0.15	JCSS (2006)
	Under-reinforced	1.0 - 1.2	0.05 – 0.15	Schlune et al. (2011)
	Under-reinforced, bending reinforcement not aligned in principal moment direction	0.9	0.05 – 0.15	Schlune et al. (2011)
	Over-reinforced, normal strength concrete	0.9 - 1.0	0.1 – 0.15	Schlune et al. (2011)
	Over-reinforced, high strength concrete	1.0	0.2 – 0.3	Schlune et al. (2011)
Shear	-	1.4	0.25	JCSS (2006)
	Failure due to yielding of the reinforcement	0.9 - 1.0	0.1 – 0.25	Schlune et al. (2011)
	Failure due to crushing of concrete, combination of compression and shear loading, large members, bending reinforcement not aligned in principal moment direction	0.7 - 1.0	0.2 – 0.4	Schlune et al. (2011)
	Resistance of slabs with inclined main reinforcement and stirrups, compressive failure of inclined concrete struts	1.15	0.05-0.3	Jaeger & Marti (2009)

Tab. 2 indicates that the available experimental data are insufficient and inconclusive for establishing the model uncertainties for FE calculations. The information provided by JCSS (2006) and Jaeger & Marti (2009) reveals considerably higher mean values of the model uncertainties than that given by Schlune et al. (2011). In addition note that for the compression, Tab. 2 provides higher coefficients of variation than those in Tab. 1 which is rather surprising. These discrepancies should be clarified by further research.

It seems to be very difficult to propose a generally applicable model for the uncertainties related to FE models. Careful consideration of the data in Tab. 2 in conjunction with other available information is advised when an appropriate model for  $\theta$  needs to be selected. As a very rough approximation  $\mu_e \approx 1$  and  $V_e \approx 0.2$  might be considered.

Reduction of variability of the model uncertainty and perhaps convergence of the mean to unity can be achieved by introduction of the guidelines for FE analysis of concrete structures. The foreseen guidelines should provide recommendations concerning assumptions, model choices and validation, and reporting results of analysis.

Differentiation of the model uncertainties with respect to complexity of the model and experience of user seems to be needed. Based on engineering judgement, a failure mode (ductile, steel, concrete in compression and tension, etc.) is an obvious additional factor for the differentiation of model uncertainty. However, the authors feel that these effects are already well covered in advanced numerical models and material uncertainties and need not to be included in the model uncertainty. Therefore, they might be considered only in simplified engineering formulas where various approximations are applied.

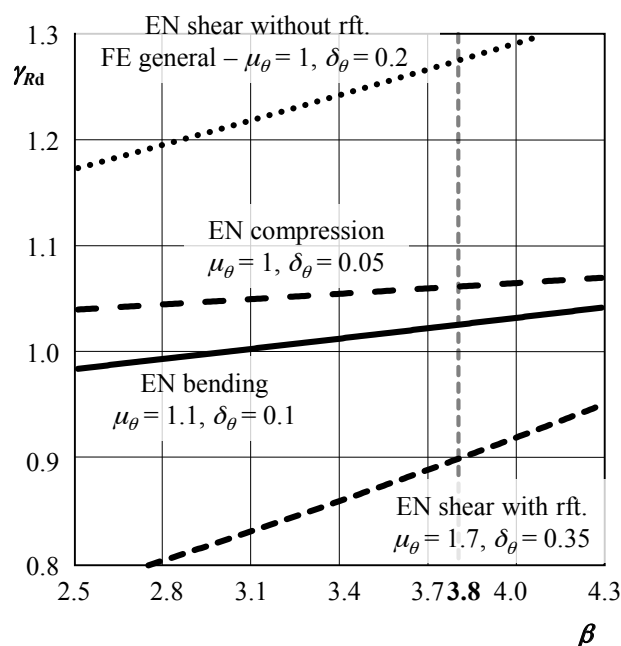


Fig. 2: Variation of the partial factor  $\gamma_{Rd}$  with  $\beta$  for  $\alpha_R = 0.32$  (rft. denoting shear reinforcement)

## 5. Model uncertainty factor for deterministic reliability verifications

In accordance with fib SAG 9 (2010) the model uncertainty factor  $\gamma_{Rd}$  for reinforced concrete structures can be obtained as a product of:

$$\gamma_{Rd} = \gamma_{Rd1} \gamma_{Rd2} \quad (5)$$

where  $\gamma_{Rd1}$  denotes the partial factor accounting for model uncertainty and  $\gamma_{Rd2}$  is the partial factor accounting for geometrical uncertainties.

EN 1992-1-1 (2004) provides no specific recommendations concerning model uncertainties. EN 1992-2 (2005) introduces the global safety format for a nonlinear analysis with the recommended model uncertainty factor of 1.06. However, it was shown by Sýkora & Holický (2011) that such a factor is rather low and should be increased in most cases depending on relevant failure mode (bending, shear, compression).

In accordance with fib SAG 9 (2010)  $\gamma_{Rd1} = 1.05$  for concrete strength and  $\gamma_{Rd1} = 1.025$  for reinforcement may be assumed in common cases. However, larger model uncertainty needs to be considered for punching shear in the case when concrete crushing is governing. A value of  $\gamma_{Rd2} = 1.05$  may be assumed for geometrical uncertainties of the concrete section size or reinforcement position. When relevant measurements of an existing structure indicate insignificant variability of geometrical properties,  $\gamma_{Rd2} = 1.0$  may be considered.

Alternatively, the partial factor  $\gamma_{Rd}$  can be obtained from the following relationship based on a lognormal distribution:

$$\gamma_{Rd} = 1 / [\mu_v \exp(-\alpha_R \beta V_v)] \quad (6)$$

where  $\alpha_R$  denotes the FORM sensitivity factor and  $\beta$  is the target reliability index according to EN 1990 (2002). Assuming the probabilistic models for the model uncertainty given in previous sections, variation of the partial factor  $\gamma_{Rd}$  obtained from Eq. (6) with the target reliability  $\beta$  for  $\alpha_R = 0.4 \times 0.8 = 0.32$  (“non-dominant resistance variable”) is indicated in Fig. 2.

It follows from Fig. 2 that the model uncertainty factor  $\gamma_{Rd}$  is close to unity for well-established models of flexural and axial compression resistances given in EN 1992-1-1 (2004). The partial factor decreases below unity for the EN shear model for the members with shear reinforcement due to the high value of  $\mu_v$ . For  $\beta = 3.8$  the model uncertainty factor of about 1.3 is obtained for the EN shear model for the members without shear reinforcement and also for FE models (see also Fig. 1). The

value for FE models shall, however, be considered indicative; further refinements with regard to different failure modes are foreseen (cf. Tab. 2).

It is interesting to observe that the results in Fig. 2 are in agreement with the findings by Taerwe (1993) who concluded (with regard to derivation of the global resistance factor, EN 1992-2 (2005) and fib (2010)) that:

- special calibration of the model uncertainty as part of the global resistance factor is warranted for coefficients of variation of 0.2 and above,
- for smaller coefficients of variation it could be tentatively suggested that the model uncertainties do not require an additional safety factor if conservative models are used.

The selection of  $\alpha_R = 0.32$  deserves additional comments. Leading and accompanying actions (with associated factors  $\alpha_E = -0.7$  and  $\alpha_E = -0.4 \times 0.7 = -0.28$ , respectively) are distinguished in Annex C of EN 1990 (2002) while  $\alpha_R = 0.8$  is recommended for resistance variables under conditions specified in the Eurocode. When the model uncertainty factor  $\gamma_{Rd}$  and material factor  $\gamma_m$  are assessed separately considering  $\alpha_R = 0.8$ , overly conservative designs may be obtained. Therefore, working draft of the bulletin on semi-probabilistic methods for verifications of reinforced concrete structures by *fib* SAG7 assumes that the model uncertainty is not a leading resistance variable and the sensitivity factor is thus reduced to  $\alpha_R = 0.4 \times 0.8 = 0.32$ . This assumption is accepted in this study.

It is worth noting that for  $\alpha_R = 0.8$  the partial factor  $\gamma_{Rd}$  increases:

- significantly for shear without shear reinforcement and FE models ( $\gamma_{Rd} \approx 1.85$  for  $\beta = 3.8$ ) and for shear with shear reinforcement ( $\gamma_{Rd} \approx 1.7$  for  $\beta = 3.8$ ),
- less significantly for axial compression and flexural resistance ( $\gamma_{Rd} \approx 1.2$  for  $\beta = 3.8$ ).

## 6. Outlook of further research

The study indicates that the uncertainties related to the resistance models for reinforced concrete structures should be further investigated. Basically two classes of models should be distinguished:

- Engineering models with strictly defined assumptions (beam and sectional approaches),
- Complex models based on general principles of structural mechanics with much wider options and potentially higher uncertainties.

In particular for FE applications outcomes of the further research should facilitate implementation of sophisticated resistance models with the full exploitation of the present know-how while limiting the model uncertainty by controlling model and user errors.

Note that the terms “modelling uncertainty” or “modelling factor” are more appropriate than “model uncertainty” when human factors are included.

The research framework is proposed as follows:

1. For structural members, reference values of the resistance will be obtained from available experimental tests. When experimental data is little or missing (large and complex structures, continuous members, members supporting distributed loads, members that fail in regions other than adjacent to a support), test values are to be found by interpretation of test results in different conditions (simpler structural members), using different software tools and verification methods. Yet, this will introduce an additional uncertainty.
2. The classes with different uncertainty levels originating from complexity of the task, underlying structural models and experience of users will be proposed. The differences between model and actual resistances will be analysed and evaluated for common types of reinforced concrete structures. The resistance at ultimate limit states will be investigated for ductile and brittle failure modes. The errors due to numerical approximations will be assessed.
3. Appropriate probabilistic description of resistance represented by models with various levels of sophistication and users' experience will be proposed. For practical applications the partial factors for model uncertainties will be derived.



4. In models based on numerical simulations, the variability of the model uncertainty can be reduced by appropriate model validation. Such validation should cover sources of uncertainties related to numerical methods and constitutive models. Rules for such improvements will be formulated and adjusted model uncertainties will be proposed.

For deteriorating structures the resistance model should inevitably incorporate degradation effects that may be a source of additional uncertainties and should be also quantified within the further research. The study by Tanner et al. (2011) provides the first insight into the model uncertainties of deteriorating concrete structures.

## 7. Concluding remarks

Previous reliability studies indicate that description of model uncertainties is a crucial problem of the design of reinforced concrete structures. Therefore, the present paper is focused on the model uncertainties; the following concluding remarks can be drawn:

1. The model uncertainty should be always clearly associated with an assumed resistance model.
2. The model uncertainty should cover the following aspects (if relevant): simplifications of known physical principles, approximations inherent to numerical methods, influence of different interpretations of users of complex software tools and related errors.
3. Relationship between the model uncertainty and resistance obtained by the model can be multiplicative or additive or combination thereof; in common cases the multiplicative form is acceptable.
4. Uncertainties related to sectional-oriented models provided in EN 1992-1-1 (2004) can be described by the following statistical characteristics and partial factors:
  - Axial compression without effects of buckling: mean  $\mu_o \approx 1$ ; coefficient of variation  $V_o \approx 0.05$ ; model uncertainty factor  $\gamma_{Rd} \approx 1.05$ ,
  - Bending:  $\mu_o \approx 1.1$ ;  $V_o \approx 0.1$  and  $\gamma_{Rd} \approx 1.05$ ,
  - Shear of the members without shear reinforcement:  $\mu_o \approx 1$ ;  $V_o \approx 0.2$  and  $\gamma_{Rd} \approx 1.3$ ,
  - Shear of the members with shear reinforcement:  $\mu_o \approx 1.7$ ;  $V_o \approx 0.35$  and  $\gamma_{Rd} \approx 0.9$ .
5. The present experimental data are insufficient and inconclusive for establishing the model uncertainties for FE calculations. As a first approximation  $\mu_o \approx 1$ ;  $V_o \approx 0.2$  and  $\gamma_{Rd} \approx 1.3$  might be considered.

Further research activities should focus on differentiation of the model uncertainties with respect to the level of the model and complexity of the task. For existing structures uncertainties of degradation models should be specified.

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