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COMPARATIVE STUDY ON NUMERICAL AND ANALYTICAL ASSESSMENT OF ELASTIC PROPERTIES OF METAL FOAMS

P. Koudelka, O. Jiroušek, T. Doktor, P. Zlámal, T. Fíla*

Abstract: *Recently, titanium metal foams are being considered as a suitable replacement for substituting trabecular bone microstructure especially for their similar pore distribution. The most common methods for determination of compressive effective elastic properties of such materials involve different approaches based on finite element analysis (FEA) of their microstructure. The internal geometry is usually modeled by two different methods - directly on the basis of a series of CT scans or using one of discretization schemes. However, all these techniques require highly specialized hardware, software and significant amount of computational time. In this paper, the effective elastic properties of the metal foam are instead obtained by analytical modulus-porosity relations and results are compared with previous FE based analysis.*

Keywords: ***metal foams, modulus-porosity relations, finite element method, compressive behavior, effective elastic properties***

1. Introduction

Metal foams are highly porous materials that possess unique combination of mechanical, acoustic and electromagnetic properties. At very low specific weights and thus high specific stiffnesses, they are able to absorb significant amount of deformation energy while guaranteeing other properties such as high fire and heat resistance, noise attenuation and shielding of electromagnetic devices (Banhart, 2001). Combination of these characteristics attracts application of metal foams in many engineering fields from interior design and equipment to civil engineering and vehicle construction.

The recent development of many cost-effective production techniques increases their potential for substitution of legacy engineering practices and/or substitution of the most commonly used materials. Particularly, metal foams will be potentially able to outperform certain types of polymer foams and honeycomb structures in light of mechanical and environmental properties respectively.

Furthermore, open-cell metal foams manufactured from biocompatible materials (i.e. pure Titanium, Ti-Ni alloys, etc.) successfully mimic natural characteristics of human bones. Usage of titanium and its alloys as fusion implants is one of the most important developments in the field of biomechanics and biomaterials. Such structural implants offer high corrosion resistance, good mechanical properties and exceptional biocompatibility among other biomaterials. Open-pore structure with mean pore dimensions of 200 – 500 μm is susceptible of transport of body fluids and also the ingrowth of new bone tissue. By modifying the morphology, amount, size and orientation of pores, mechanical characteristics can be adjusted to particular bone tissue, which gives the opportunity to assess optimal characteristics compatible with the bone. This is the key parameter in bone implant applications because it prevents the stress-shielding problem existing in the implantation of bulk materials due to the mismatch of mechanical properties.

During the last decades, there has been much effort dedicated to understanding of the porosity dependence of the effective elastic constants of cellular metals. Numerous relations derived from various constitutive laws have been developed as a result of extensive theoretical and experimental work. The most of the relationships give the variation of elastic constants in terms of porosity. However, many of the suggested laws contain fitting parameters whose mechanical and physical explanation is either

*Bc. Petr Koudelka, Doc. Ing. Ondřej Jiroušek, PhD., Ing. Tomáš Doktor, Ing. Petr Zlámal, Bc. Tomáš Fíla: Institute of Theoretical and Applied Mechanics AS CR, Prosecká 76; 190 00, Praha 9; CZ, e-mail: [koudelkap,jirousek,doktor,zlamal,fila]@itam.cas.cz

unclear or values of the parameters do not correspond to microstructural characteristics of the material. This indicates that the derivation of the microstructure-property relationships for porous metals (with both open- and closed-cell pores) is still an open issue and in many cases other parameters (i.e. pore shape and distribution) have to be taken also into account.

In this paper, effective elastic properties in compression predicted by various models for modulus-porosity relations are compared to experimental and numerical results of compressive behavior of Alporas aluminium closed-cell foam and Optinium open-cell biocompatible titanium foam. Experimental values of elastic modulus and porosity of Alporas foam were assessed on the basis of quasi static compressional loading and weighting of the sample respectively.

2. Materials

2.1. Alporas

Alporas® is a closed-cell aluminium foam developed in late 80's and produced by Japan manufacturer Shinko Wire Co., Ltd. Structure of this material is typically constituted by large inner pores of polyhedral shape with average size 4.5 mm. Cell walls that create complex random inner structure are typically $100\ \mu$m thick with overall porosity approximately $90\ \%$ (Miyoshi, 1998), although it can be manufactured at different levels of porosity saying that polyhedron cells become spherical at porosities under $70\ \%$. Foam is manufactured using special unnormalized alloy containing $97\ \%$ of aluminium, $1.5\ \%$ of calcium and $1.5\ \%$ of titanium (Miyoshi, 1998). Because material properties of this alloy are not provided by the manufacturer, the material models typically use mechanical properties of $98\ \%$ aluminium as stated in (Konstantinidis, 2005).

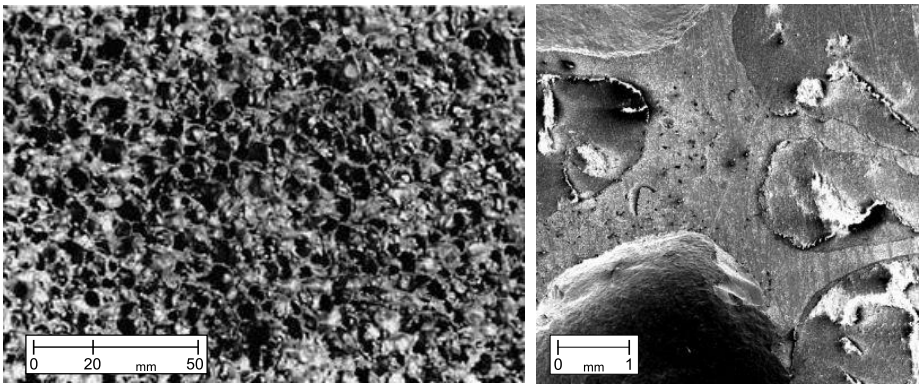

Fig. 1: Alporas - macroscopic structure (left), surface of the cell wall captured using SEM - specimen prepared for nanoindentation experiment (right)

2.2. Optinium

Optinium© foam is manufactured using commercial grade CP4 titanium powder which is transformed to open-pore structure using a propellant (space holder technique, see Singh (2009b)). Final porous structure reaches effective porosity of $60 - 65\ \%$ and as has been shown in a permeability study of these materials (Singh, 2009a), the pore and interconnect sizes are almost independent of relative density, but foam strut thickness decreases with increasing porosity. Implants manufactured from this material are ideal for their bone integration and ongrowth potential. The combination of bone-like mechanical properties with the interconnecting porosity leads to excellent biological fixation, a method that has already been proven in endoprosthetics. Main usage of the implants is the surgical treatment of degenerate discs of the lumbar spine and restoration of initial height of the intervertebral disc space.

3. Finite element analysis

Determination of mechanical characteristics of cellular materials using finite element (FE) analysis is strongly dependent on microstructure modeling scheme. The internal structure can be generated directly

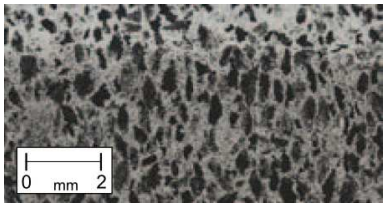

Fig. 2: Macroscopic structure of Optinium

from samples of the real material or using one of several discretization methods. Direct modeling of the internal structure is based on computed-tomography scanning of specimens. This approach facilitates FE representation of the internal structure including all of its irregularities and defects (Vavřík, 2011). Simultaneously, material properties at the level of the individual cells can be assessed by nanoindentation with high reliability and reproducibility (Jiroušek, 2009; Králík, 2011); moreover Digital Volumetric Correlation (DVC) method can be used to identify the three-dimensional strain field in the loaded microstructure (Jiroušek, 2011). Instead of modeling the complex internal structure of metal foam directly, a unit cell approach is often used. Internal structure of cellular materials can be then represented by miscellaneous two- and three-dimensional models according to the various cellular forms. In the preceding FE study (Koudelka, 2011) the beam-only discretization with cubic cells (Fig. 3) was used intentionally to investigate its suitability for modeling of closed-cell foams and trabecular bones. It has been shown that this discretization scheme, originally developed for modeling of open-cell foams, is suitable for also assessment of elastic characteristics of Alporas closed-cell foam (Fig. 4). Declared elastic modulus of Alporas (in the range from 0.4 GPa to 1.0 GPa) was acquired with model relative density in interval $0.08 - 0.13$, which is consistent with real mechanical characteristics while significantly reducing amount of computational time.

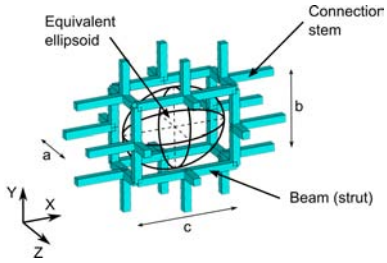

Fig. 3: Gibson-Ashby's cell with corresponding equivalent ellipsoid

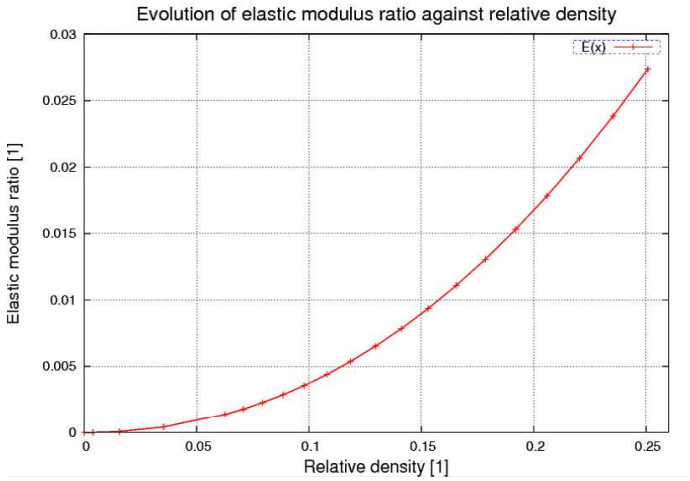

Fig. 4: Evolution of relative elastic modulus plotted against relative density - FE simulations of Alporas

4. Effective elastic moduli of porous metals

From the view of micromechanics, cellular metals can be considered as a special case of multiphase mixtures or materials with microstructure. The effective elastic properties can be predicted when mechanical characteristics of material components and microstructural details are known. Microstructural information of the lowest order involve volume fractions of constituent phase whereas higher order information stands for pore size, shape and orientation including their statistical characteristics (Markov, 2000; Milton, 2003; Nemat-Nasser, 1999; Torquato, 2002).

Effective elastic modulus E_{e} can be then considered as a function of phase moduli E_{i} and the lowest-order only (porosity ϕ_{i}) microstructural information (Pabst, 2004a):

$$E_{\mathrm{e}} = f(E_{\mathrm{i}}, \phi_{\mathrm{i}}) \tag{1}$$

where E_{i}, $i \in (0, 1, \ldots, n)$ are the phase moduli of all n constituent phases and ϕ_{i} are the volume fractions of the n phases.

For porous metals where the void phase exhibits approximately zero mechanical properties one can assume that the void phase is redundant and therefore write the following basic assumptions: $\phi_1 \equiv 1-\phi$, $\phi_2 \equiv \phi$ and for void phase with zero elastic modulus: $E_2 = 0$, $E_1 \equiv E_0$.

This provides for definition of Hashin-Shtrikman (HS) bound for the effective elastic modulus as:

$$E_{\mathrm{HS}} = \frac{9K_{\mathrm{HS}}G_{\mathrm{HS}}}{3K_{\mathrm{HS}} + G_{\mathrm{HS}}} \tag{2}$$

where G_{HS} ans K_{HS} are HS bounds for the shear and bulk modulus respectively (Hashin, 1963). In general, HS bounds are valid for microstructure called Hashin assemblage, which consists of polydisperse composite spheres containing concentric spherical inclusions. In the case of macroscopically isotropic porous material, Hashin assemblage would be approximated by a material containing hollow spheres with an infinitely wide size distribution.

In the case of porous materials relative elastic modulus is the most frequently defined as:

$$E_{\mathrm{r}} = \frac{E_{\mathrm{e}}}{E_0} \tag{3}$$

where E_{e} is the effective elastic modulus and E_0 is the elastic modulus of matrix phase or solid skeleton phase (for closed-cell and open-cell materials respectively). Linear dependence between the relative elastic moduli and porosity can be considered at very low porosities:

$$E_{\mathrm{r}} = 1 - [E]\,\phi \tag{4}$$

where $[E]$ is the intrinsic elastic modulus defined as (Pabst, 2003):

$$[E] \equiv -\lim_{\phi \to 0} \frac{E_{\mathrm{r}} - 1}{\phi} \tag{5}$$

In the special case of a cellular material with spherical pores and solid phase Poisson's ratio $\upsilon_0 = 0.2$ or $\upsilon_0 = 0.33$ the intrinsic elastic modulus is equal to two ($[E] = 2$) and any deviations from this value may be inflicted by discrepancies from sphericity of pore shapes or discrepancies of the solid phase Poisson's ratio (Pabst, 2006a). This model also predicts a critical porosity parameter $\phi_{\mathrm{c}} = [E]^{-1} \leq 1$ representing a point where the effective elastic modulus reaches zero and the material looses its integrity.

4.1. Nonlinear models

Porosity dependence of the effective elastic moduli is usually nonlinear. The applicability of modulus-porosity relations is often stated to be controlled by the approximate isometry and not only by the sphericity of pores. This is supported by the fact that Coble-Kingery relation in the form (Coble, 1956):

$$E_{\mathrm{r}} = 1 - [E]\,\phi + ([E] - 1)\phi^2 \tag{6}$$

which reduces to:

$$E_{\mathrm{r}} = (1 - \phi)^2 \tag{7}$$

for porous material with spherical pores and solid phase Poisson ratio $\upsilon_0 = 0.2$ is in the same form as a special case of power-law relations, functional equation approach, differential approach and semi-empirical Gibson-Ashby model derived by fitting large set of experimental data irrespective to spherical pore shape or isolated-pore topology. As a result, the connectedness of pores itself does not exclude the use of any modulus-porosity relation.

The mechanical properties of foams can be modeled by considering deformation mechanisms of individual cells (Ashby, 2004). Open-cell foams deform by bending followed at large loads by the formation of plastic hinges within the cell walls. Relative elastic modulus is then defined as:

$$E_{\mathrm{r}} = \alpha \left(\frac{\rho_{\mathrm{f}}}{\rho_0}\right)^{\eta} \tag{8}$$

where ρ_{f} is density of the foam, ρ_0 is density of the matrix material, α and η are constants related to cell geometry.

In closed-cell foams, bending of the cell walls is accompanied by stretching of the cell faces (Gibson, 1997). The relation for the relative elastic modulus has then a linear density term, which is related to face stretching, and square term related to edge bending:

$$E_{\mathrm{r}} = C\varphi^2 \left(\frac{\rho_{\mathrm{f}}}{\rho_0}\right)^2 + C(1 - \varphi)\left(\frac{\rho_{\mathrm{f}}}{\rho_0}\right) \tag{9}$$

where C is a constant related to cell geometry and φ is the fraction of solid contained in the cell edges. It should be noted that experimental verification of these relations exhibits problems with the determination of φ for high foam's relative densities (above $30\,\%$) because there is no distinction between cell edges and faces. Furthermore, either the cell size effect and cell anisotropy effect on mechanical properties are not incorporated in the Gibson-Ashby's model.

Other considered relations based on exponential and power laws are the most commonly used for determination of modulus-porosity dependence of ceramic materials. In case of porous metals considered in this paper, the modified exponential relation, Mooney-type exponential relation, Archie type relation, Phani-Niyogi relation and Pabst-Gregorová relation are studied.

The modified exponential relation appears in the form (Mooney, 1951):

$$E_{\mathrm{r}} = \exp\left(\frac{-[E]\phi}{1 - \phi}\right) \tag{10}$$

Derivation of this equation has been given via the functional equation approach (Pabst, 2004b) and trivial case for porous material with spherical pores leads to the prediction:

$$E_{\mathrm{r}} = \exp\left(\frac{-2\phi}{1 - \phi}\right) \tag{11}$$

This model results in zero relative modulus only in the case of $100\,\%$ porosity. To allow for zero relative modulus at porosities lower than $100\,\%$, additional parameter of critical porosity has to be introduced, which leads to the Mooney-type exponential relation (Mooney, 1951):

$$E_{\mathrm{r}} = \exp\left(\frac{-[E]\phi}{1 - \frac{\phi}{\phi_{\mathrm{c}}}}\right) \tag{12}$$

Power-law relations are in the simplest form represented by the Archie-type relation (Archie, 1942):

$$E_{\mathrm{r}} = (1-\phi)^{[E]} \tag{13}$$

This equation does not exhibit problem with zero relative modulus at $100\,\%$ porosity and again the critical porosity parameter can be introduced to allow zero relative modulus at porosities lower than $100\,\%$. This results in a Krieger-type power-law relation (Krieger, 1972), often called Phani-Niyogi relation (Phani, 1987):

$$E_{\mathrm{r}} = \left(1-\frac{\phi}{\phi_{\mathrm{c}}}\right)^{[E]\phi_{\mathrm{c}}} \tag{14}$$

All these exponential and power-law relations can be derived via the functional equation approach and their semi-empirical character is given by the fact that intrinsic elastic modulus may not be reliably known due to variations in pore shape and difficulties in assessment of pore size distribution and connectivity. Thus, reliable estimates of critical porosities are not available.

Pabst-Gregorová model (Pabst, 2004c) has been found heuristically and appears in the form:

$$E_{\mathrm{r}} = \left(1-[E]\phi+([E]-1)\phi^2\right)\frac{\left(1-\frac{\phi}{\phi_{\mathrm{c}}}\right)}{(1-\phi)} \tag{15}$$

This relation ensures zero relative elastic moduli for the case $\phi = \phi_{\mathrm{c}}$. When critical porosity of the material with spherical isometric pores is considered to be equal to 1, the relation reduces to the Coble-Kingery relation (Eq. 6).

5. Results

The hereinbefore mentioned relations were applied to prediction of relative elastic moduli of porous aluminium and titanium. Elastic characteristics of pore-free macroscopically isotropic matrix materials which are necessary input information for calculation of the relative moduli were taken from literature (Miyoshi, 1998; Singh, 2009b) and are listed in Tab. 1.

Tab. 1: Elastic characteristics of pore-free aluminium and titanium alloys used for manufacturing of Alporas and Optinium respectively (CP stands for "Commercially Pure")

	Alporas	Optinium
Elastic property	CP Aluminium	CP4 Titanium
E [GPa]	69	112.3
υ [1]	0.33	0.317

Tab. 2: Elastic characteristics of reference materials

Material property	Alporas	Optinium
E [GPa]	0.826	10.4
porosity [1]	0.914	0.639

Internal structure of the studied materials is constituted of large overlapping void inclusions (closed pores) of polyhedral shape and skeleton-like open pore structure with considerable variation in the size

and orientation of the pores and hence the foam struts in case of Alporas and Optinium respectively. This may lead to preliminary estimates of intrinsic elastic moduli significantly divergent from benchmark value of 2.

For Alporas, however, when the data are fitted with master curve using the Archie-type relation (Eq. 13) which seems to be the most promising among other models, the resulting intrinsic tensile modulus obtained via this equation is $[E] = 1.84$ for experimental results and $[E] = 2.11$ for FE results. When the Phani-Niyogi relation (Eq. 14) is used for fitting with critical porosity as a additional parameter, the intrinsic tensile modulus is $[E] = 2.36$, the critical porosity exceeds unity giving $\phi_{\mathrm{C}} = 1.13$ for experimental data and $[E] = 2.11$, $\phi_{\mathrm{C}} = 1.01$ for FE results. Similarly, when the Pabst-Gregorová (Eq. 15) model is considered in its complex form with both fitting parameters (intrinsic elastic modulus and critical porosity), fitted values are again close to benchmark ones with experimental $[E] = 1.95$, $\phi_{\mathrm{C}} = 1.01$ and numerical $[E] = 2.03$, $\phi_{\mathrm{C}} = 1.00$. It is apparent that if one lets vary freely the intrinsic elastic modulus in the power-law equations (making it adjustable fit parameter) the resulting intrinsic tensiel modulus is always approximately 2. Although internal structure of Alporas is composed of overlapping polyhedral cells and not spherical ones (that are indicated by the intrinsic elastic modulus equal to 2), it can be interpreted as a clear confirmation of the approximate isometricity of pores and also foam's isotropic mechanical characteristics. As contrasted to porosity dependence of porous ceramics, exponential-law models are completely unsatisfactory for metal foams with modified exponential model (Eq. 10)resulting in value $[E] = 0.44$ a and $[E] = 0.88$ for experimental and numerical results respectively. This result indicates significant deviation from spherical pore shape but visual comparison of fitting curve and numerical results shows diametrically different evolution of the fit model. By contrast, evolution of the Mooney-type model (Eq. 12)fits the numerical curve well with intrinsic elastic moduli $[E] = 0.88$ and $[E] = 1.6$ for experimental and numerical results respectively. Nevertheless, the corresponding critical porosity values amount to experimental $\phi_{\mathrm{C}} = 1.01$ and numerical $\phi_{\mathrm{C}} = 1.29$ that is clearly nonsense from the physical reality (Tab. 3, Fig. 5).

Tab. 3: Fit parameters determined for the master curve of the porosity dependence of the elastic modulus of Alporas and Optinium metal foams using power- and exponential-law models

		Alporas		Optinium	
Relation	Fit model	$[E]$	ϕ_{C}	$[E]$	ϕ_{C}
Archie	$E_{\mathrm{r}} = (1-\phi)^{[E]}$	2.11	–	2.20	–
Phani-Niyogi	$E_{\mathrm{r}} = \left(1 - \frac{\phi}{\phi_{\mathrm{c}}}\right)^{[E]\phi_{\mathrm{c}}}$	2.11	1.01	1.89	0.92
Pabst-Gregorová	$E_{\mathrm{r}} = \left(1 - [E]\phi + ([E]-1)\phi^2\right) \frac{\left(1-\frac{\phi}{\phi_{\mathrm{c}}}\right)}{(1-\phi)}$	2.03	1.00	2.08	1.01
Modified exponential	$E_{\mathrm{r}} = \exp\left(\frac{-[E]\phi}{1-\phi}\right)$	0.88	–	1.37	–
Mooney	$E_{\mathrm{r}} = \exp\left(\frac{-[E]\phi}{1-\frac{\phi}{\phi_{\mathrm{c}}}}\right)$	1.60	1.29	1.37	1.01

Analogous results have been obtained for Optinium foam with exponential models predicting large deviations from actual curves representing modulus-porosity dependence of this type of foam. Foam exhibits different relative stiffnesses in directions perpendicular and parallel to compaction direction due to eccentric spaceholders used during the production process. This kind of pore geometry and macroscopic material characteristics is essential according to intended usage of this material. PlivioPore implant system is indicated for posterior lumbar intercorporeal fusion using osteoconductive titanium implants from Optinium ensuring excellent both primary and secondary stability. This is primarily achieved by virtue of rotation principle during embedding of the implant into spinal column. Firstly, the implant is fully inserted into the intervertebral disc space using the implant holder and consequently rotated by 90 degrees to its desired position (Synthes, 2007). Such utilization principle and surgical technique require highest material stiffness in directions perpendicular to transverse and sagital plane where most loads are transferred. These material characteristics can be predicted by power-law equations, as in the case of Alporas (see Tab. 3). Moreover, both exponential-law models are suitable for prediction of modulus-porosity

dependence of this material with intrinsic elastic moduli $[E] = 1.37$ and critical porosity $\phi_{\mathrm{C}} = 1.01$ indicating anisometry of the internal structure due to nonspherical spaceholders.

Table 4 summarizes results obtained by fitting the master curves using Gibson-Ashby's equations (Eqs. 8, 9). For Optinium, if one lets vary both fitting parameters freely, optimal master curve characterizing porosity-dependence of this foam is achieved with $\alpha = 1.78$ and $\eta = 2.82$. This is in contrast with previous theoretical and experimental works with $\alpha = 2.03$ and $\eta = 2$ according to experimental study of Dillard (2004) and $\alpha = 0.6$ and $\eta = 1.6$ according to theoretical model of strut bending of Ashby (2004). Values of constants in these experimental and numerical studies were obtained for nickel-based open-cell foam indicating that any of the models is not applicable in general and must be adapted to particular studied material. When $\eta = 2$ is considered, the optimal value of α amounts to 1.01 which is consistent with more detailed structural mechanics analysis of a low density, open-cell, Kelvin foam with tetrakaidecahedral cells and struts with a Plateau border shape (Warren, 1997). FE simulations of Kelvin foam lead to prediction of $\alpha = 0.98$ and comparison with experimental data suggests that $\alpha = 1$ can be used for a wide variety of open-cell foams. By contrast, application of Gibson-Ashby's equation for porosity dependence of elastic modulus for closed-cell foams is limited due to difficulties in determination of fraction of solid contained in the cell edges. FE simulations of a unit tetrakaidecahedral closed-cell with flat faces give $\varphi = 0.32$ for relative densities less than 0.2 (Simone, 1998). For such low relative densities, the second linear density term dominates, implying that cell face stretching is the more significant mechanism of deformation in closed-cell foams. Similar simulations on tetrakaidecahedral closed-cells and Weaire-Phelan closed-cells give $\varphi = 0.311$ (Kraynik, 1999). Fitting using these values give optimal constant equal to approximately 1.5 but, as was observed at exponential-law models, comparison of fitting curve and numerical results shows diametrically different evolution of the fit model. If one lets fraction of solid contained in the cell edges vary freely as a additional fitting parameter, master curve fits the actual modulus-porosity dependence well. Yet, optimal fit model values are $C = 2.14$ and $\varphi = 1.02$ which is nonreasonable value for closed-cell foams corresponding to all of the foam's solid contained in the cell edges (Fig. 6).

Tab. 4: Fit parameters determined for the master curve of the porosity dependence of the elastic modulus of Alporas and Optinium metal foams using Gibson-Ashby's models

		Alporas		Optinium	
Structure	Fit model	α	η	C	φ
Open-cell	$E_{\mathrm{r}} = \alpha \left(\frac{\rho_{\mathrm{f}}}{\rho_0}\right)^{\eta}$	1.78	2.82	–	–
Closed-cell	$E_{\mathrm{r}} = C\varphi^2 \left(\frac{\rho_{\mathrm{f}}}{\rho_0}\right)^2 + C(1-\varphi)\left(\frac{\rho_{\mathrm{f}}}{\rho_0}\right)$	–	–	1.03	2.14

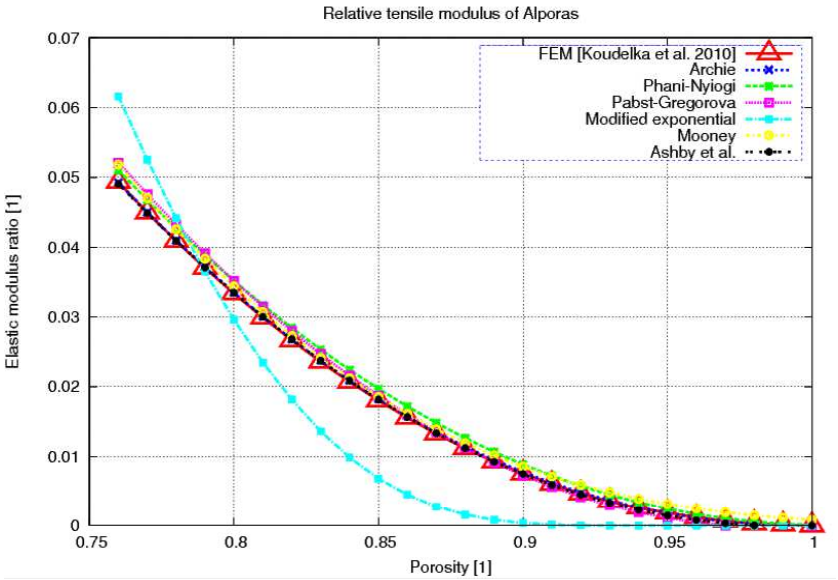

Fig. 5: Evolution of relative elastic modulus plotted against porosity for Alporas showing master curves determined by fitting using equations listed in Tabs. 3 and 4

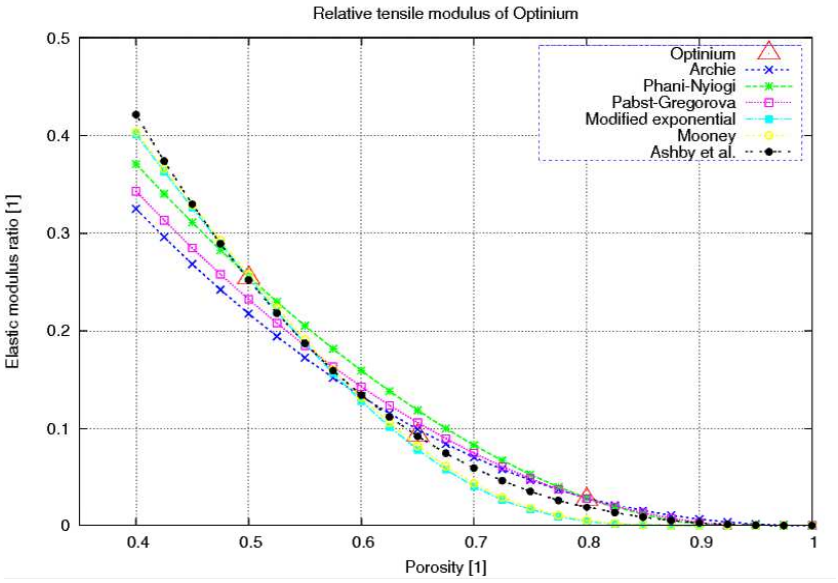

Fig. 6: Evolution of relative elastic modulus plotted against porosity for Optinium showing master curves determined by fitting using equations listed in Tabs. 3 and 4

6. Conclusion

Several exponential- and power-law equations have been introduced in order to obtain a master curve characterizing as precisely as possible the general trend of the porosity dependence of the elastic moduli. Predictions given by these mathematical models have been applied to mechanical behavior of Alporas closed-cell aluminium foam and Optinium open-cell titanium biocompatible foam. Based on the concept of intrinsic elastic moduli a brief overview of modulus-porosity relations has been given including exponential and power-law expressions as well as Gibson-Ashby's relations for both types of studied foams, among them also the relations with critical porosity. It has been shown that for materials with microstructure types investigated here, both the modified exponential relation and Mooney type relation provide unsatisfactory predictions of the actual porosity dependence of Alporas. Power-law models proved to be well suitable for determination of modulus-porosity dependence of both studied foams whereas Gibson-Ashby's semi-empirical equations are only suitable for predictions of modulus-porosity dependence of

Optinium. From the view of computational resources, modulus-porosity predictions assessed by fitting of aforementioned equations is incomparably simpler than performing sets of FE simulations - irrespective of direct or indirect microstructural modeling. Experimental verification of modulus-porosity predictions implies custom manufactured samples of cellular materials as both Alporas and Optinium foams are produced only at 90 % and 65 % levels of porosity respectively.

Acknowledgment

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