18th International Conference
ENGINEERING MECHANICS 2012
Svratka, Czech Republic, May 14 – 17, 2012

pp. 383–389
Paper **#225**

EVALUATION OF WEDGE-SPLITTING TEST RESULTS FROM QUASI-BRITTLE PRISMATIC SPECIMENS USING THE DOUBLE-*K* FRACTURE MODEL

I. Havlíková*, T. Pail, V. Veselý***, D. Vořechovská****, Z. Keršner*****

Abstract: . *The fracture-mechanical parameter values of concrete, a quasi-brittle composite material, are determined from records of experiments on specimens with stress concentrators. One of the fracture models applicable to concrete is the double-K model. This model combines the concept of cohesive forces acting on the effective crack increment with a criterion based on the stress intensity factor. The outputs of the model are critical crack tip opening displacement and fracture toughness values, including the initiation stress intensity factor value corresponding to the beginning of stable crack propagation. In this paper, a method of calculation by means of the double-K fracture model is verified using published data and the results of a pilot wedge-splitting test performed by the authors.*

Keywords: ***Double-K fracture model, concrete, prismatic specimen, wedge-splitting test.***

1. Introduction

Cement-based composites are one of the most widely used building materials. Concrete may be classified as a so-called quasi-brittle material. Studying the mechanical response of specimens made of such composites under static and dynamic/fatigue loading is complicated due to their highly nonlinear nature. Numerical tools for modelling both elastic (elastic-plastic) behaviour and also the fracture process are commonly used to predict or assess the response of structures fabricated from quasi-brittle materials. Such tools − often based on the finite element method (Červenka et al., 2007) or physical discretization of the continuum (Frantík, 2007) − are usually equipped by exploiting a type of nonlinear fracture model simulating the cohesive nature of cracking of quasi-brittle material (Bažant & Planas, 1998; Karihaloo, 1995; Shah et al., 1995). The parameters of this fracture model are determined from records of fracture tests; this is carried out either using evaluation methods built on the principle of the used non-linear fracture model, e.g. the work of fracture method (RILEM, 1985) or the size effect method (RILEM, 1990), or using inverse analysis with the possible application of advanced identification methods (Štafa & Frantík, 2010).

The utilization of existing methods for the evaluation of test records can result in the obtaining of fracture parameter values influenced by both the size and shape of the test specimen and the test geometry (the boundary conditions of the test). Such parameters cannot be used as relevant inputs to an analysis using the above-mentioned numerical tools. A similarly distorted description of the fracture may be indirectly caused by utilization of the methods for evaluation of the fracture model parameters – through the identification methods used – if this procedure is applied to the results of only one type of test and specimen size/shape. The effects of the specimen's size/geometry/free boundaries directly affect the recorded load–deflection or the load–crack mouth opening displacement diagram by means of which the inverse analysis is carried out. Note that both groups of methods for determination of the parameters of quasi-brittle fracture models have been studied by the authors' team for several years (see e.g. Veselý et al., 2007, 2009, 2010, and 2011).

* Ing. Ivana Havlíková: Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics, Veveří 331/95; 602 00, Brno; CZ, e-mail: havlikova.i@fce.vutbr.cz

** Ing. Tomáš Pail: ditto, e-mail: pail.t@fce.vutbr.cz

*** Ing. Václav Veselý, Ph.D.: ditto, e-mail: vesely.v1@fce.vutbr.cz

**** Ing. Dita Vořechovská, Ph.D.: ditto, e-mail: vorechovska.d@fce.vutbr.cz

***** doc. Ing. Zbyněk Keršner, CSc.: ditto, e-mail: kersner.z@fce.vutbr.cz

Research into the above-mentioned effects of the characteristics of the specimen or the test has been the subject of considerable attention in recent decades. Various methods are used for the determination of the characteristics of fracture models for concrete test geometries on notched specimens; the three-point bending of notched beams or wedge-splitting of compact notched specimens (Brühwiler & Wittmann, 1990; Karihaloo, 1995) are among the most common. The model referred to as the "double *K*" (double-*K* or double-*G* – see Reinhardt & Xu, 1999, Xu & Reinhardt, 1999a, b, c; Xu et al., 2003, Xu et al. 2006, Zhao et al., 2007, Xu & Zhang, 2008; Kumar & Bara, 2009; Zhang & Xu, 2011) is similar in this respect. In principle, this model combines the concept of cohesive forces acting on the faces of the fictitious (effective) crack increment with a criterion based on the stress intensity factor (SIF). This model can determine the critical crack tip opening displacement and the fracture toughness and is capable of describing different levels of crack propagation: an initiation part, which corresponds to the beginning of stable crack growth (at the level of reaching the stress intensity factor, $K_{Ic,ini}$), and a part featuring unstable crack propagation (after reaching the unstable fracture toughness, $K_{Ic,un}$).

In this paper, a selected method of calculation exploiting the double-*K* fracture model parameters is verified using published data (Zhang & Xu, 2011). Subsequently, it is employed in processing the results of the authors' own pilot wedge-splitting test performed on a prismatic concrete specimen. Note that the shape function of the wedge-splitting test specimen used in the evaluation procedure was prepared from data published in Seitl et al. (2011).

2. Evaluation of the wedge-splitting test on concrete specimens

The geometry of a prismatic-shaped specimen for use in wedge-splitting tests (WST) is shown in Fig 1, where D is specimen depth, $2H$ is specimen width, B is specimen thickness and a_0 is the initial notch length. A sketch of the loading force decomposition is shown in Fig. 1 right and consists of applied vertical load P_V, normal reaction N, friction force F_f, horizontal force applied on the specimen P_H and wedge angle θ.

The horizontal splitting force P_H can be calculated as:

$$P_H = \frac{P_V}{2\tan\theta} \tag{1}$$

where $\theta = 15°$ in our case.

The execution of this test on a selected concrete specimen is shown in Fig. 2. The input data are summarized in Tab. 1 for two tested concrete specimens: i) data from the literature (Zhang & Xu, 2011), denoted as Specimen 1, and ii) data from the authors' own afore-mentioned pilot wedge-splitting experiment (Specimen 2).

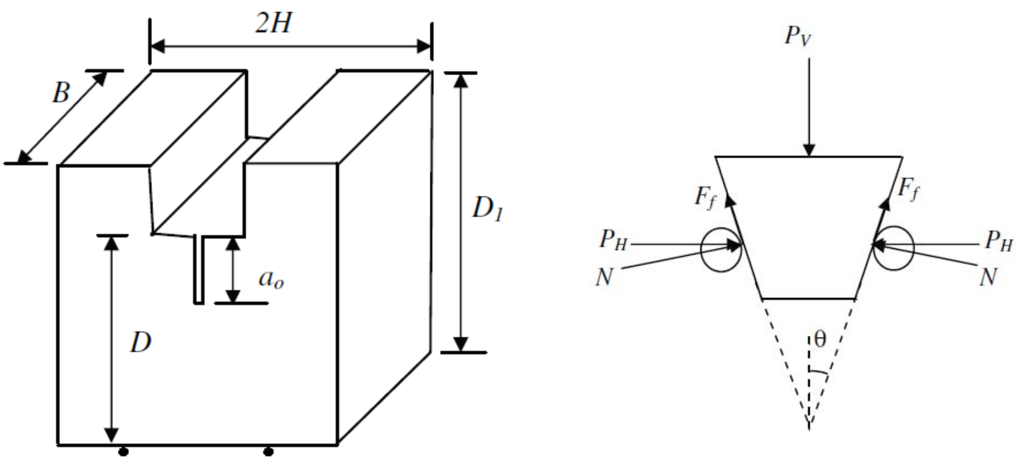

Fig. 1: Specimen geometry and force diagram.

Fig. 2: Wedge-splitting test configuration and a detail of a cracked concrete specimen (photo by Táňa Holušová).

Tab. 1: Input parameters.

Parameter	Symbol	Unit	Specimen 1	Specimen 2
Depth	D	mm	200	130
Width	$2H$	mm	200	150
Thickness	B	mm	199	70
Notch depth	a_0	mm	80	25
Thickness of holder	H_0	mm	2	5
Load from the linear part	P_i	N	7856	2799
Crack mouth opening displacement for P_i	$CMOD_i$	mm	0.026	0.117
Maximum load	$P_{\max}$	N	10023	4443
Crack mouth opening displacement for $P_{\max}$	$CMOD_c$	mm	0.0898	0.2386

2.1. Calculation of double-*K* fracture parameters

The analytical method for WST was developed (Xu & Reinhardt, 1999c) as an alternative to the experimental approach (Xu & Reinhardt, 1999a) used to assess double-*K* fracture toughness parameters. This procedure is based on a linear asymptotic superposition assumption (Xu & Reinhardt, 1999b) and requires numerical investigation of the cohesive toughness $K_{\mathrm{I}c,c}$, which represents the growth of unstable fracture toughness $K_{\mathrm{I}c,un}$ above the level of the initiation fracture toughness $K_{\mathrm{I}c,ini}$ due to cohesive stress transfer in the fictitious crack zone. This relation is given by the equation:

$$K_{\mathrm{I}c}^{un} = K_{\mathrm{I}c}^{ini} + K_{\mathrm{I}c}^{c} \tag{2}$$

and can be explained as an increase in the crack resistance caused by aggregates interlocking in the fracture process zone (FPZ) located in front of the stress-free crack tip. Once the cohesive toughness $K_{\mathrm{I}c,c}$ is solved analytically, the value of $K_{\mathrm{I}c,ini}$ can be evaluated by Eq. (2) and therefore it is only necessary to obtain a smaller number of measured parameters from a recorded *P–CMOD* diagram in comparison with the experimental approach.

2.1.1. Determination of the unstable fracture toughness $K_{\mathrm{I}c,un}$

The unstable fracture toughness $K_{\mathrm{I}c,un}$ is defined as the critical stress intensity factor created by the maximum load $P_{\max}$ at the effective crack tip and can be expressed as the resistance to unstable crack propagation. To evaluate this parameter one can use the following linear elastic fracture mechanics (LEFM) formula (Xu & Reinhardt, 1999a, b, c):

$$K_{\mathrm{I}c}^{un} = \frac{P_{\max}}{BD}\sqrt{D}F\left(\alpha_c = \frac{a_c + H_0}{D + H_0}\right) \tag{3}$$

$$F(\alpha_c) = \frac{(2+\alpha_c)(0.886 + 4.64\alpha_c - 13.32\alpha_c^2 + 14.72\alpha_c^3 - 5.6\alpha_c^4)}{(1-\alpha_c)^{3/2}} \tag{4}$$

where the maximum load $P_{\max}$ and the critical effective crack length a_c are the input parameters readily obtained from the measured *P–CMOD* diagram, H_0 represents the thickness of the clip extensometer holder and B, D are the specimen dimensions according to Fig. 1.

To evaluate Eq. (4) it is necessary to evaluate the critical effective crack length a_c at the unstable loading condition ($P_{\max}$) by solving the following nonlinear equation proposed by Murakami (1987):

$$CMOD_c = \frac{P_{\max}}{BE}V\left(\alpha_c = \frac{a_c + H_0}{D + H_0}\right) \tag{5}$$

$$V(\alpha_c) = \left[2.163 + 12.219\alpha_c - 20.065\alpha_c{}^2 - 0.9925\alpha_c{}^3 + 20.609\alpha_c{}^4 - 9.9314\alpha_c{}^5\right] \times \left(\frac{1+\alpha_c}{1-\alpha_c}\right)^2 \tag{6}$$

where $CMOD_c$ is the critical crack mouth opening displacement due to the maximum load $P_{\max}$ and E is Young's modulus, which can be determined either through a compressive cylinder test or through the calculation method given in the RILEM recommendation (RILEM, 1990).

2.1.2. Determination of cohesive fracture toughness $K_{\mathrm{I}c,c}$

The cohesive fracture toughness can be described as the energy dissipated by cohesive forces distributed along the critical fictitious crack zone. To calculate the cohesive fracture toughness $K_{\mathrm{I}c,c}$, bilinear distribution of the cohesive stress is assumed. Generally, the cohesive stress function $\sigma(w)$ expresses the relation between the cohesive stress σ and the fictitious crack opening displacement w. Four parameters are necessary to uniquely define this function, namely the tensile strength f_t, the cohesive stress σ_s, the crack opening displacement w_s at the slope discontinuity of the bilinear curve, and the critical crack opening displacement w_0 at which the cohesive stress σ drops to zero. These four parameters can be determined using expressions given e.g. in the CEB-FIP design code. The following parameter values of the bilinear cohesive stress function were used in this paper: f_t = 3.546 MPa, σ_s = 0.599 MPa, w_s = 0.02363 mm and w_0 = 0.16177.

At the maximal load $P_{\max}$ the crack becomes unstable and the corresponding opening at the tip of the stress-free crack (the origin of the fictitious crack) is termed the critical crack tip opening displacement, $CTOD_c$. Depending on the value of $CTOD_c$, two cases exist for the cohesive stress distribution along the fictitious crack length at the critical state (Zhang & Xu, 2011).

Case I is supposed to hold for $CTOD_c < w_s$, and the corresponding distribution of cohesive stress in the FPZ is linearly distributed according to the formula:

$$\sigma_1(x) = \sigma(CTOD_c) + \frac{x - a_0}{a_c - a_0}\left(f_t - \sigma(CTOD_c)\right) \tag{7}$$

where $\sigma(CTOD_c)$ is the cohesive stress at the tip of the initial crack with the length a_0 at the critical state. This can be obtained using Eq. (8) according to the cohesive stress function:

$$\sigma(CTOD_c) = \sigma_s(w_s) + \frac{w_s - CTOD_c}{w_s}\left(f_t - \sigma_s(w_s)\right) \tag{8}$$

Case II is supposed to hold when $w_s < CTOD_c < w_0$, and in this case, the corresponding cohesive stress distribution is bilinear and can be expressed for $a_0 \le x \le a_s$ as:

$$\sigma_2(x) = \sigma(CTOD_c) + \frac{x - a_0}{a_s - a_0}\left(\sigma_s(w_s) - \sigma(CTOD_c)\right) \tag{9}$$

or for $a_s \leq x \leq a_c$ this can be rewritten as:

$$\sigma_3(x) = \sigma_s(w_s) + \frac{x - a_s}{a_c - a_s}\left(f_t - \sigma_s(w_s)\right) \tag{10}$$

where, in both equations, $\sigma_s(w_s)$ is the cohesive stress corresponding to the crack opening displacement w_s, f_t is the tensile strength, a_0 is the initial notch depth (initial crack length) and a_s is the effective crack length corresponding to $CTOD = w_s$, which can be obtained by solving the following nonlinear equation:

$$w_s = CMOD_c\left(\left(1 - \frac{a_s}{a_c}\right)^2 + \left(1.081 - 1.149\frac{a_c}{D}\right)\left(\frac{a_s}{a_c} - \left(\frac{a_s}{a_c}\right)^2\right)\right)^{1/2} \tag{11}$$

In Eq. (11) $CMOD_c$ is the crack opening mouth displacement at the critical point, D is the specimen depth according to Fig. 1 and a_c is the critical effective length mentioned above.

Once the distribution of cohesive stress in the FPZ is known, one can calculate the cohesive fracture toughness $K_{\mathrm{Ic},c}$ by integrating the following expression:

$$K_{Ic}^c = \int_{a_0/a_c}^{1} \frac{2\sqrt{a_c}}{\sqrt{\pi}}\sigma(U)F(U,V)dU \tag{12}$$

where $$F(U,V) = \frac{3.52(1-U)}{(1-V)^{3/2}} - \frac{4.35 - 5.28U}{(1-V)^{1/2}} + \left(\frac{1.30 - 0.30U^{3/2}}{\left(1-U^2\right)^{1/2}} + 0.83 - 1.76U\right)\left[1 - (1-U)V\right] \tag{13}$$

and where the substitutions $U=x/a_c$, $V=a_c/D$ are used: $\sigma(U)$ is the cohesive stress according to formulas (7, 9, 10) and $F(U,V)$ is the characteristic Green function. To evaluate Eq. (12) a special numerical integration method is necessary to handle the singularity at the integral boundary.

2.2. Results of calculations

This paper is primarily focused on the functionality of the double-*K* fracture model. The results of the calculations of the values of selected parameters of this model can be found in Tab. 2. The values in parentheses for "Specimen 1" are taken from Zhang & Xu (2011).

Tab. 2: The resulting values.

Parameter	Symbol	Unit	Specimen 1	Specimen 2
Critical effective crack length	a_c	mm	119.59 (119.84)	31.62
SIF (unstable fracture)	K_{Ic}^{un}	MPa·m$^{1/2}$	1.557 (1.557)	0.918
SIF (cohesive toughness)	K_{Ic}^{c}	MPa·m$^{1/2}$	0.868 (0.885)	0.141
SIF (initiation toughness)	K_{Ic}^{ini}	MPa·m$^{1/2}$	0.689 (0.672)	0.777
Critical crack tip opening	$CTOD_c$	mm	0.03992 (0.03938)	0.101

3. Conclusions

The concept of the double-*K* fracture model is not currently used in the Czech Republic (except for in a study reported in the paper Keršner & Matesová (2001), which focused on three-point bending of notched prismatic concrete specimens), but the worldwide scientific and professional public interest in this model has recently been increasing. A technical committee (TC) of the RILEM international fellowship (International Union of Laboratories and Experts in Construction Materials, Systems and Structures), related to the double-*K* concept and headed by Prof. Shilang Xu and Dr. Shailendra Kumar, was established in October 2011. The task of the new technical committee is to examine the crack stability criteria related to the double-*K* fracture model, whose use has been shown to be independent of the sample size, and on the basis of further extensive experimental and numerical verification, to attempt to prepare a RILEM document which will be usable for international standardization activities in the field of concrete and concrete structures. Note that the double-*K* model criteria are used to assess the safety of large concrete structures (dams) in Chinese national standard No. DL/T5332-2005, issued in 2005.

In this paper, the utilization and results of a method for calculation of the double-*K* fracture model parameters were shown. The procedure was programmed and verified using published data and the results of the authors' own pilot wedge-splitting test. The applicability of the used approach was demonstrated via the comparison of the evaluated results of WST experiments on prismatic-shaped concrete specimen with published data.

Acknowledgement

This outcome has been achieved with the financial support of the Czech Scientific Foundation, project No. P104/11/0833, and of the Ministry of Education of the Czech Republic, project No. MSM0021630519.

References

Bažant, Z. P. & Planas, J. (1998) *Fracture and Size Effect in Concrete and other Quasibrittle Materials.* CRC Press, Boca Raton, Florida.

Červenka, V., Jendele, L. & Červenka, J. (2007) *ATENA Program documentation – Part 1: theory*. Červenka Consulting, Prague.

Brühwiler, E. & Wittmann, F. H. (1990) The wedge splitting test, a new method of performing stable fracture mechanics tests. *Engineering fracture mechanics*, Vol. 35, 117–125.

Frantík, P. (2007) FyDiK application, http://www.kitnarf.cz/fydik, 2007–2011.

Karihaloo, B. L. (1995) *Fracture mechanics of concrete*. Longman Scientific & Technical, New York.

Keršner, Z. & Matesová, D. (2001) Jak funguje model „dvojí K“ u betonových vzorků. Sborník *Problémy lomové mechaniky*, Brno, 60–63, ISBN 80-214-1906-7. In Czech.

Kumar, S. & Barai, S. V. (2009) Equivalence between stress intensity factor and energy approach based fracture parameters of concrete. *Engineering Fracture Mechanics.* 76:1357–1372.

Murakami, Y. (1987) *Stress Intensity Factor Handbook I, II, III*. Pergamon Press, Oxford.

Reinhardt, H. W. & Xu, S. (1999) Crack extension resistance based on the cohesive force in concrete. *Engineering Fracture Mechanics.* 64:563–587.

RILEM Committee FMT 50 (1985) Determination of the fracture energy of mortar and concrete by means of three-point bend test on notched beams. *Mater. Struct*., 18, 285–290.

RILEM Committee FMT 89 (1990) Size effect method for determining fracture energy and process zone size of concrete. *Mater. Struct*., 23, 461–465.

Seitl, S., Veselý, V. & Řoutil, L. (2011) Two-parameter fracture mechanical analysis of a near-crack-tip stress field in wedge splitting test specimen. *Computers and Structures*, 89, 1852–1858

Shah, S. P., Swartz, S.E. & Ouyang, Ch. (1995) *Fracture mechanics of structural concrete: applications of fracture mechanics to concrete, rock, and other quasi-brittle materials*. John Wiley & Sons, Inc., New York.

Štafa, M. & Frantík, P. (2010) Model for high precision approximation of load deflection diagrams. In: Proc. of conference *Engineering Mechanics 2010*, Svratka. ISBN 978-80-87012-26-0.

Veselý, V., Frantík, P. & Keršner, Z. (2009) Cracked volume specified work of fracture. In B.H.V. Topping, L.F. Costa Neves and R.C. Barros (eds), *Proc. of 12th Int. Conf. on Civil, Structural and Environmental Engineering Computing*, Funchal, 2009. Civil-Comp Press.

Veselý, V., Keršner, Z., Němeček, J., Frantík, P., Řoutil, L. & Kucharczyková, B. (2010) Estimation of fracture process zone extent in cementitious composites. *Chemické Listy*, vol. 104 (2010), s382–s385. ISSN 0009-2770.

Veselý, V., Řoutil, L. & Keršner, Z. (2007) Structural geometry, fracture process zone and fracture energy. In: Carpinteri Al., Gambarova, P., Ferro, G., Plizzari, G. (eds.), *Proc. of Fracture Mechanics of Concrete and Concrete Structures (FraMCoS-6)*, Catania, Italy, 2007. Taylor & Francis/Balkema, vol. 1, 111–118.

Veselý, V., Řoutil, L. & Seitl, S. (2011) Wedge-Splitting Test – Determination of Minimal Starting Notch Length for Various Cement Based Composites. *Key Engineering Materials*, Vol. 452-453, 77–80.

Xu, S. & Reinhardt, H. W. (1999a) Determination of double-K criterion for crack propagation in quasi-brittle fracture, Part I: Experimental investigation of crack propagation. *International Journal of Fracture.* 98:111–149.

Xu, S. & Reinhardt, H. W. (1999b) Determination of double-K criterion for crack propagation in quasi-brittle fracture, Part II: Analytical evaluating and practical measuring methods for three-point bending notched beams. *International Journal of Fracture.* 98:151–177.

Xu, S. & Reinhardt, H. W. (1999c) Determination of double-K criterion for crack propagation in quasi-brittle fracture, Part III: Compact tension specimens and wedge splitting specimens. *International Journal of Fracture.* 98:179–193.

Xu, S., Reinhardt, H. W., Wu, Z. & Zhao, Y. (2003) Comparison between the double-K fracture model and the two parameter fracture model. *Otto-Graf-Journal.* Vol. 14, 131–158.

Xu, S. & Zhang, X. (2008) Determination of fracture parameters for crack propagation in concrete using an energy approach. *Engineering Fracture Mechanics.* 75:4292–4308.

Xu, S., Zhao, Y. & Wu, Z. (2006) Study on the average fracture energy for crack propagation in concrete. *Journal of Materials in Civil Engineering.* 817–824.

Zhang, X. & Xu, S. (2011) A comparative study on five approaches to evaluate double-K fracture toughness parameters of concrete and size effect analysis. *Engineering Fracture Mechanics.* 78:2115–2138.

Zhao, Y., Xu, S. & Wu, Z. (2007) Variation of fracture energy dissipation along evolving fracture process zones in concrete. *Journal of Materials in Civil Engineering.* 625–633.