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STRUCTURES WITH UGLI IMPERFECTIONS

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Abstract: *Derivation of the basic formulae for determination of the flexural buckling resistance of frames with members with non-uniform cross-sections and/or non-uniform axial compression forces. Similar formulae given in EN 1993-1-1 (2005) are limited to frames with uniform cross-sections and compression forces. Detailed description of the procedure of iterative calculation (Baláž, I., 2008). Graphical interpretation of the method proposed by the authors and numerical examples for members with uniform cross-sections and uniform axial compression forces.*

Keywords: ***stability, metal structures, flexural buckling resistance, imperfections in the form of first buckling mode.***

1. Introduction

The proposed procedure was the first time published in Baláž, I. (2008) and was verified by calculating of several numerical examples. The procedure is based on Chladný`s method. Derivation of the basic formulae used in this paper differ from the ones published by Chladný in publication Baláž, I. et al. (2007, 2010). The way of calculation proposed by Baláž, I. (2008) was used also in PhD thesis written by Kováč, M. (2010).

Prof. Chladný developed his method with the aim to derive a formula for the lateral forces acting on U-frames of open truss bridges in Chladný, E. (1958) and in Chladný, E. (1974). He showed there the importance of the curvature of the initial imperfection. The results were used in Czechoslovak Bridge Standard ČSN 73 6205 (1984), cl. 39. See also Chladný, E. (1998).

In 2000 proposed Prof. Chladný his method in more generalized form for Eurocode 3, and it was the first time accepted in draft prEN 1993-1-1 (5 June 2002). His contribution is acknowledged in publication (Sedlacek, G. et al., 2004). See there item [52]. Chladný derived the formula for $e_{0,\mathrm{d}}$ and is the author of the method given in EN 1993-1-1 (2005), 5.3.2(11). He later generalized it also for non-uniform cross-sections and non-uniform compression forces. This generalization is used in STN EN 1993-1-1/NA (2007) and in EN 1999-1-1 (2007), 5.3.2(11). Chladný applied his method in the design of bridges in practice, e.g. in design of basket handle arch type Apollo bridge in Bratislava, Pentele bridge in Dunaújváros and in investigations of continuous truss bridges. He further modified his method to be convenient for basket handle arch type bridges in the National Annex STN EN 1993-2/NA (2009). Chladný described details of his method and published numerical examples in Baláž, I. – Ároch, R. – Chladný, E. – Kmeť, S. – Vičan, J. (2007, 2010).

2. Flexural buckling resistance of frames with non-uniform members and non-uniform compression normal forces

Flexural buckling resistance of the frame, which consists of members with variable cross-sections, with any boundary conditions, supports and/or variable foundation and under variable axial forces may be verified by the following condition

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$$\left| \frac{N_{\mathrm{Ed}}(x)}{N_{\mathrm{Rd}}(x)} + \frac{M_{\mathrm{Ed,ugli}}^{\mathrm{II}}(x)}{M_{\mathrm{Rd}}(x)} \right|_{\max} \le 1 \tag{1}$$

where

$N_{\mathrm{Ed}}(x)$ is the axial force distribution, positive if compression, which is effect of the actions. The design values of the axial forces may be calculated by the 1st order theory,

$M_{\mathrm{Ed,ugli}}^{\mathrm{II}}(x)$ is the bending moment distribution, which is the result of axial forces acting in members of frame having "unique global and local initial imperfection" („ugli" imperfection). The design values of this bending moment shall be calculated by the 2nd order theory. The „ugli" imperfection is an equivalent geometrical imperfection, which purpose is to cover in numerical model all imperfections (geometrical and structural) of real structure.

$N_{\mathrm{Rd}}(x)$ is the distribution of the axial force resistance depending on the cross-section class,

$M_{\mathrm{Rd}}(x)$ is the distribution of the bending moment resistance depending on the cross-section class.

The characteristics relating to critical cross-section, which is the cross-section relevant for assessment of flexural buckling resistance of the frame, are below denoted by index „m". The most onerous condition (1) occurring in critical cross-section „m", may be then rewritten in the form

$$\frac{N_{\mathrm{Ed,m}}}{N_{\mathrm{Rd,m}}} + \frac{M_{\mathrm{Ed,ugli,m}}^{\mathrm{II}}}{M_{\mathrm{Rd,m}}} \le 1 \tag{2}$$

The "ugli" imperfection is defined as follows

$$\eta_{\mathrm{ugli}}(x) = \eta_{0,\mathrm{ugli,m}} \eta_{\mathrm{cr}}(x) \tag{3}$$

where

$\eta_{\mathrm{cr}}(x)$ is the first elastic critical buckling mode, with the amplitude $|\eta_{\mathrm{cr}}(x)|_{\max} = 1$.

$\eta_{0,\mathrm{ugli,m}}$ is the amplitude of „ugli" imperfection depending on characteristics of critical cross-section „m". Index „0" will in this paper indicate that a value is amplitude of a deflection. The amplitude of „ugli" imperfection may be determined from the condition requiring the following: the critical member of the frame, when under compression axial force, should have the same flexural buckling resistance as its "generalized equivalent member" („gem"). The „gem" is the member simply supported on its ends, having the same cross-section properties (EI_{m}, A_{m}) and axial force ($N_{\mathrm{Ed,m}}$) as the critical member of the frame in its critical cross-section „m", and having such buckling length L_{cr}, that its elastic critical axial force is the same as the elastic critical axial force $N_{\mathrm{cr,m}}$ of the critical member of the frame in its critical cross-section „m".

The „ugli" imperfection amplitude depending on the characteristics of the critical cross-section „m" is then defined by

$$\eta_{0,\mathrm{ugli,m}} = \frac{N_{\mathrm{cr,m}} e_{0,\mathrm{d,m}}}{EI_{\mathrm{m}} \left| \eta''_{\mathrm{cr,m}} \right|} = \alpha_{\mathrm{cr}} \frac{N_{\mathrm{Ed,m}} e_{0,\mathrm{d,m}}}{\left| M_{\eta\mathrm{cr,m}} \right|} \tag{4}$$

$$e_{0,\mathrm{d,m}} = e_{0,\mathrm{k,m}} \frac{1 - \dfrac{\chi_\mathrm{m} \bar{\lambda}_\mathrm{m}^2}{\gamma_{\mathrm{M1}}}}{1 - \chi_\mathrm{m} \bar{\lambda}_\mathrm{m}^2} \tag{5}$$

$$e_{0,\mathrm{k,m}} = \alpha_\mathrm{m} \left(\bar{\lambda}_\mathrm{m} - 0.2 \right) \frac{M_{\mathrm{Rk,m}}}{N_{\mathrm{Rk,m}}}, \quad \text{for } \bar{\lambda}_\mathrm{m} > 0.2 \tag{6}$$

where

$\left| M_{\eta\mathrm{cr,m}} \right|$ is the absolute value of the fictitious bending moment at the critical cross-section „m“, due to $\eta_\mathrm{cr}(x)$,

$N_{\mathrm{Ed,m}}$ is the design value of compression axial force at the critical cross-section „m“, positive if compression,

$M_{\mathrm{Rk,m}}$ is the characteristic value of bending moment resistance of the critical cross-section „m“,

$N_{\mathrm{Rk,m}}$ is the characteristic value of axial force resistance of the critical-cross section „m“,

$e_{0,\mathrm{d,m}}$, $e_{0,\mathrm{k,m}}$ are the design (index „d“) and characteristic (index „k“) values of amplitude of "local initial" („li“) imperfection of the "gem" depending on the characteristics of the critical cross-section „m“. It can be easily shown, that „li“ imperfection is used when analysis of individual member is done,

α_m is the imperfection factor for the critical cross-section „m“ and the relevant buckling curve, see Table 6.1 and Table 6.2 in EN 1995-1-1 (2005),

γ_{M1} is the partial factor, which should be applied to the various characteristic values of resistance of members to instability,

$$\bar{\lambda}_\mathrm{m} = \sqrt{\frac{N_{\mathrm{Rk,m}}}{N_{\mathrm{cr,m}}}} \tag{7}$$

is the relative slenderness of the structure, relating to the critical-cross section „m“,

χ_m is the reduction factor depending on the relevant imperfection factor α_m and the relative slenderness $\bar{\lambda}_\mathrm{m}$, see 6.3.1 in EN 1995-1-1 (2005),

$N_\mathrm{cr}(x)$ is the distribution of elastic critical force,

α_cr is the minimum force amplifier for the values of the axial force configuration $N_\mathrm{Ed}(x)$ in members to reach the values of elastic critical force configuration $N_\mathrm{cr}(x)$. For the given frame, α_cr is constant. The ratio $\alpha_\mathrm{cr} = N_\mathrm{cr}(x)/N_\mathrm{Ed}(x)$ gives for all cross-sections „x“ the same numerical value.

The location of the critical cross-section „m“ is generally not known, because it depends on the location of maximum of the sum of two functions in the left side of the condition (1). The value of the second term of the sum in (1) depends on the characteristics of the critical cross-section „m“. The location of the maximum of the first function in (1): $N_\mathrm{Ed}(x)/N_\mathrm{Rd}(x)$ usually does not coincide with the location of maximum of the second function in (2): $M_{\mathrm{Ed,ugli}}^{\mathrm{II}}(x)/M_\mathrm{Rd}(x)$, which is given by the location of the maximum of the function $\left|\eta''_{\mathrm{cr}}(\mathrm{x})/I(x)\right|_{\max}$. Generally it is therefore necessary to use iterative calculation.

In the special case, when $N_{\mathrm{Ed}}(x)/N_{\mathrm{Rd}}(x)$ is constant, the location of critical cross-section „m" is determined by the location of $\left|M^{\mathrm{II}}_{\mathrm{Ed,ugli}}(x)/M_{\mathrm{Rd}}(x)\right|_{\max}$ or $\left|\eta''_{\mathrm{cr}}(\mathrm{x})/I(x)\right|_{\max}$ and when also $EI(x)$ is constant, by the location of $\left|\eta''_{cr}(\mathrm{x})\right|_{\max}$.

Distribution of bending moment $M^{\mathrm{II}}_{\mathrm{Ed,ugli}}(x)$, which is the effect of axial forces acting in members of frame having the „ugli" imperfection $\eta_{\mathrm{ugli}}(x)=\eta_{0,\mathrm{ugli,m}}\eta_{\mathrm{cr}}(x)$, may be calculated in the following way:

1) The first eigen-value α_{cr} and the first buckling mode $\eta_{\mathrm{cr}}(x)$ and its derivates $\eta'_{\mathrm{cr}}(x)$ and $\eta''_{\mathrm{cr}}(x)$ are obtained by numerical methods using a computer program.

2) The „ugli" imperfection amplitude $\eta_{0,\mathrm{ugli,m}}$ depending on the characteristics of the critical cross-section „m" is calculated for the estimated location of the critical cross-section „m"

$$\eta_{0,\mathrm{ugli,m}} = \alpha_{\mathrm{cr}} \frac{N_{\mathrm{Ed,m}} e_{0,\mathrm{d,m}}}{\left|M_{\eta\mathrm{cr,m}}\right|} \tag{8}$$

3) The distribution of the "ugli" imperfection is then

$$\eta_{\mathrm{ugli}}(x) = \eta_{0,\mathrm{ugli,m}}\eta_{\mathrm{cr}}(x) \tag{9}$$

4) The amplitude $\eta_{0,\mathrm{m}}$ of the additive deflection $\eta(x)$, which is the result of axial forces acting in the members of frame with „ugli" imperfection, may be calculated as

$$\eta_{0,\mathrm{m}} = \frac{\eta_{0,\mathrm{ugli,m}}}{\alpha_{\mathrm{cr}} - 1} \tag{10}$$

5) The distribution of the additive deflection $\eta(x)$ is then

$$\eta(x) = \eta_{0,\mathrm{m}}\eta_{\mathrm{cr}}(x) = \frac{\eta_{0,\mathrm{ugli,m}}}{\alpha_{\mathrm{cr}} - 1}\eta_{\mathrm{cr}}(x) \tag{11}$$

6) The distribution of the bending moment $M^{\mathrm{II}}_{\mathrm{Ed,ugli}}(x)$ due to „ugli" imperfection having shape of $\eta_{\mathrm{cr}}(x)$, may be calculated from the formula

$$M^{\mathrm{II}}_{\mathrm{Ed,ugli}}(x) = -EI(x)\eta''(x) = -EI(x)\frac{\eta_{0,\mathrm{ugli,m}}}{\alpha_{\mathrm{cr}} - 1}\eta''_{\mathrm{cr}}(x) = kN_{\mathrm{Ed,m}}e_{0,\mathrm{d,m}}\frac{-EI(x)\eta''_{\mathrm{cr}}(x)}{\left|M_{\eta\mathrm{cr,m}}\right|} \tag{12}$$

where

k is the well known ratio of the bending moment calculated according to the 2nd order theory to the bending moment calculated according to the 1st order theory, which is in the case of using elastic critical buckling mode $\eta_{\mathrm{cr}}(x)$ constant value for the whole frame

$$k = \frac{\alpha_{\mathrm{cr}}}{\alpha_{\mathrm{cr}} - 1} = \frac{1}{1 - \frac{1}{\alpha_{\mathrm{cr}}}} \tag{13}$$

It may be also written

$$M^{\mathrm{II}}_{\mathrm{Ed,ugli}}(x) = kN_{\mathrm{Ed,m}}e_{0,\mathrm{d,m}}\frac{-EI(x)\eta''_{\mathrm{cr}}(x)}{EI_{\mathrm{m}}\left|\eta''_{\mathrm{cr,m}}\right|} = kN_{\mathrm{Ed,m}}e_{0,\mathrm{d,m}}\frac{-EI(x)\eta''_{\mathrm{C,cr}}(x)}{EI_{\mathrm{m}}\left|\eta''_{\mathrm{C,cr,m}}\right|} = $$
$$= kN_{\mathrm{Ed,m}}e_{0,\mathrm{d,m}}\frac{M_{\eta\mathrm{cr}}(x)}{\left|M_{\eta\mathrm{cr,m}}\right|} = M^{\mathrm{II}}_{0,\mathrm{Ed,ugli,m}}\frac{M_{\mathrm{C}\eta\mathrm{cr}}(x)}{\left|M_{\mathrm{C}\eta\mathrm{cr,m}}\right|} \quad (14)$$

where

$$\eta_{\mathrm{C,cr}}(x) = C_0\eta_{\mathrm{cr}}(\mathrm{x}) \quad (15)$$

is the first elastic critical buckling mode with amplitude C_0, which may have any numerical value, and

$$M^{\mathrm{II}}_{0,\mathrm{Ed,ugli,m}} = kN_{\mathrm{Ed,m}}e_{0,\mathrm{d,m}} \quad (16)$$

From (14) it may be seen that, the first elastic critical buckling mode $\eta_{\mathrm{C,cr}}(x)$ with any value of the amplitude C_0 may be used, and not only $\eta_{\mathrm{cr}}(\mathrm{x})$ having $C_0 = 1$, when computing ratio of bending moments

$$M_{\eta\mathrm{cr}}(x)/\left|M_{\eta\mathrm{cr,m}}\right|. \quad (17)$$

7) After the distribution of the function on the left side of the condition (1) is known, the condition (2) may be evaluated and checked, if the location of maximum of this function will coincide with estimated location of the critical cross-section „m“ from the first iteration. If the answer is no, the procedure shall be repeated in an iterative way.

8) If the answer is yes, the condition (2) may be evaluated and the frame verified.

3. Numerical examples

Example 1: Given input values: uniform member with cross-section HE 260 B (ARBED), steel grade S 355, partial safety factor $\gamma_{\mathrm{M1}} = 1.1$, member length $L = 4.6\,\mathrm{m}$, action: axial normal force N_{Ed} equals to the resistance, which means that for N_{Ed} utilization grade $U = 1$. Two cases are investigated:

a) flexural buckling about major axis *y-y* (buckling curve “b”, $\alpha = 0.34$),

b) flexural buckling about minor axis *z-z* (buckling curve “c”, $\alpha = 0.49$).

Boundary conditions are the same in both planes: left end is fixed, right end is simple supported.´

a) Flexural buckling about major axis *y-y* (Figure 1-4)

HE 260 B S 355 $f_y = 355 \cdot MPa$ $\gamma_{M1} = 1.1$ $f_{y.d} = 322.727 \cdot MPa$

$h = 0.26\,m$ $b = 0.26\,m$ buckling curve "b" $\alpha := 0.34$

$A = 11.84 \times 10^3 \cdot mm^2$ $I_y = 149.2 \times 10^6 \cdot mm^4$ $W_{el.y} = 1.148 \times 10^6 \cdot mm^3$

$N_{Rk} = 4.203 \cdot MN$ $N_{Rd} = 3.821 \cdot MN$ $M_{y.Rk} = 407.54 \cdot kN \cdot m$

$N_{Ed} = 3.576 \cdot MN$ $N_{cr} = 29.897 \cdot MN$ $L_{cr} = 3.216\,m$

$\beta = 0.699$ $\varepsilon := \dfrac{\pi}{\beta}$ $\alpha_{cr} := \dfrac{N_{cr}}{N_{Ed}} = 8.36$ $k := \dfrac{1}{1 - \dfrac{1}{\alpha_{cr}}} = 1.136$

$\lambda_m := \sqrt{\dfrac{N_{Rk}}{N_{cr}}} = 0.375$ $\phi := 0.5 \cdot \left[1 + \alpha \cdot (\lambda_m - 0.2) + \lambda_m^{\,2}\right] = 0.6$

$\chi := \dfrac{1}{\phi + \sqrt{\phi^2 - \lambda_m^{\,2}}} = 0.936$ $e_{o.k} := \alpha \cdot (\lambda_m - 0.2) \cdot \dfrac{M_{Rk}}{N_{Rk}} = 5.768 \cdot mm$

$e_{o.d} := e_{o.k} \cdot \dfrac{1 - \dfrac{\chi \cdot \lambda_m^{\,2}}{\gamma_{M1}}}{1 - \chi \cdot \lambda_m^{\,2}} = 5.847 \cdot mm$ $\dfrac{L}{e_{o.d}} = 786.727$

$\eta_{cr}(x) := \dfrac{\left[\left(1 - \cos\left(\dfrac{\varepsilon \cdot x}{L}\right)\right) \cdot \varepsilon + \sin\left(\dfrac{\varepsilon \cdot x}{L}\right) - \dfrac{\varepsilon \cdot x}{L}\right]}{C}$ $C = 6.283$

$\eta2_{cr}(x) := \dfrac{\cos\left(\varepsilon \cdot \dfrac{x}{L}\right) \cdot \dfrac{\varepsilon^3}{L^2} - \sin\left(\varepsilon \cdot \dfrac{x}{L}\right) \cdot \dfrac{\varepsilon^2}{L^2}}{C}$ $x_m = 2.992\,m$

$\eta(x) := \dfrac{\eta_{o.ugli.m}}{\alpha_{cr} - 1} \cdot \eta_{cr}(x)$ $\eta_{o.ugli.m} := \dfrac{\alpha_{cr} \cdot e_{o.d} \cdot N_{Ed}}{E \cdot I \cdot \left|-\eta2_{cr}(x_m)\right|} = 7.981 \cdot mm$

$M_{II}(x) := -E \cdot I \cdot \eta2(x)$

$M_{II}(x_m) = 23.751 \cdot kN \cdot m$ $k \cdot N_{Ed} \cdot e_{o.d} = 23.751 \cdot kN \cdot m$

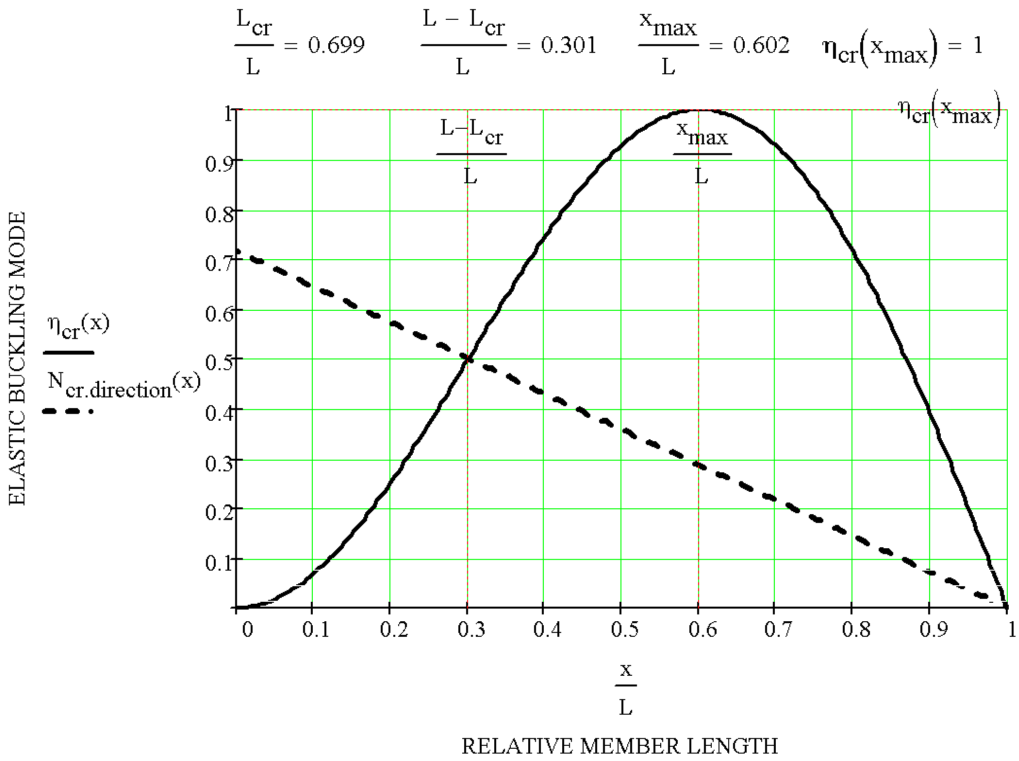


Fig. 1: The first elastic buckling mode $\eta_{cr}(x)$ valid for both cases: a) buckling about y-y and b) buckling about z-z

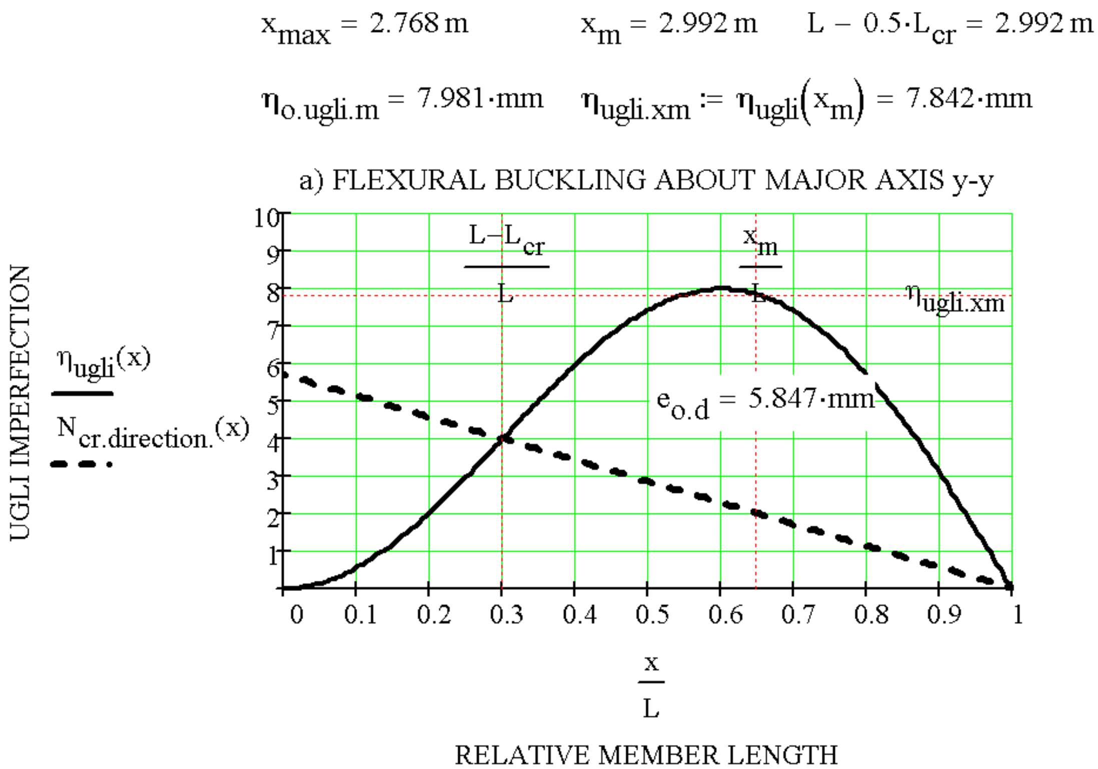


Fig. 2: Uniform global and local initial ("ugli") imperfection valid for buckling about y-y

$\eta_{o.ugli.m} = 7.981 \cdot mm$ $\quad x_{max} = 2.768\,m$ $\quad \eta(x_{max}) = 1.084\,mm$

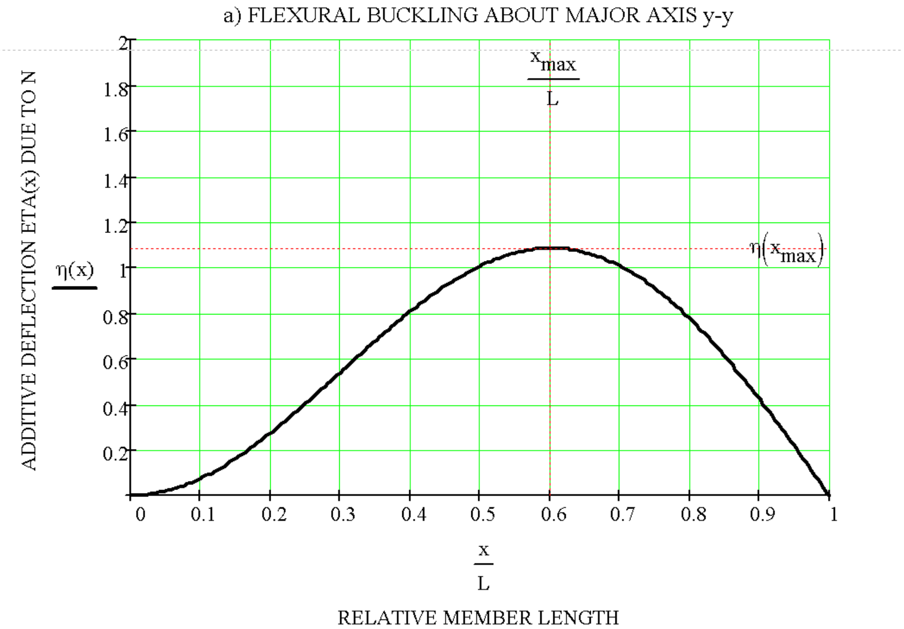


Fig. 3: Additive deflection η(x) due to N_{Ed} for flexural buckling about major axis y-y

$M_{II}(0) = -23.18349 \cdot kN \cdot m$ $\quad N_{Ed} = -3576.136 \cdot kN$ $\quad M_{II}(x_m) = 23.75066 \cdot kN \cdot m$

$\frac{N_{Ed}}{A} + \frac{M_{II}(0)}{W} = -322.233 \cdot MPa$ $\quad x_m = 2.992\,m$ $\quad \frac{N_{Ed}}{A} - \frac{M_{II}(x_m)}{W} = -322.727 \cdot MPa$

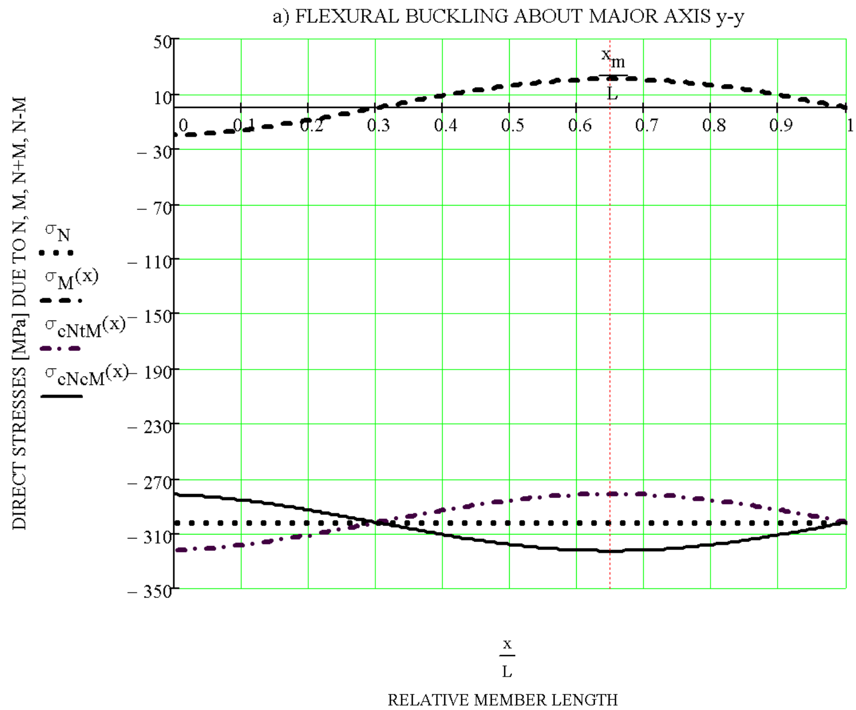


Fig. 4: Direct stresses for flexural buckling about major axis y-y

$U_1 := \frac{N_{Ed}}{N_{Rd}} + \frac{M_{II}(x_m)}{M_{Rd}}$ $\quad U_1 = 1$ $\quad \chi = 0.936$ $\quad f_{y.d} = 322.727 \cdot MP$ $\quad U_2 := \frac{N_{Ed}}{\chi \cdot A \cdot f_{y.d}}$ $\quad U_2 = 1$

b) Flexural buckling about minor axis z-z

HE 260 B S 355 $f_y = 355 \cdot MPa$ $\gamma_{M1} = 1.1$ $f_{y.d} = 322.727 \cdot MPa$

$h = 0.26\,m$ $b = 0.26\,m$ buckling curve "c" $\alpha := 0.49$

$A = 11.84 \times 10^3 \cdot mm^2$ $I_z = 51.35 \times 10^6 \cdot mm^4$ $W_{el.z} = 395 \times 10^3 \cdot mm^3$

$N_{Rk} = 4.203 \cdot MN$ $N_{Rd} = 3.821 \cdot MN$ $M_{z.Rk} = 140.225 \cdot kN \cdot m$

$N_{Ed} = 2.911 \cdot MN$ $N_{cr} = 10.29 \cdot MN$ $L_{cr} = 3.216\,m$

$$\beta = 0.699 \qquad \varepsilon := \frac{\pi}{\beta} \qquad \alpha_{cr} := \frac{N_{cr}}{N_{Ed}} = 3.534 \qquad k := \frac{1}{1 - \frac{1}{\alpha_{cr}}} = 1.395$$

$$\lambda_m := \sqrt{\frac{N_{Rk}}{N_{cr}}} = 0.639 \qquad \phi := 0.5 \cdot \left[1 + \alpha \cdot (\lambda_m - 0.2) + \lambda_m^{\,2}\right] = 0.812$$

$$\chi := \frac{1}{\phi + \sqrt{\phi^2 - \lambda_m^{\,2}}} = 0.762 \qquad e_{o.k} := \alpha \cdot (\lambda_m - 0.2) \cdot \frac{M_{Rk}}{N_{Rk}} = 7.179 \cdot mm$$

$$e_{o.d} := e_{o.k} \cdot \frac{1 - \frac{\chi \cdot \lambda_m^{\,2}}{\gamma_{M1}}}{1 - \chi \cdot \lambda_m^{\,2}} = 7.473 \cdot mm \qquad \frac{L}{e_{o.d}} = 615.508$$

$$\eta_{cr}(x) := \frac{\left[\left(1 - \cos\left(\frac{\varepsilon \cdot x}{L}\right)\right) \cdot \varepsilon + \sin\left(\frac{\varepsilon \cdot x}{L}\right) - \frac{\varepsilon \cdot x}{L}\right]}{C} \qquad C = 6.283$$

$$\eta2_{cr}(x) := \frac{\cos\left(\varepsilon \cdot \frac{x}{L}\right) \cdot \frac{\varepsilon^3}{L^2} - \sin\left(\varepsilon \cdot \frac{x}{L}\right) \cdot \frac{\varepsilon^2}{L^2}}{C} \qquad x_m = 2.992\,m$$

$$\eta(x) := \frac{\eta_{o.ugli.m}}{\alpha_{cr} - 1} \cdot \eta_{cr}(x) \qquad \eta_{o.ugli.m} := \frac{\alpha_{cr} \cdot e_{o.d} \cdot N_{Ed}}{E \cdot I \cdot \left|-\eta2_{cr}(x_m)\right|} = 10.201 \cdot mm$$

$$M_{II}(x) := -E \cdot I \cdot \eta2(x)$$

$$M_{II}(x_m) = 30.346 \cdot kN \cdot m \qquad k \cdot N_{Ed} \cdot e_{o.d} = 30.346 \cdot kN \cdot m$$

$x_{max} = 2.768\,m \qquad x_m = 2.992\,m \qquad L - 0.5 \cdot L_{cr} = 2.992\,m$

$\eta_{o.ugli.m} = 10.201 \cdot mm \quad \eta_{ugli.xm} := \eta_{ugli}(x_m) = 10.024 \cdot mm$

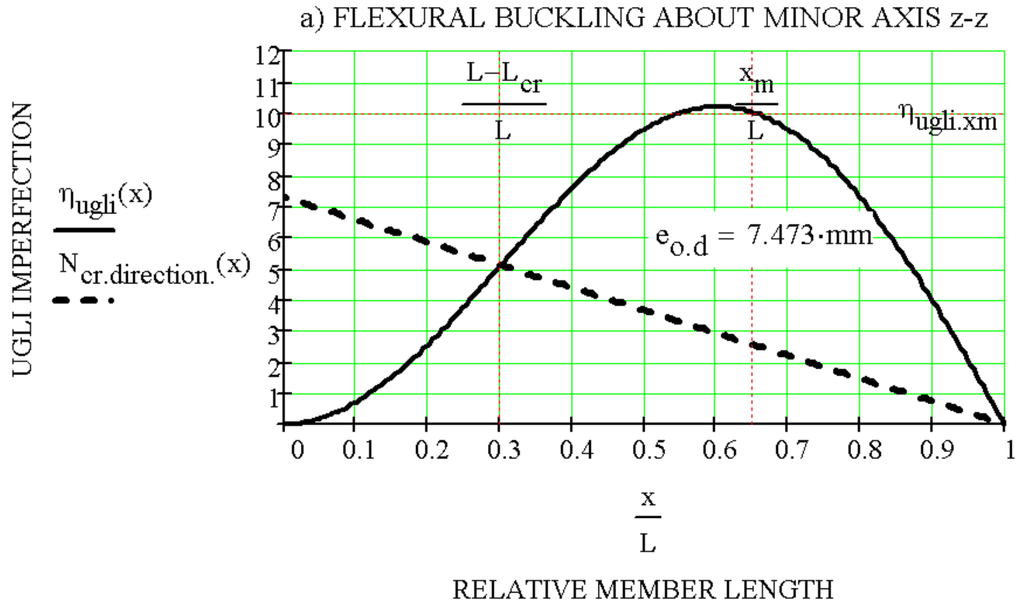


Fig. 5: Uniform global and local initial ("ugli") imperfection valid for buckling about z-z

$\eta_{o.ugli.m} = 10.201 \cdot mm \qquad x_{max} = 2.768\,m \qquad \eta(x_{max}) = 4.025\,mm$

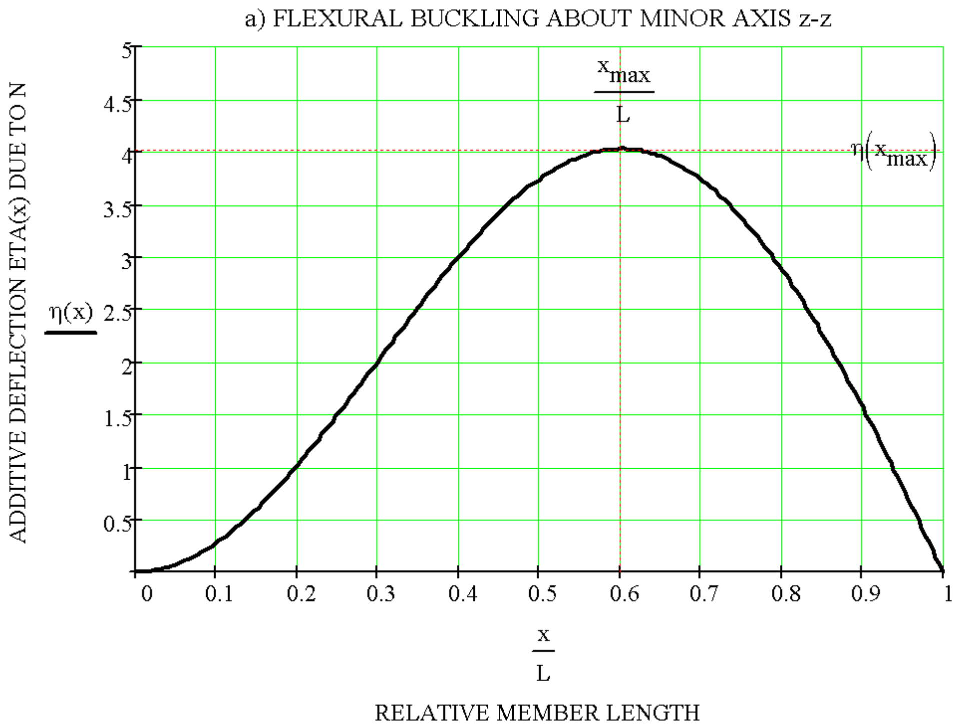


Fig. 6: Additive deflection η(x) due to N for flexural buckling about minor axis z-z

$M_{II}(0) = -29.62084 \cdot kN \cdot m$ $\quad N_{Ed} = -2911.494 \cdot kN$ $\quad M_{II}(x_m) = 30.3455 \cdot kN \cdot m$

$\frac{N_{Ed}}{A} + \frac{M_{II}(0)}{W} = -320.893 \cdot MPa$ $\quad x_m = 2.992\,m$ $\quad \frac{N_{Ed}}{A} - \frac{M_{II}(x_m)}{W} = -322.727 \cdot MPa$

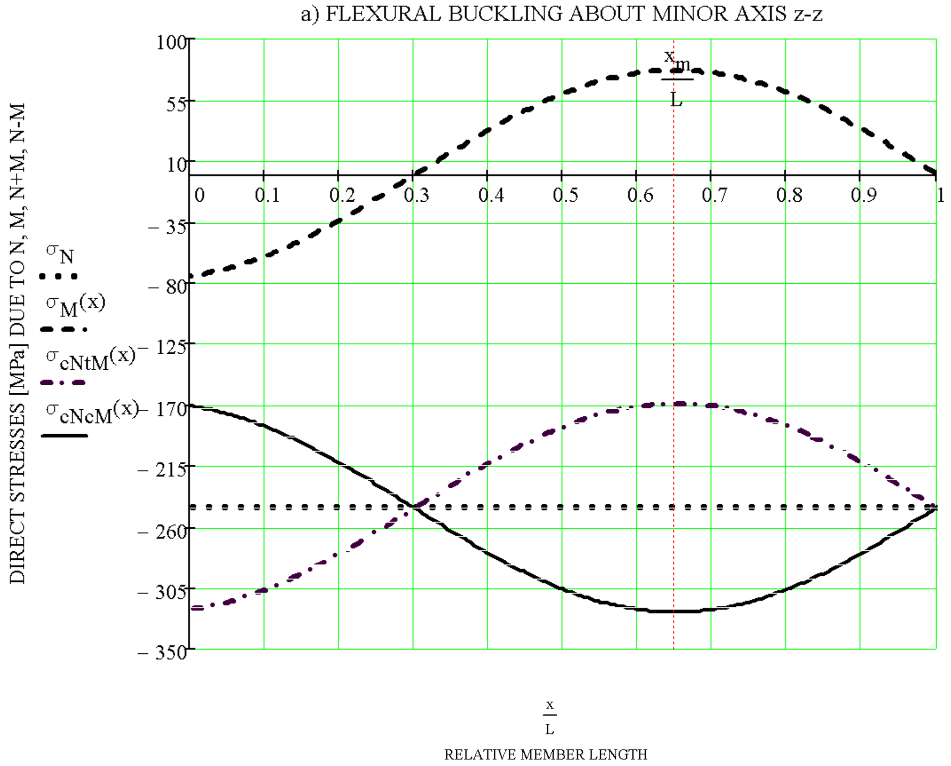


Fig. 7: Direct stresses for flexural buckling about minor axis z-z

$U_1 := \frac{N_{Ed}}{N_{Rd}} + \frac{M_{II}(x_m)}{M_{Rd}}$ $\quad U_1 = 1$ $\quad \chi = 0.762$ $\quad f_{y.d} = 322.727 \cdot MPa$ $\quad U_2 := \frac{N_{Ed}}{\chi \cdot A \cdot f_{y.d}}$ $\quad U_2 = 1$

Example 2: Input data see in Lindner, J. – Heyde, S. (2009) in Fig. 42. Fig. 43 and results on page 305-306 in Lindner, J. – Heyde, S. (2009) are incorrect. Correct values may be obtained very easy by proposed procedure as follows:

$A = 11.84 \times 10^3\,mm^2$ $\quad W_{pl.y} = 1.283 \times 10^6\,mm^3$ $\quad f_y = 355\,MPa$ $\quad \gamma_{M1} = 1$

$N_{Rk} = 4.2032 \cdot MN$ $\quad M_{pl.y.Rk} = 455.465\,kN \cdot m$ $\quad N_{Ed} = 620\,kN$ $\quad \alpha_{cr} = 4.441$

$\alpha = 0.34$ $\quad \beta = 2.30385$ $\quad \lambda_p = 1.23555$ $\quad \chi = 0.459$

$$k := \frac{1}{1 - \frac{1}{\alpha_{cr}}} = 1.291$$

$$e_{o.d} := \alpha \cdot (\lambda_p - 0.2) \cdot \frac{M_{el.y.Rk}}{N_{Rk}} \cdot \frac{1 - \frac{\chi \cdot \lambda_p^2}{\gamma_{M1}}}{1 - \chi \cdot \lambda_p^2} = 34.138\,mm$$

$\eta_{o.ugli.m} := e_{o.d} = 34.138\,mm$ $\quad M_{II.Ed.imp} := k \cdot N_{Ed} \cdot e_{o.d} = 27.31674\,kN \cdot m$

Comparison with results obtained by computer program IQ 100:

$EI\eta_{II.cr} = 2.7537 \cdot MN$ $\quad \eta_{o.ugli.m} := \frac{e_{o.d} \cdot N_{cr}}{EI\eta_{II.cr}} = 34.134\,mm$ $\quad M_{II.Ed.imp} := 27.314 \cdot kN \cdot m$

Example 3: Verification of in plane stability of steel large span arches using three different methods given in Eurocodes EN 1993-1-1 (2005), EN 1993-2 (2006), EN 1999-1-1 (2007): a) by equivalent column method and by global analysis taking into account the second order effects and relevant imperfections b) according to 5.3.2(11) in EN 1993-1-1 (2005) and 5.3.2(11) in EN 1999-1-1 (2007) and c) according to Tab.D.8 in EN 1993-2 (2006). The internal forces were calculated by IQ 100 (Rubin, H. – Aminbaghai, M. – Weier, H., (2004)) using 1st and 2nd order analysis with and without imperfections. Details of calculation and distributions of internal forces for hingeless arch are presented here (see Fig. 11-14). Comparisons of all results including ones valid for the same but two-hinged arch are given in Table 1. Details of calculation and internal forces distributions for two-hinged arch are not presented here.

Characteristics of given structure: steel parabolic arch with span $L = 320\,\text{m}$, rise/span ratio $\frac{f}{L} = \frac{40\,\text{m}}{320\,\text{m}} = 0.125$. The shape of arch $\frac{y}{L}\left(\frac{f}{L},\frac{x}{L}\right)$ and relative half length of the arch $\frac{s}{L}\left(\frac{f}{L}\right)$, which are defined by expressions

$$\frac{y(x)}{L} = 4\frac{f}{L}\frac{x}{L}\left(1-\frac{x}{L}\right) \text{ and } \frac{s}{L} = \frac{1}{4}\left[\sqrt{\left(4\frac{f}{L}\right)^2+1}+\frac{L}{4f}\ln\left[4\frac{f}{L}+\sqrt{\left(4\frac{f}{L}\right)^2+1}\right]\right] = 0.52\text{, respectively.}$$

Material properties: Young modulus $E = 210\,\text{GPa}$, steel grade $S355$, $f_y = 335\,\text{MPa}$ $(t > 40\,\text{mm})$, $\gamma_{\text{M1}} = 1.1$. Properties of uniform cross-section: area $A = 0.381\,\text{m}^2$, in plane second moment of area $I = 1.714\,\text{m}^4$, height of the cross-section $h = 5\text{m}$, in plane elastic section modulus $W = 0.686\,\text{m}^3$, in plane radius of inertia $i = 2.121\,\text{m}$. Class 3 cross-section; buckling curve „c“. Boundary conditions of hingeless arch:

a) in plane: translation fixed and rotation fixed on both ends,

b) the arch is laterally supported and no lost of stability out of plane may occur.

Design values of actions:

a) Permanent action uniform along the length of arch span $g_d = 0.13\,\text{MN/m}$,

b) Variable action along the left half of arch $q_d = 0.02\,MN/m$ (real value would be greater).

c) In plane imperfection according to table D.8 in EN 1993-2 (2006): the shape of two asymmetric waves with design value of amplitude $e_o = \pm L/400 = \pm 320\,\text{m}/400 = \pm 0.8\,\text{m}$ (hingeless arch, buckling curve „c“ in table D.8). Mean value of uniform action along the length of arch span $q_{\text{m,d}} = g_d + 0.5q_{\text{d}} = 0.13 + 0.5 * 0.02 = 0.14\,\text{MN/m}$.

Influence of shortening of arch centre line due to normal force for hingeless arch $\nu = \frac{45}{4}\left(\frac{i}{f}\right)^2 = \frac{45}{4}\left(\frac{2.121}{40}\right)^2 = 0.032$

Horizontal component of thrust $H = \frac{q_{\text{m.d}}L^2}{8f}\frac{1}{1+\nu} = \frac{0.14*320^2}{8*40}\frac{1}{1+0.032} = 43.426\,\text{MN}$.

Replacement of initial imperfections according to table D.8 by equivalent action $q_{\text{equ.eo}} = \pm\frac{8He_{\text{o}}}{(0.5L)^2} = \pm\frac{8*43.426*0.8}{(0.5*320)^2} \pm 0.0109\,\text{MN/m}$.

d) Combination of design values of actions:

for 1st and 2nd order analysis without imperfections (results see in Fig. 11 and Fig. 13 respectively):

in left half of arch span length $q_{\text{l.d}} = g_{\text{d}} + q_{\text{d}} = 0.13 + 0.02 = 0.15\,\text{MN/m}$, in right half of arch span length $q_{\mathrm{r.d}} = g_{\mathrm{d}} = 0.13\,\mathrm{MN/m}$,

for 1st and 2nd order analysis with imperfections (results see in Fig. 12 and Fig. 14 respectively)

in left half of arch span $q_{\mathrm{l.d}} = g_{\mathrm{d}} + q_{\mathrm{d}} + q_{\mathrm{equ.eo}} = (0.13 + 0.02 + 0.0109)\,\mathrm{MN/m} \approx 0.16\,\mathrm{MN/m}$

in right half of arch span length $q_{\mathrm{r.d}} = g_{\mathrm{d}} - q_{\mathrm{equ.eo}} = (0.13 - 0.0109)\,\mathrm{MN/m} \approx 0.12\,\mathrm{MN/m}$.

Internal forces

Parabolic arch was replaced by structure having form of polygon. Arch span was divided into 100 equal parts. Uniform loading q was replaced by point loads Q. Structure, values of point loads, reactions, deformations, distributions and values of internal forces N, M, V for hingeless arch may be found in Fig. 11-14.

Values in Fig. 11 and 12 were calculated by 1st order analysis without and with imperfections respectively, and values in Fig. 13 and 14 by 2nd order analysis without and with imperfections respectively. Computer program IQ 100 (Rubin, H. et al., 2004) was used to obtain results. In all calculations the influence of normal force deformations was taken into account.

Verification of arch stability

Characteristic and design values of cross-section resistances

$N_{\mathrm{Rk}} = Af_{\mathrm{y}} = 0.381\,\mathrm{m}^2\,335\,\mathrm{MPa} = 127.619\,\mathrm{MN}$, $\quad M_{\mathrm{Rk}} = Wf_{\mathrm{y}} = 0.686\,\mathrm{m}^3\,335\,\mathrm{MPa} = 229.714\,\mathrm{MNm}$

$N_{\mathrm{Rd}} = Af_{\mathrm{y}} / \gamma_{\mathrm{M1}} = 0.381\,\mathrm{m}^2\,335\,\mathrm{MPa} / 1.1 = 116\,\mathrm{MN}$,

$M_{\mathrm{Rd}} = Wf_{\mathrm{y}} / \gamma_{\mathrm{M1}} = 0.686\,\mathrm{m}^3\,335\,\mathrm{MPa} / 1.1 = 208.8\,\mathrm{MNm}$.

a) Stability verification by using equivalent column method for hingeless arch

Internal forces (Fig. 11):

$N_{\mathrm{Ed,I,q}}(a) = -49.2122\,\mathrm{MN}$, $M_{\mathrm{Ed,I,q}}(a) = 66.7498\,\mathrm{MNm}$.

Buckling length factor calculated using academic Dinnik`s values for critical loading (Dinnik, A. N., 1939)

$$q_{\mathrm{cr}}\left(\frac{f}{L} = 0.1\right) = K_{\mathrm{cr}}\left(\frac{f}{L} = 0.1\right)\frac{EI}{L^3} = 60.7\frac{EI}{L^3}, \quad q_{\mathrm{cr}}\left(\frac{f}{L} = 0.2\right) = K_{\mathrm{cr}}\left(\frac{f}{L} = 0.2\right)\frac{EI}{L^3} = 101\frac{EI}{L^3},$$

$$\beta_{\mathrm{H}}\left(\frac{f}{L} = 0.1\right) = \pi\sqrt{\frac{8f/L}{K_{\mathrm{cr}}\left(\frac{f}{L} = 0.1\right)}} = \pi\sqrt{\frac{8*0.1}{60.7}} = 0.361,$$

$$\beta_{\mathrm{H}}\left(\frac{f}{L} = 0.2\right) = \pi\sqrt{\frac{8f/L}{K_{\mathrm{cr}}\left(\frac{f}{L} = 0.2\right)}} = \pi\sqrt{\frac{8*0.2}{101}} = 0.395,$$

$$\beta_{\mathrm{N(a)}}\left(\frac{f}{L} = 0.1\right) = \beta_{\mathrm{H}}\left(\frac{f}{L} = 0.1\right)\frac{\sqrt{\cos[\arctan(4f/L)]}}{s/L(f/L = 0.1)} = 0.361\frac{\sqrt{\cos[\arctan(4*0.1)]}}{0.513} = 0.678,$$

$$\beta_{\mathrm{N(a)}}\left(\frac{f}{L} = 0.2\right) = \beta_{\mathrm{H}}\left(\frac{f}{L} = 0.2\right)\frac{\sqrt{\cos[\arctan(4f/L)]}}{s/L(f/L = 0.2)} = 0.395\frac{\sqrt{\cos[\arctan(4*0.2)]}}{0.549} = 0.636,$$

$$\beta_{\mathrm{N(a)}}\left(\frac{f}{L} = 0.125\right) = \beta_{\mathrm{N(a)}}\left(\frac{f}{L} = 0.1\right) - \frac{0.125 - 0.1}{0.2 - 0.1}\left[\beta_{\mathrm{N(a)}}\left(\frac{f}{L} = 0.1\right) - \beta_{\mathrm{N(a)}}\left(\frac{f}{L} = 0.2\right)\right] = 0.667$$

In plane buckling length factor according to table D.4 in EN 1993-2 (2006) $\beta = 0.67$.

In plane critical buckling force $N_{\mathrm{cr}} = \dfrac{\pi^2 EI}{(\beta s)^2} = \dfrac{\pi^2 210000*1.714}{(0.67*0.52*320)^2} = 285.729\,\mathrm{MN}$,

More exact value was taken into account $N_{\mathrm{cr}} = \alpha_{\mathrm{cr}} N_{\mathrm{Ed,I,q}}(a) = 5.8982*49.2122 = 290.257\,\mathrm{MN}$,

where $\alpha_{cr} = 5.8982$ is minimum force amplifier, which was calculated by IQ 100 (Rubin, H. et al., 2004).

Relative slenderness $\bar{\lambda} = \sqrt{\dfrac{N_{\mathrm{Rk}}}{N_{\mathrm{cr}}}} = \sqrt{\dfrac{127.619}{290.257}} = 0.663$.

Measure of imperfection $\alpha = 0.49$ (buckling curve „c“).

Factor $\Phi = 0.5\left[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2\right] = 0.5\left[1 + 0.49(0.663 - 0.2) + 0.663^2\right] = 0.833$.

Reduction factor $\chi = \dfrac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} = \dfrac{1}{0.833 + \sqrt{0.833^2 - 0.663^2}} = 0.747$.

In plane buckling resistance $N_{\mathrm{b,Rd}} = \chi A \dfrac{f_y}{\gamma_{\mathrm{M1}}} = 0.747*0.381\dfrac{335}{1.1} = 86.711\,\mathrm{MN}$.

Equivalent uniform moment factor for sway buckling mode obtained by using method 2 from

Annex B in EN 1993-1-1 (2005). $C_{m,y} = 0.9$ and interaction factor is as follows

$$k_{\mathrm{yy}} = C_{\mathrm{m,y}}\left(1 + 0.6\bar{\lambda}\frac{|N_{\mathrm{Ed,I,q}}(a)|}{N_{\mathrm{b,Rd}}}\right) = 0.9\left(1 + 0.6*0.663\frac{|-49.2122|}{86.711}\right) = 1.103\,.$$

Utilization grade

$$U_{\mathrm{I}} = \frac{N_{\mathrm{Ed,I,q}}(a)}{N_{\mathrm{b,Rd}}} + \frac{k_{\mathrm{yy}} M_{\mathrm{Ed,I,q}}(a)}{M_{\mathrm{Rd}}} = \frac{|-49.2122|}{86.411} + \frac{|-1.103*66.7498|}{208.8} = 0.568 + 0.353 = 0.920 < 1.0\,.$$

b) Verification of strength by using 2nd order analysis with imperfections according to table D.8 EN 1993-2 (2006) for hingeless arch

Internal forces (Fig. 14):

$N_{\mathrm{Ed,II,q,eo}}(a) = -50.0777\,\mathrm{MN}$, $M_{\mathrm{Ed,II,q,eo}}(a) = 103.621\,\mathrm{MNm}$,

Utilization grade

$$U_{\mathrm{II}} = \frac{N_{\mathrm{Ed,II,q,eo}}(a)}{N_{\mathrm{Rd}}} + \frac{M_{\mathrm{Ed,II,q,eo}}(a)}{M_{\mathrm{Rd}}} = \frac{|-50.0777|}{116} + \frac{|-103.621|}{208.8} = 0.432 + 0.496 = 0.928 < 1.0\,.$$

c) Verification of strength by using 2nd order analysis with imperfections having shape of 1st buckling mode according to clause 5.3.2(11) in EN 1993-1-1 (2005) and EN 1999-1-1 (2007) for hingeless arch

The total maximum normal stress $(\sigma_{\mathrm{N}} + \sigma_{\mathrm{M}})$ acts in support *a*, therefore critical point *m* is located in support *a.* In critical point *m* we have (index *m* is omitted in the following quantities):

Internal forces (Fig. 13)

$N_{\mathrm{Ed,II,q}}(a) = -49.6309\,\mathrm{MN}$, $M_{\mathrm{Ed,II,q}}(a) = 67.9147\,\mathrm{MNm}$,

Minimum force amplifier for hingeless arch $\alpha_{\mathrm{cr}} = 5.8982$ was calculated by IQ 100 (Fig. 8).

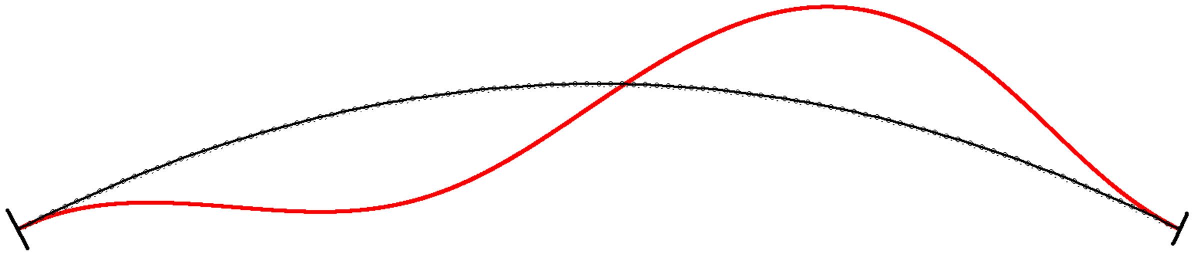


Fig. 8: First buckling mode $\eta_{cr,max}(29-30)=1$, $\eta_{cr}(70-71)=-0.999267$. *Minimum force amplifier for the axial force configuration N_{Ed} to reach the elastic critical buckling force* $\alpha_{cr}=5.8982$.

In plane critical force $N_{cr}=\alpha_{cr}N_{Ed,II,q}(a)=5.8982*49.6309=292.733\,\mathrm{MN}$.

Relative slenderness $\bar{\lambda}=\sqrt{\frac{N_{Rk}}{N_{cr}}}=\sqrt{\frac{127.619}{292.733}}=0.66$.

Measure of imperfection $\alpha=0.49$ (buckling curve „c“).

Factor $\Phi=0.5\left[1+\alpha(\bar{\lambda}-0.2)+\bar{\lambda}^2\right]=0.5\left[1+0.49(0.68-0.2)+0.68^2\right]=0.831$.

Reduction factor $\chi=\frac{1}{\Phi+\sqrt{\Phi^2-\bar{\lambda}^2}}=\frac{1}{0.831+\sqrt{0.831^2-0.66^2}}=0.749$.

Characteristic value of imperfection amplitude

$$e_{o,k}=\alpha(\bar{\lambda}-0.2)\frac{M_{Rk}}{N_{Rk}}=0.49(0.66-0.2)\frac{229.714}{127.619}=0.406\,\mathrm{m}.$$

Design value of imperfection amplitude

$$e_{o,d}=e_{o,k}\frac{1-\frac{\chi\bar{\lambda}^2}{\gamma_{M1}}}{1-\chi\bar{\lambda}^2}=0.406\frac{1-\frac{0.749*0.66^2}{1.1}}{0.749*0.66^2}=0.424\,\mathrm{m}$$

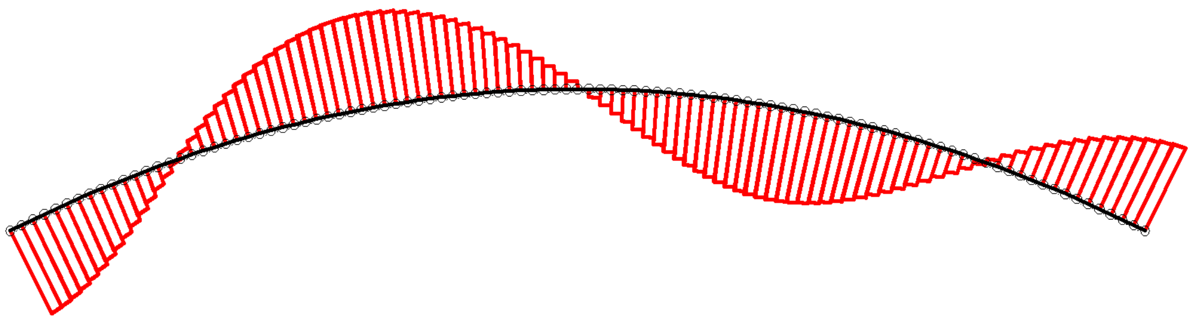


Fig. 9: Bending moment due to $\eta_{cr}(x)$ ($\left|\eta_{cr}(x)\right|_{max}=1$). *In the critical section m (located in left support a) the value* $\left|M^{II}_{\eta cr,m}\right|=EI\left|\eta_{cr,m}''\right|=196.071\,\mathrm{kNm}$. *It was calculated by IQ 100 (Rubin, H. et al., 2004).*

Amplitude of unique global and local initial ("ugli") imperfection depending on quantities in critical point *m*

$$\eta_{\text{ugli,m}} = \frac{e_{\text{o,d}} N_{\text{cr}}}{\left| EI \eta_{\text{cr,m}}'' \right|} = \frac{0.424 * 292.733}{196.071} = 632.815$$

Unique global and local initial ("ugli") imperfection

$$\eta_{\text{init}}(x) \equiv \eta_{\text{ugli}}(x) = \eta_{\text{ugli,m}} \eta_{\text{cr}}(x) = 632.815 \eta_{\text{cr}}(x)$$

Deflection of the structure calculated using 2nd order analysis for the structure with imperfection

$$\eta^{\text{II}}(x) = \frac{1}{\alpha_{\text{cr}} - 1} \eta_{\text{ugli}}(x) = \frac{1}{\alpha_{\text{cr}} - 1} \eta_{\text{ugli,m}} \eta_{cr}(x) = \frac{1}{5.8982 - 1} 632.815 \eta_{\text{cr}}(x) = 129.193 \eta_{\text{cr}}(x) \,.$$

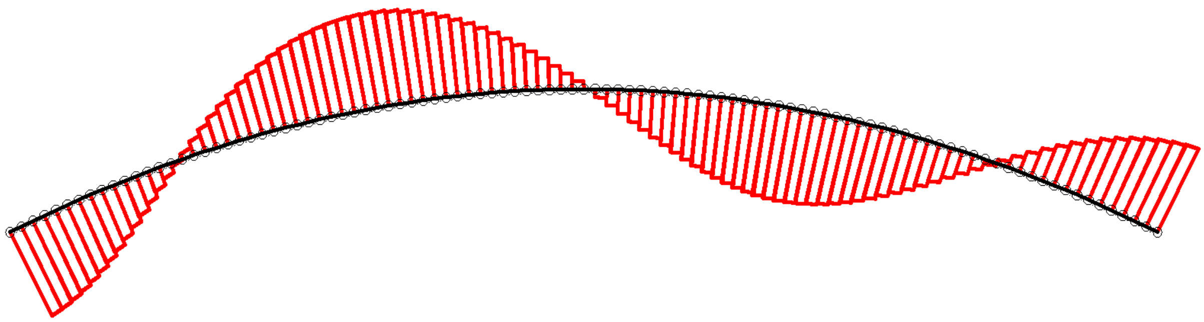


Fig. 10: Bending moment [MNm] *in the structure due to* $\eta_{\text{init}}(x) \equiv \eta_{\text{ugli}}(x)$ *with allowing for 2nd order effects* $M^{\text{II}}_{\eta\text{init,m}} \equiv$ *M(a) = - 25.3311, M(32) = 25.5385, M(68) = - 25.5267, M(b) = 25.2534 (note opposite signs to signs in moment line distribution).*

Bending moment in the structure due to $\eta_{\text{init}}(x) \equiv \eta_{\text{ugli}}(x)$ with allowing for 2nd order effects may be calculated from the formula

$$M^{\text{II}}_{\eta\text{init,m}} = \frac{\eta_{\text{ugli,m}}}{\alpha_{\text{cr}} - 1} \left| M^{\text{II}}_{\eta\text{cr,m}} \right| = \frac{632.815}{5.8982 - 1} 196.071 \,\text{kNm} = 129.193 * 196.071 \,\text{kNm} = 25.3311 \,\text{MNm} \,.$$

Utilization grade

$$U_{\text{II}} = \frac{N_{\text{Ed,II,q}}(a)}{N_{\text{Rd}}} + \frac{M_{\text{Ed,II,q}}(a)}{M_{\text{Rd}}} + \frac{M^{\text{II}}_{\eta\text{init,m}}}{M_{\text{Rd}}} = \frac{|-49.6309|}{116} + \frac{|-67.9147|}{208.8} + \frac{|-25.3311|}{208.8} =$$
$$= 0.428 + 0.325 + 0.121 = 0.874 < 1.0$$

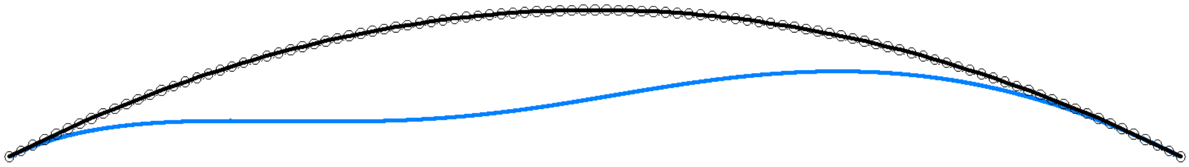
The similar calculation as for hingeless arch, which is presented in details, was done also for the same but two-hinged arch. Numerical results of stability verification of hingeless and two-hinged arches obtained by using three different Eurocode methods are given in Table 1. Comparisons show good agreement of utilityzation grades. It is necessary to mention that safety margin of method given in 5.3.2(11) is a little bit greater comparing with safety margin of equivalent member method and that the closer we are to utilitization grade $U = 1$ the lesser is difference between safety margins. Method proposed by Chladný for 5.3.2(11) in EN 1993-1-1 (2005) and for 5.3.2(11) in EN 1999-1-1 (2007) uses characteristic value of imperfection amplitude $e_{\text{o,k}}$. The value $e_{\text{o,k}}$, should be based on statistical evaluations of measurements on real structures. In example presented here the value $e_{\text{o,k}}$ is taken according to EN 1993-1-1 (2005), because such evaluations are not available.

Tab. 1: Results of analysis of steel large span parabolic hingeless arch and the same but two-hinged arch using three different procedures of Eurocodes EN 1993-1-1 (2005), EN 1999-1-1 (2007)

	1st order analysis		**2nd order analysis**		
	according to 5.2.2(3)c) in EN 1993-1-1(2005) without imperfections (they are hidden in χ). Method 2, Annex B	with imperfections according to Tab.D.8 in EN 1993-2 $e_0 = L/400 = 800\,\text{mm}$	without imperfections	according to 5.2.2(3)a) in EN 1993-1-1 (2005) with imperfections according to 5.3.2(11) in EN 1993-1-1(2005) EN 1999-1-1(2007) $\eta_{\text{ugli,m}}$	according to 5.2.2(3)a) in EN 1993-1-1 (2005) with imperfections according to Tab.D.8 in EN1993-2(2006) $e_0 = L/400 = 800\,\text{mm}$
HINGELESS ARCH ($\alpha_{cr} = 5.8982$, $\eta_{\text{ugli,m}} = 632.8\,\text{mm} \approx L/506$, $e_0 = L/400 == 800\,\text{mm}$)					
$H\,[\text{MN}]$	43.4493		43.906		
$N(a)$	- 49.2122	- 49.6467	- 49.6309		- 50.0777
$M(a) \equiv \equiv M_{\min}$ [MNm]	- 66.7498	- 98.0327	- 67.9147 (common); - 67.9147	$M_{\min}$= - 67.9147 - - 25.3311= = - 93.2458	- 103.621
$M_{\max}$ [MNm]	$M(37)$ = = 31.4032	$M(34)$ = = 48.714	$M(36-37) = 35.6816$ (common)	$M^{\text{II}}_{\eta\text{init}}(32)$ = = 25.5385, $M_{\max} < 61.22$ = = (35.68+25.54)	$M(34)$ = = 56.9085
Utilitiz. grade	**in left half of arch 0.920**			**in point m ≡ a 0.874**	**in point a 0.928**
TWO-HINGED ARCH ($\alpha_{cr} = 2.7187$, $\eta_{\text{ugli,m}} = 742.1\,\text{mm} \approx L/431$, $e_0 = L/400 == 800\,\text{mm}$)					
$H\,[\text{MN}]$	44.5596		44.926		
$N(a)$	- 50.1205	- 50.4683	-50.4376		- 50.7848
$N\,[\text{MN}]$	$N(27)$ = = - 45.7754	$N(26)$ = = - 45.8839	$N(25)$ = - 46.3515		$N(25)$ = = - 46.359
$M_{\max}$ [MNm]	$M(27)$ = = 39.3754	$M(26)$ = = 71.297	$M(25) = 58.2369$ (common)	$M^{\text{II}}_{\eta\text{init}}(25)$ = = 57.80, M(25) = = 58.24+57.80 = = 116 .04	$M(25)$ = = 109.323
Utilitiz. grade	**in left half of arch 1.019**			**in point (25) ≈ m 0.955**	**in point (25) 0.923**

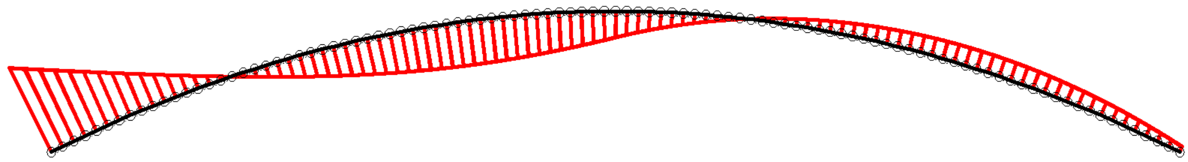
11a) Point loads: 240 kN + 49 x 480 kN + 448 kN + 49 x 416 kN + 208 kN,

reactions [MN, MNm]: H = 43.4493, V_a = 23.3955, V_b = 21.4045, M_a = 66.7498, M_b = 4.18383



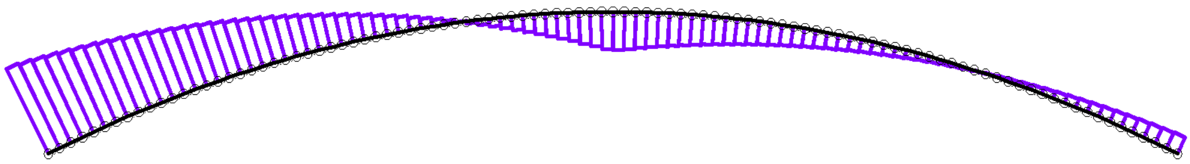
11b) Maximum deformations [m]:

vertical deflection v(40) = 0.40449, (horizontal deflection u(30) = 0.059595)

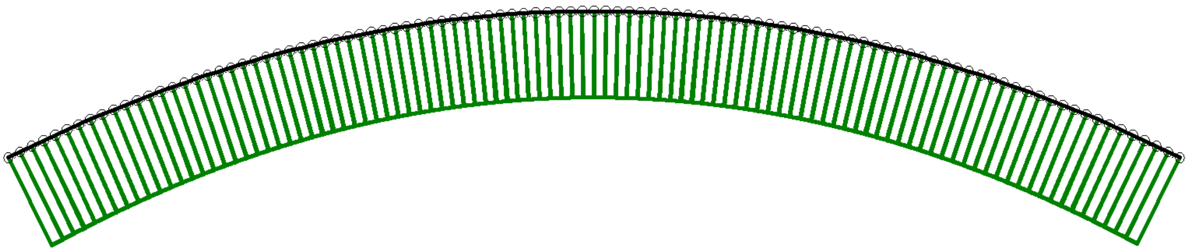


11c) Bending moments [MNm]:

M(a) = - 66.7498, M(25) = 21.4133, M(37) = 31.4032, M(83) = - 13.0495, M(b) = - 4.18383



11d) Shear force [MN]: maximum value V(a) = 1.47707

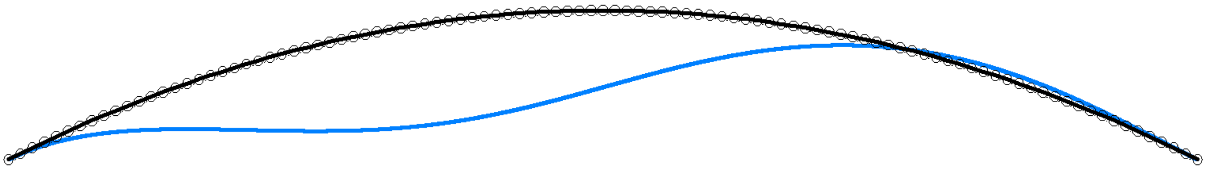


11e) Normal force [MN]: N(a) = - 49.2122, N(.25) = - 44.9711, N(50.) = - 43.4528, N(b) = - 48.3431

Fig. 11: Hingeless parabolic arch. Span L = 320 m, rise 40 m, rigidities EA = 80 000 MN, EI = 360 000 MNm^2. 1st order analysis without imperfections. Shortening due to normal force is taken into account. Vertical loads q_{left} = 0.15 MN/m in left side of arch and q_{right} = 0.13 MN/m in right side of arch were replaced by vertical point loads in 101 points.

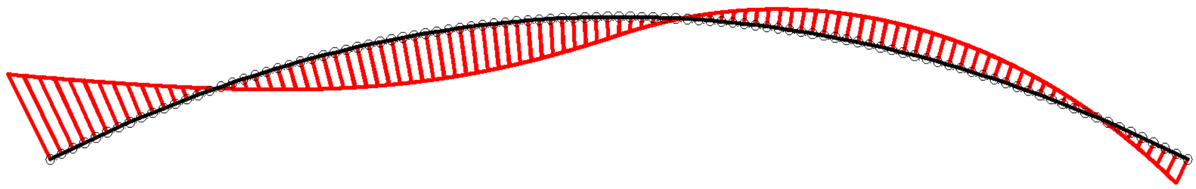
12a) Point loads: 256 kN + 49 x 512 kN + 448 kN + 49 x 384 kN + 192 kN,

reactions [MN, MNm]: H = 43.4493, V_a = 24.391, V_b = 20.409, M_a = 98.0327, M_b = - 27.0991



12b) Maximum deformations [m]:

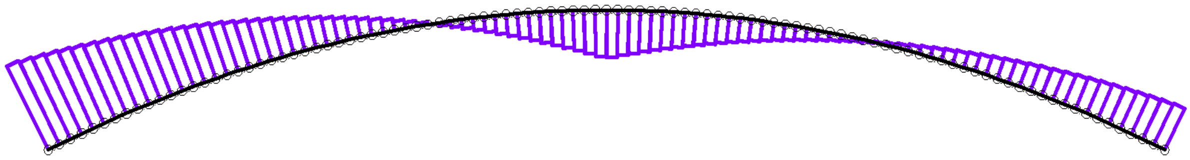
vertical deflection v(35-36) = 0.48801, (horizontal deflection u(29-30) = 0.097073)



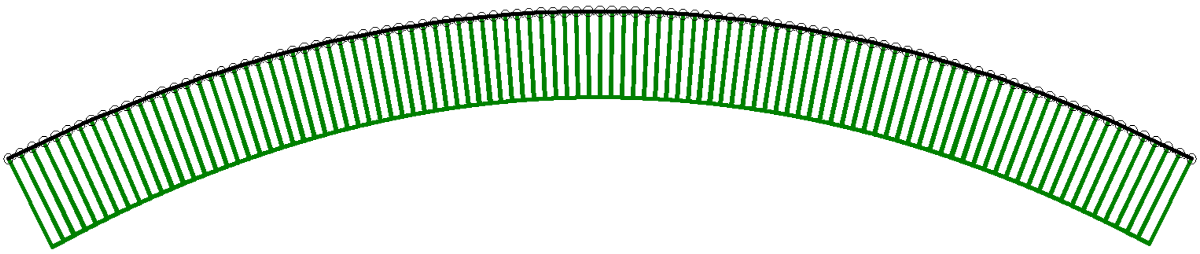
12c) Bending moments [MNm]:

M(a) = - 98.0327, M(25) = 37.7718, M(34) = 48.714, M(74) = - 27.7523, M(b) = 27.0091

.



12d) Shear force [MN]: maximum value V(a) = 2.35493

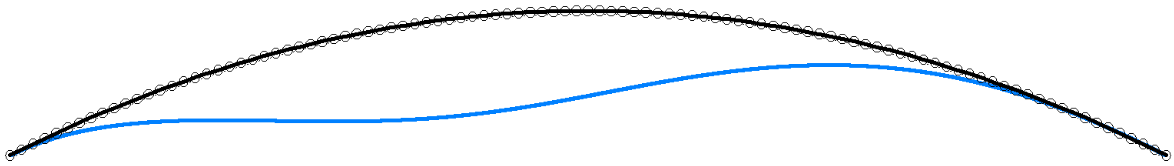


12e) Normal force [MN]: N(a) = - 49.6467, N(.25) = - 45.0293, N(50.) = - 43.4547, N(b) = - 47.9085

Fig. 12: Hingeless parabolic arch. Span L = 320 m, rise 40 m, rigidities EA = 80 000 MN, EI = 360 000 MNm^2. 1st order analysis with imperfections. Buckling curve „c“. Shortening due to normal force is taken into account. Vertical loads q_{left} = 0.16 MN/m in left side of arch and q_{right} =0.12 MN/m in right side of arch were replaced by vertical point loads in 101 points.

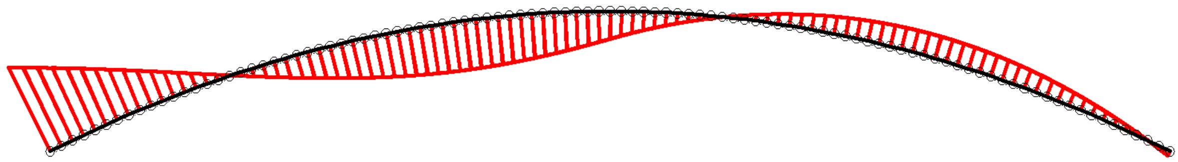
13a) Point loads: 240 kN + 49 x 480 kN + 448 kN + 49 x 416 kN + 208 kN,

reactions [MN, MNm]: H = 43.9052, V_a = 23.4183, V_b = 21.3817, M_a = 67.9147, M_b = - 3.4664



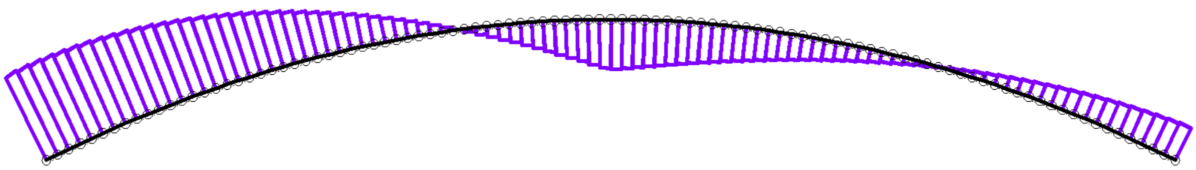
13b) Maximum deformations [m]:

vertical deflection v(39) = 0.4295, (horizontal deflection u(30) = 0.06689)

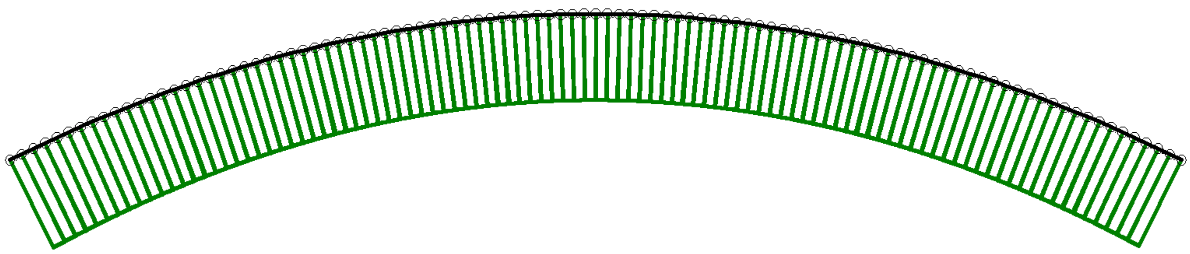


13c) Bending moments [MNm]:

M(a) = - 67.9147, M(25) = 23.9468, M(36-37) = 35.6816, M(79) = - 16.5234, M(b) = 3.4664



13d) Shear force [MN]: maximum value V(.1) = 1.3276

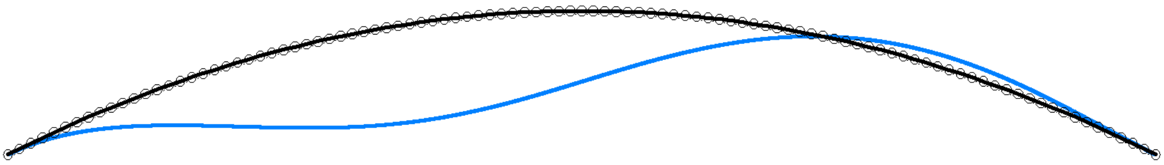


13e) Normal force [MN]: N(a) = - 49.6309, N(.25) = - 45.4245, N(50.) = - 43.9086, N(b) = - 48.7416

Fig. 13: Hingeless parabolic arch. Span L = 320 m, rise 40 m, EA = 80 000 MN, EI = 360 000 MNm^2. 2nd order analysis without imperfections. Shortening due to normal force is taken into account. Vertical loads q_{left} = 0.15 MN/m in left side of arch and q_{right} = 0.13 MN/m in right side of arch were replaced by vertical point loads in 101 points.

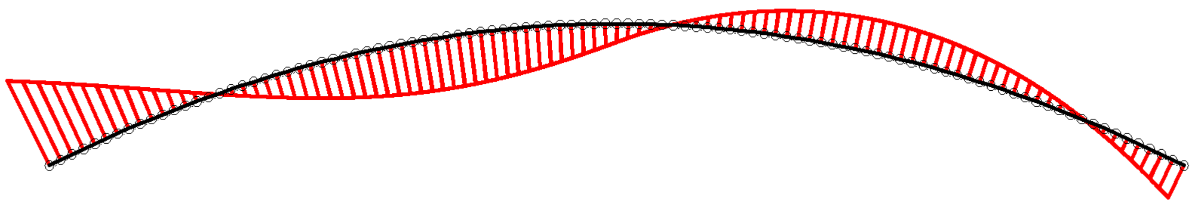
14a) Point loads: 256 kN + 49 x 512 kN + 448 kN + 49 x 384 kN + 192 kN,

reactions [MN, MNm]: H = 43.9075, V_a = 24.4384, V_b = 20.3616, M_a = 103.621, M_b = - 39.1536



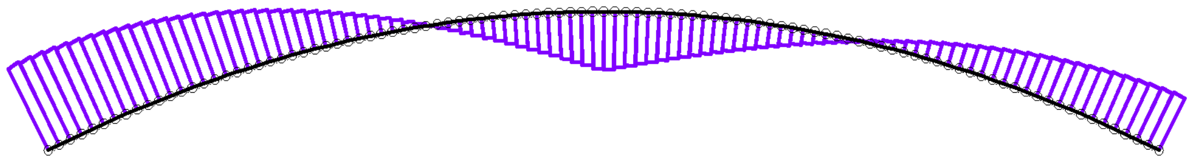
14b) Maximum deformations [m]:

vertical deflection v(35) = 0.5307, (horizontal deflection u(29-30) = 0.111524)

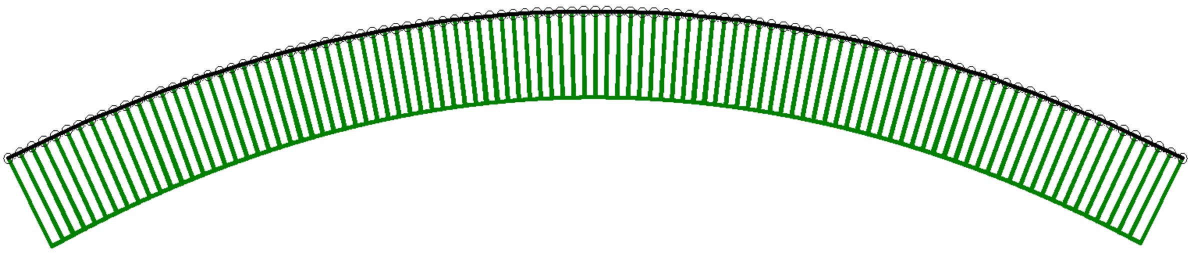


14c) Bending moments [MNm]:

M(a) = - 103.621, M(25) = 43.7785, M(34) = 56.9085, M(73) = - 35.8893, M(b) = 39.1536



14d) Shear force [MN]: maximum value V(a) = 2.19176



14e) Normal force [MN]: N(a) = - 50.0777, N(.25) = - 45.4846, N(50.) = - 43.9137, N(b) = - 48.2988

Fig. 14: Hingeless parabolic arch. Span L = 320 m, rise 40 m, rigidities EA = 80 000 MN, EI = 360 000 MNm^2. 2nd order analysis with imperfections. Buckling curve „c“. Shortening due to normal force is taken into account. Vertical loads q_{left} = 0.16 MN/m in left side of arch and q_{right} = 0.12 MN/m in right side of arch were replaced by vertical point loads in 101 points.

Example 4: The analysis of two-hinged arch with geometrical and loading parameters of Žďákov bridge was published in Baláž, I. – Koleková, Y. (2011 b).

Example 5: Show how it is easy to find location "m" and to calculate the value of maximum bending moment due to equivalent uniform global and local initial imperfection $\eta_{\rm init}(x) \equiv \eta_{\rm ugli}(x)$ acting in the structure in the point "m". The authors developed graphical interpretation of the method valid for members or frames with uniform cross-section and/or with uniform normal force distribution. The graphical interpretation of the method is shown in Fig. 15-17 for 14 structures. The steps of the graphical method:

- draw the first buckling mode,
- identify the buckling length $L_{\rm cr}$. Keep in the mind that the elastic critical force N_{cr} is transmitted in the direction of the line, which connects two neighbouring inflexion points,
- find the location of the point "m", where the maximum normal stress acts. This stress consists from the normal stress due to uniform normal force and from the normal stress due to maximum bending moment due to equivalent "ugli" imperfection $\eta_{\rm init}(x) \equiv \eta_{\rm ugli}(x)$. The point "m" is located: (i) in the middle of the buckling length $L_{\rm cr}$ (see 6 cases in Fig. 15 and 2 non-sway frames in Fig.16 and 2 in Fig.17), or (ii) in the cross-section of the sway frames, where the part of the sinus wave defining buckling length has the maximum displacement (see 2 sway frames in Fig.16 and 2 in Fig.17).
- the value of amplitude $e_{0,\rm d}$ is defined by the formula derived by Chladný. This formula is today in EN 1993-1-1, (2005), 5.3.2(11), formula (5.10) and in EN 1999-1-1, (2007), 5.3.2(11), formula (5.7). Not convenient symbol e_0 instead of $e_{0,\rm d}$ is used in Eurocodes. The amplitude $e_{0,\rm d}$ is located in the point "m".
- calculate factors

$$\alpha_{cr} = \frac{N_{cr}}{N_{Ed}}, \quad k = \frac{1}{1 - \dfrac{1}{\alpha_{cr}}} = \frac{\alpha_{cr-1}}{\alpha_{cr}} \tag{18}$$

- calculate bending moment due to equivalent "ugli" imperfection $\eta_{\rm init}(x) \equiv \eta_{\rm ugli}(x)$ from simple formula (16, 19) by hand

$$M^{\rm II}_{0,\rm Ed,ugli,m} = kN_{\rm Ed,m}e_{0,\rm d,m} \tag{19}$$

Examples 6, 7, 8: relating to the structures with the non-uniform cross-sections and/or the non-uniform normal force distribution were published by Chladný in (Baláž, I. et al., 2007, 2010) and by Kováč in Kováč, M. (2010).

4. Conclusions

New very promissing and useful method for design and verification of stability and flexural buckling resistance of metal members and frames with equivalent uniform global and local initial imperfections is presented. The original method was developed by Chladný and today is used in Eurocodes EN 1993-1-1 (2005) and EN 1999-1-1 (2007). It may be used also for frames with non-uniform cross-sections and/or non-uniform axial force distribution. In the paper new way of derivation of basic formulae of the method and clear step by step description of its application based on this derivation are presented. The original graphical interpretation of the method developed by authors is valid for frames with uniform cross-sections under uniform axial compression forces and enables to obtain very easy the maximum value of bending moment due to equivalent "ugli" imperfection. Several numerical examples show in details application of this method, which may be further developed and used also for lateral torsional buckling of beams.

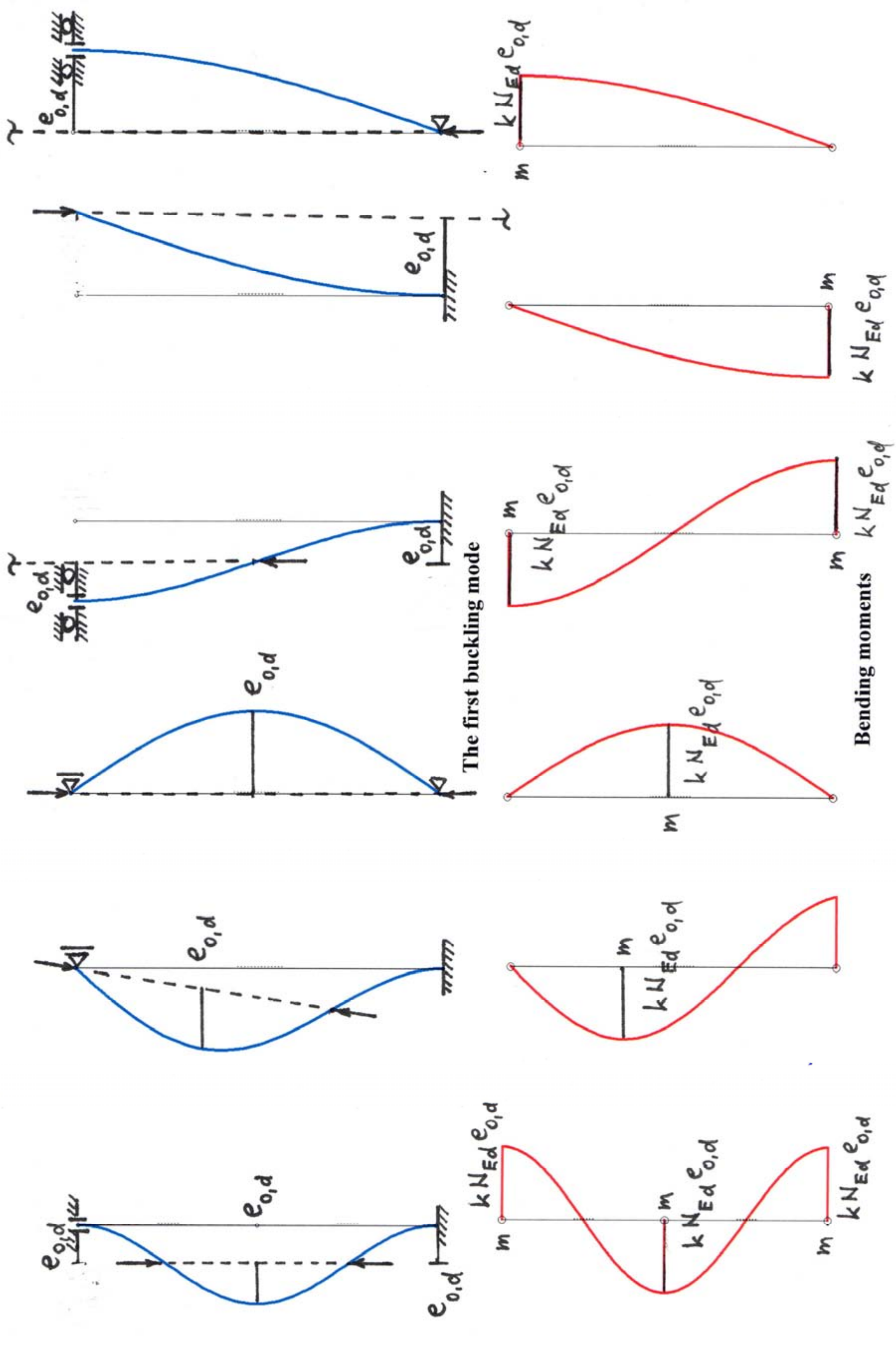


Fig. 15: Bending moments due to equivalent “ugli” imperfections

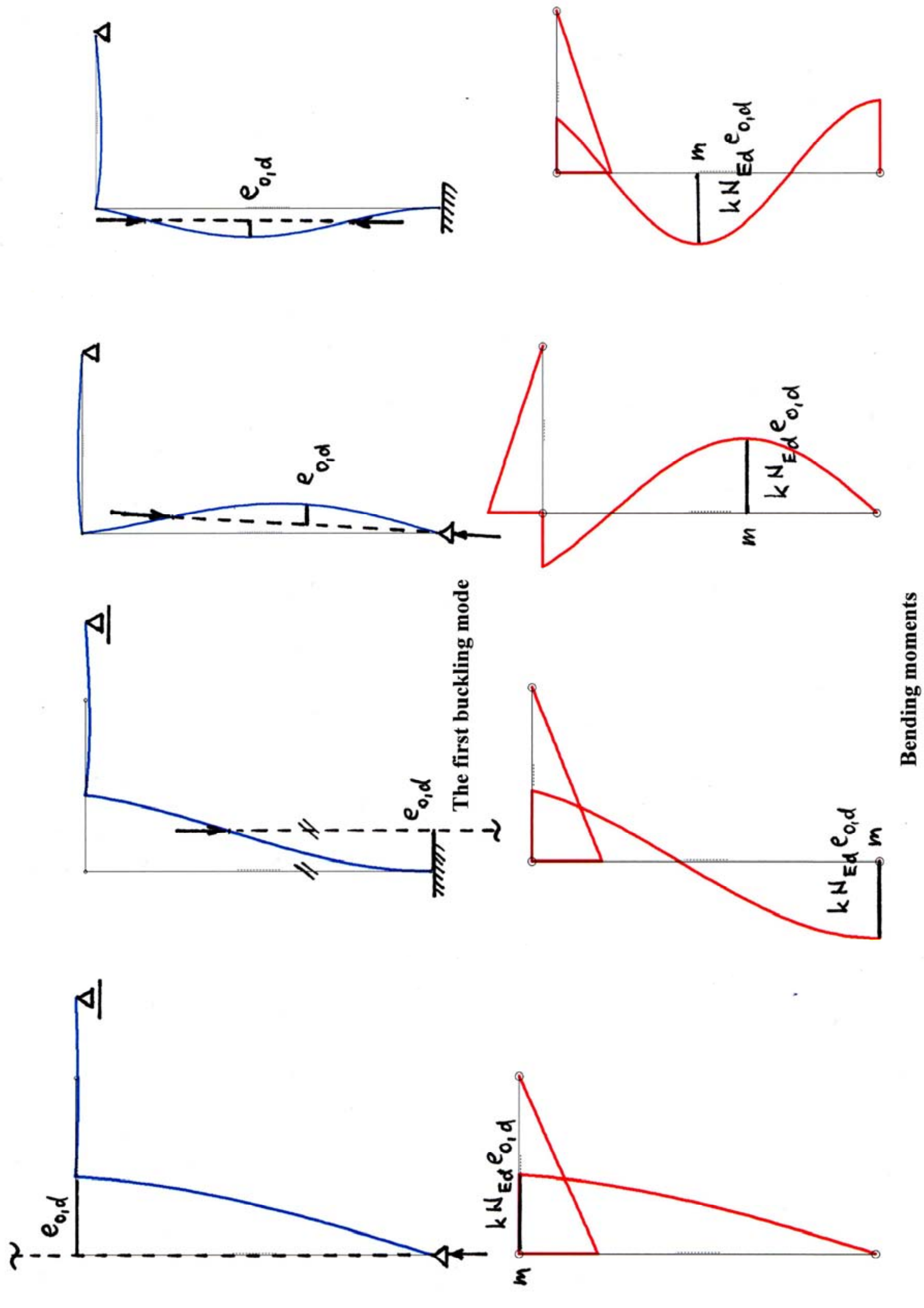


Fig. 16: Bending moments due to equivalent "ugli" imperfections

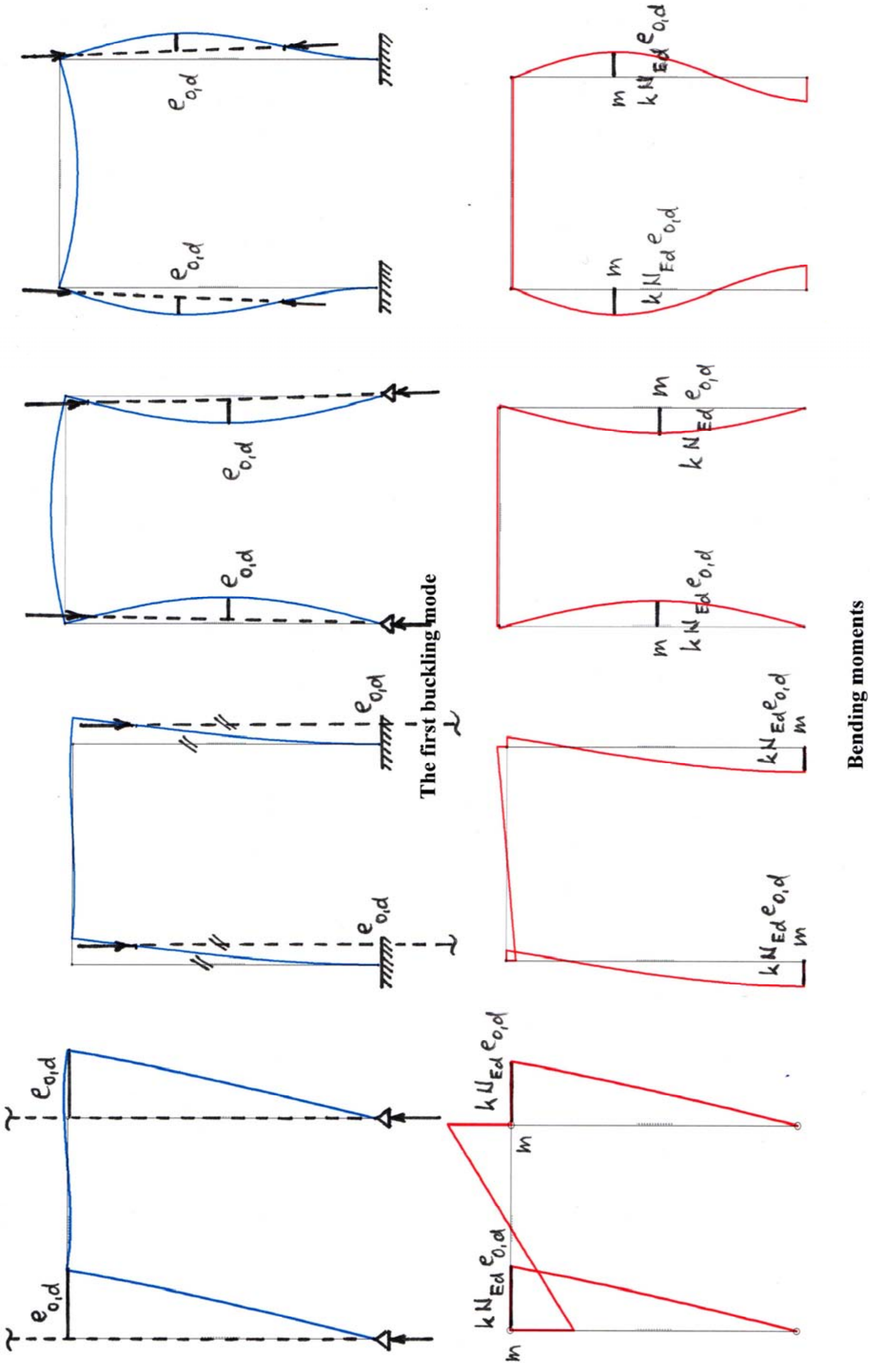


Fig. 17: Bending moments due to equivalent "ugli" imperfections

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