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DISCRETE TOPOLOGY OPTIMIZATION OF PLANAR CABLE-TRUSS STRUCTURES BASED ON GENETIC ALGORITHMS

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Abstract: *This paper demonstrates the application of Genetic Algorithms to design optimal lightweight Cable-Truss structures, which are structures composed by bars and prestressed cables and offer a high potential in robotics. The optimal lightweight structure shape is determined through a discrete topology optimization process which starts from a ground grid of nodes and interconnect them using cables or bars in order to obtain optimal results. The optimal solution is considered to have the lower mass and highest stiffness, such relation is expressed in the parameter stiffness-to-mass ratio. The objective function of the optimization problem evaluates the bending stiffness and the mass of the feasible solutions searching for the maximum stiffness-to-mass ratio. Symmetric structural response is desired once that in movable machines the majority of the structures are moving parts in which forces can assume different directions during working cycle, as a result the algorithm must find solutions with are symmetric in the axis perpendicular to the loading direction. Simulations are also presented showing comparisons between Cable-Truss and Truss structures under the same boundary conditions, population and iterations. Structural static response is computed using nonlinear finite element iterative procedure. Examples with optimized modular layout of a 2D robotic arm are shown, which presents improvement of Cable-Truss structures in comparison with Truss structures in all cases that have been simulated.*

Keywords: ***Cable-Truss structures, Genetic Algorithms, Discrete Topology Optimization, lightweight design.***

1. Introduction

Lightweight structures research is based not only in material science but also in structural mechanics, processing and design. Is important noting that the scientific aim is to develop knowledge for the specific phenomena occurring in these areas and in the interface between them, in order to achieve increased performance for a wide range of structural applications (LsAA,2010). In addition, the main lightweight structures include tensile/tension structures, frame supported, air supported, air inflated, cable net, cable-and-strut, geodesic domes, and grid shells. In lightweight designing, different structural elements are used, which can be optimized, combined or substituted using different methods. However, assigning a single all-encompassing definition for all applications is an extremely complex task and consequently the meaning of the term lightweight structure varies accordingly to application and field of research. In the structural field, lightweight structures can be defined as those which shape is determined through an optimization process to efficiently carry the loads from a critical loading case regardless of the type of material employed (LsAA, 2010).

In the research of lightweight structures, trusses have attracted tremendous interest due to their extensive application in the contracture of infrastructures and space structures. Research works have focused on material characteristics, truss joint design, processing and construction of structural components. In recent years, influence of cables in such structures has also been investigated (Liao, 2009).

The term cable-truss is often taken to describe a structural member consisting of bars and prestressed cables. Cable elements can only withstand tension forces and are used not only to maintain stability and strength of truss system but also to decrease the structure weight, since the weight density of truss members is usually much higher than in cable elements (Liao, 2009). Cable elements are in

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essence non-linear elements which undergo large displacements, in order to take this feature into account nonlinear finite element analysis is performed in order to compute static structural response of cable trusses.

Lightweight design of cable-trusses aims to obtain optimal mass reduction with minimal losses in stiffness, therefore structural optimization is essential. Such task can be divided into sizing, shape and topology optimization, among these, the later yields more material savings and greater complexity (Su, 2009).

Discrete topology optimization approaches are typically based on Evolutionary Algorithms (EAs), which are adaptive methods used for stochastic search and optimization (Hajela, 1995). Among several EAs, Genetic Algorithms (GAs) have been widely used in discrete topology optimization (Rozvany, 2009, Hajela, 1995, Balling, 2006), they consist of an adaptive heuristic search algorithms based on the principles of natural biological evolution. As such, they represent an intelligent exploitation of a random search used to solve optimization problems.

The objective of this work is to provide a topology optimization method for cable-truss structures. Differently than common approaches for discrete topology optimization, the proposed method decides not only the interconnection between nodes, but also if this interconnection is going to be performed by bar or cable. For that, ground structure approach is used, which consists of using a fix grid of nodes and explore the combinations of interconnection between nodes using cable and bar elements.

In addition, genetic algorithm is used for searching for the best solution and nonlinear finite element procedure is applied for computing static structural response. Stiffness-to-mass ratio is adopted as optimization criteria since it relates the stiffness performance in a determined direction and mass, which are both aimed when designing lightweight structures.

Furthermore, differently than previous researches, symmetric structural static response is aimed once the main target is to use cable-trusses for designing lightweight movable machines. Such application differs from civil engineering as forces can assume different directions during working cycle. This behavior demands that structural stiffness cannot rely only in cable elements since changes in force direction can lead to their compression.

Examples with optimal layout of a 2D cantilever beam are presented. The simulations aim to compare the stiffness-to-mass ratio of cable-trusses and trusses for different slenderness ratios. The obtained results indicate that Cable-Trusses reached improvement in all cases.

2. Cable-truss Structures

Cable-truss structures can be described as a system of straight bars and cables joined at their ends from a rigid framework. Similarly to trusses the objective is to transfer applied loads to the supports in the form of axial forces. Although trusses and cable-trusses are actually three-dimensional structures, most can be reduced to planar cases, such approximation is adopted in this work since it reduces computational cost and also brings a deeper insight of the structure dependences.

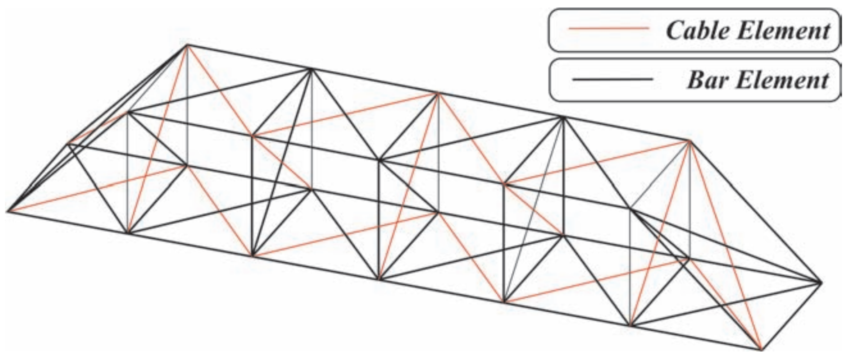


Fig. 1: Cable-Truss Structure.

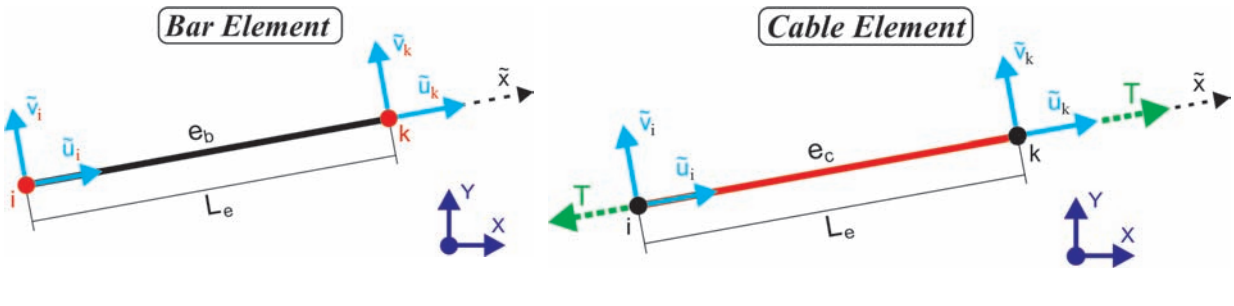
As for trusses, planar cable-trusses idealization is subject to three main assumptions: (1) all external forces are applied at joints; (2) joints are considered frictionless hinges; (3) each element is only subjected to axial stress, which are constant along its length.

Planar cable-truss structures having simple configurations and fewer members can be solved analytically, however for complex cable-truss systems numerical procedures are needed. Structural static response is usually computed by nonlinear finite element iterative procedures, where the numerical model is based on the characteristics of structural members, which can be simulated as bar element (compression-tension) or cable element (tension-only). The formulation of such elements is presented along the following lines.

The stiffness of a bar element, shown in Figure 2, is given by $E_b A_b / L_b$, where E_b is the modulus of elasticity, $A_b$is the cross-sectional area and L_b is the length. This stiffness is projected into two-dimensional space composing a 4x4 element matrix. Considering that the material is linearly elastic and loads are applied at nodes, the elastic stiffness matrix, $[k_b]_e$, for one node of the bar element can be written as:

$$[k_b]_e = \frac{E_b A_b}{L_b} \begin{bmatrix} l^2 & lm \\ lm & m^2 \end{bmatrix}, \tag{1}$$

where, l and m are direction cosine values of the angles between local axis, $\tilde{x}$, and global axes, X and Y, as depicted in Figure 2a. This way, there is no need of coordinates transformation.



(a) (b)

Fig. 2: Bar and Cable Elements

Note that, though, cables exhibit geometrically nonlinear behavior which demands nonlinear analysis as strains are small but displacements are large as a reason of high flexibility. Moreover, cables cannot resist shear, axial pressure forces or bending moments (Nuholgu, 2010, Karoumi, 1999). Therefore, structural analysis of systems having cable elements is relatively complex since linear analysis, where elastic deformations and displacements are assumed to be small, is not often applicable and calculations divergence occurs rather frequently (Nuholgu, 2010).

If the material is linearly elastic material and forces are applied just in nodes, the stiffness matrix for only one node of planar cable element, $[k_c]_e$, can be written as the sum of its elastic and geometrical stiffness matrices, $[k_E]_e$ and $[k_G]_e$, respectively. In order to avoid coordinate transformations the stiffness matrix can also be expressed in terms of direction cosines between local and global axes as shown in Eq.2.

$$[k_c]_e = [k_E]_e + [k_G]_e = \frac{E_c A_c}{L_{c0}} \begin{bmatrix} l^2 & lm \\ lm & m^2 \end{bmatrix} + \frac{T}{L_c} \begin{bmatrix} (1 - l^2) & -lm \\ -lm & (1 - m^2) \end{bmatrix}, \tag{2}$$

where E_c is the cable elasticity modulus, A_c is the cable cross-sectional area, L_{c0} is the initial length of the element and L_c is the current length of the cable element. Note that, T relates to the applied tightening force usually applied to the system in order to increase cable rigidity. Sag effect in cables is taken into account by considering null tension on the element once it is submitted to compression.

The global stiffness matrix, $[K_g]$, of the cable-truss structure is then assembled by combining cable and bar elements for a determined topology, as defined in Eq.3.

$$[K_g] = \sum_{e=1}^{N} [k_i]_e, \tag{3}$$

where $[k_i]_e$ is the stiffness matrix of bar or cable element and N is total number of elements. After the assembly of global stiffness matrix and inclusion of support conditions the global set of equilibrium equations is formulated as:

$$[K_g]\{q\} = \{F_e\} + \{F_T\} = \{F\}, \tag{4}$$

where $\{q\}$ is the displacement vector, $\{F_e\}$ is the external imposed loads, and $\{F_T\}$ is the vector of initial nodal forces, which is computed based on the tightening forces applied to cable elements. It is important noting that the vector of initial nodal forces is design-dependent through the change of element length, positioning and initial strain applied to each element.

The solution of the nonlinear equations is not trivial because the stiffness of cable elements can be affected by its displacement. Thus, algorithms are needed to compute static structural response of the cable-truss structure. The most commonly used method is the so called Newton-Raphson method, which corresponds to an iterative procedure that uses the estimation of the structural response from previous steps, requiring several iterations before the system attains equilibrium. At the beginning of $(i-1)$ steps the imbalance force (residual force), R, can be written as:

$$\{R(\{q\}^{i-1})\} = [K_g]\{q\}^{i-1} - \{F\}, \tag{5}$$

If the imbalance force at the beginning of the i-th step, $\{R(\{q\}^{i})\}$, is expanded in a low order Taylor series, hence:

$$\begin{aligned} \{R(\{q\}^{i})\} &= \{R(\{q\}^{i-1})\} + \left.\frac{\partial R}{\partial \{q\}}\right|_{\{q\}^{i-1}} \delta\{q\}, \\ &= \{R(\{q\}^{i-1})\} + [K_g]_{\{q\}^{i-1}} + \delta\{q\}, \end{aligned} \tag{6}$$

where $\{q\}^{i-1}$ is the displacement vector in the *(i-1)th* step. Setting $\{R(\{q\}^{i})\} = 0$ it is possible to find the increment in displacement created by the imbalance force by solving the system:

$$\delta\{q\} = -\left([K_g]_{\{q\}^{i-1}} \right)^{-1} \left([K_g]\{q\}^{i-1} - \{F\}\right). \tag{7}$$

Nodal displacements are then updated using increments obtained in equation (7):

$$\{q\}^{i} = \{q\}^{i-1} + \delta\{q\}. \tag{8}$$

The above steps are repeated until the ratio of magnitude of the displacement increment vector to the previous displacement is met, which corresponds to the convergence criteria usually adopted. For further details of Newton-Raphson procedures see (Bathe, 1996).

3. Discrete Topology Optimization

The proposed discrete topology optimization method searches for the best interconnectivity between nodes, and also for the elements in each interconnection. For that, ground structure approach is used, in which all possible interconnections are performed based on an initial grid of nodes, as illustrated in Fig.3.

Note that, increasing the amount of nodes in the ground structure sharply increases the number of feasible solutions since more structural elements are used for forming the cable-truss system. Moreover, the nodes from ground structure can be deactivated when they are not used, however, these nodes cannot be moved during the optimization process.

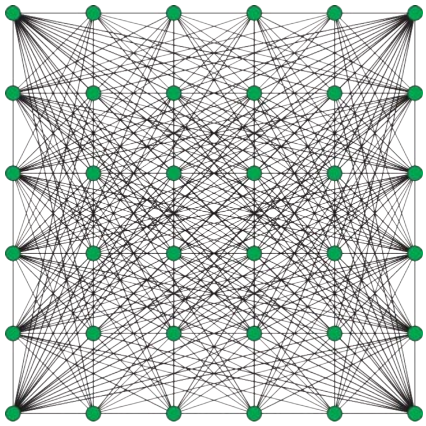
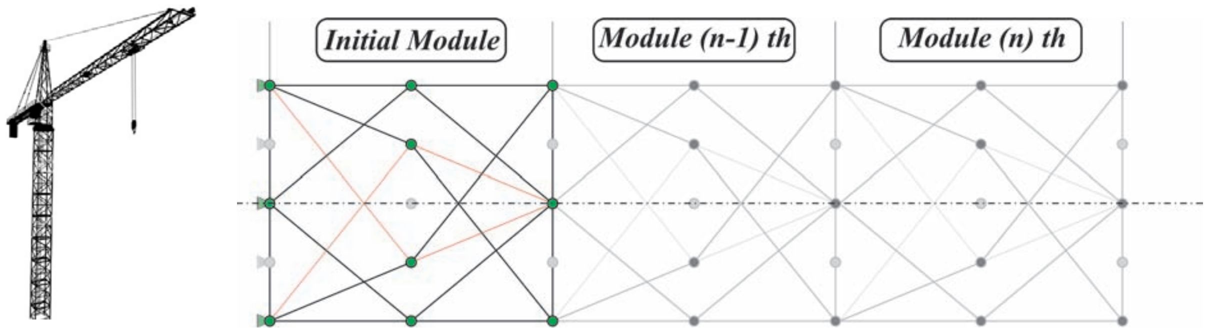


Fig. 3: Planar Gournd Structure

Since the main target in the lightweight design is to obtain optimal mass reduction with minimal losses in stiffness, topology optimization has been adopted in this work. Such method not only presents benefits regarding material saving, but also can be used as a first step in a multi-level optimization process (Carlos, 2000).

To achieve more complex structures, modular design can be used decrease design parameters when modeling cable-trusses. Such approach is commonly used in truss design, e.g., crane structures as illustrated in Fig.4a. Modularity in design consists in patterning basic modules to form more complex structures, as shown in Fig.4b. It can be also noticed that building time and cost is reduced by using a limited number of cross-sections in its structural members.



(a) (b)

Fig. 4: Modular Design in Truss(a) and Cable-truss structures(b).

In order to attend lightweight requirements, the optimization process criteria should take into account the stiffness of the structure as well as its mass. Stiffness analysis can be performed through the definition of the overall system stiffness using the Cartesian stiffness matrix (Carbone, 2010). One of the possible uses of this approach consists in: a) selecting a specific node which characterize the structural behavior, b) loading the structure, and c) evaluating the displacements on the selected node to determine in which direction the structure presents higher stiffness. By this approach, the structural stiffness-to-mass ratio can be written as follows:

$$stm = \left(\frac{F_{jl}}{q_{il}}\right) m_t^{-1}, \tag{9}$$

where F_{jl} is the force applied at node j in the direction l, q_{il} is the displacement of node i in the direction l, and m_t is the total mass of the structure. Such index for stiffness performance has the advantage of having full physical meaning for single loading cases. Nonetheless, it does not take into consideration structural stiffness in different directions, which may lead to structures with reduced isometric stiffness properties.

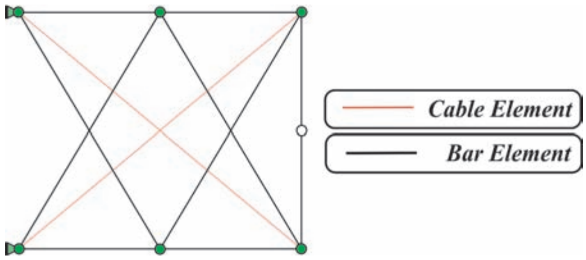


Fig. 5: Kinematically stable cable-truss structure

In addition, during the stochastic search of the optimal topology several kinematically instable or structurally invalid structures appear (Su 2009). Such structures, exemplified in Fig.5, should be filtered in order to increase convergence and decrease computational cost. Consequently, the filter presented in Eq.10, evaluates the degrees of freedom (dof) of the structure, which must be lesser or equal to 0.

$$dof_s = 2n_s - m_s - c_s, \tag{10}$$

where n is the number of nodes being actively used in the structure, m_s is the number of members, and c_s is the number of degrees of freedom. Note that, the subscript s implies that the aforementioned equation refers to each structure being evaluated during the stochastic search instead of the ground structure. In this sense, the optimization problem can be formulated as:

$$\begin{cases} \max \Rightarrow stm_s = (stiffness - to - mass\ ratio) \\ \qquad\qquad s.t. \\ dof_s \leq\ 0\ (Kinematical\ Stability\ Constraints) \end{cases}. \tag{11}$$

4. Genetic Algorithms

Genetic Algorithms (GAs) are adaptive methods which can be used for searching and optimization problems, performing a stochastic search and optimization. Compared to traditional optimization methods, such as calculus-based and enumerative strategies, GAs are robust, global, and may be applied generally without recourse to domain-specific heuristics. Although performance is affected by these heuristics, EAs operate on a population of potential solutions, applying the principle of survival of the fittest to produce successively better approximations to a solution (Mitchell, 1996, Han, 2002).

Individuals in a population compete for resources and mates. Those individuals most successful in each competition, e.g., the structures with higher stiffness to mass ratio, will produce more offspring than those that performed poorly. Genes from fittest individuals propagate throughout the population so that two good parents will sometimes produce offspring that are better than either parent. As a consequence, each successive generation will become more suited to their environment. To sum up, it can be said that GAs aim to use selective breeding of the solutions to produce offspring better than the parents by combining information from the chromosomes (Mitchell, 1996). In addition, GAs uses highly customizable genetic operators; selection, cross over and mutation, to perform the search for the optimal solution.

Traditional binary-vector encoding is commonly used for discrete topology optimization of trusses (Hajela, 1995, Balling, 2006). In order to encode cable-truss structures element properties have been encoded into integer numbers (0, 1 and 2), where 0 represents disconnection between two nodes, 1 and 2 relates to bar and cable element, accordingly.The GA with Nonlinear finite element solver used in this work was programmed in MATLAB. The logic of the program flows as shown in Fig.6 and in the procedure below:

[1.Start]: User provides number of nodes used, slenderness ratio of the structure, applied loads, and geometrical information about bars and cables.

[2.Generation of Ground Structure]: Symmetric ground structure is build using nodal matrix provided by user.

[3.Initial population]: Generate random population of n chromosomes (suitable solutions for the problem).

[4.Stability and Validity Checking]: Eq.10 is used to determine kinematically instable structures, which are repaired and reinserted into the population. The repair operation consists in stochastically adding bars to the original structure until stability is achieved.
[5.Decoding] : Population is decoded into bars, cables and disconnections.

[6.Nonlinear Structural Analysis]: Newton-Raphson procedure, see section 2, is used for computing static structural response and stiffness-to-mass ratio for each individual of the population.

[7.Evaluation] If the best solution of the population is better than the previous overall solution found by the algorithm, then the overall solution is updated. Otherwise, the worst individual of the current population is substitute by previous best solution.

[8.Test] If the end condition is satisfied, go to step [11]. If not, go to next step.

[9.New population] Create a new population by repeating following steps until the new population is complete:

> **[Selection]** Select two parent chromosomes from a population according to their fitness (the better fitness, the bigger chance to be selected).
>
> **[Crossover]** With a crossover probability of 0.9, cross over the parents to form a new offspring (children).
>
> **[Mutation]** With a mutation probability of 0.1, mutate new offspring at each locus (position in chromosome).

[10.Loop] Return to step [4].

[11.End] Best structure is stored.

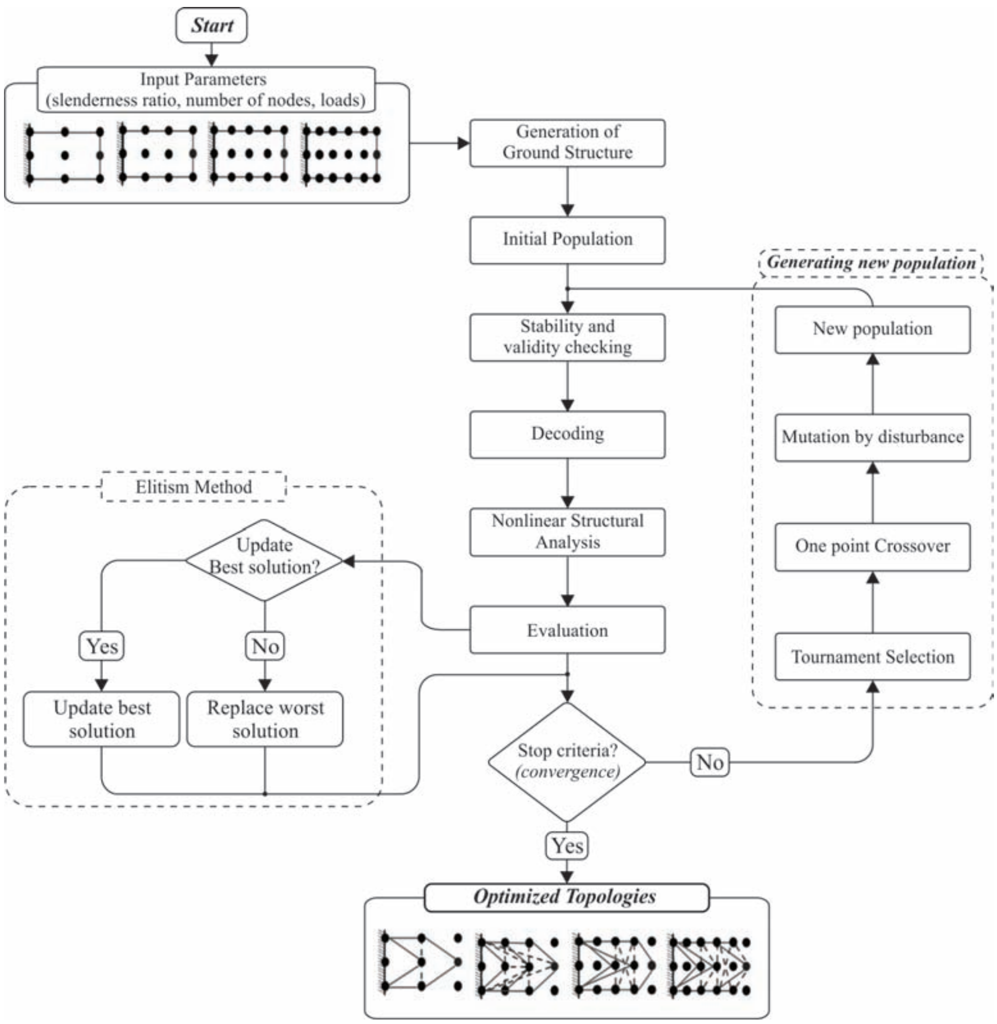


Fig. 6: GA applied to topology optimization of Cable-Trusses

5. Study Case - Cantilever Optimization

The design of optimal planar cable-trusses comprises the choice of several design parameters, such as, the ratio between cable and bar cross section areas, pre-stress, slenderness ratio (sl) of the structure, materials, among others. Discrete topology optimization was performed for a robotic arm which is approximated by a horizontal cantilever beam, an additional constraint was used for maintaining a minimum set of nodes which represents the structure. After obtaining the optimal topology, the structure is analyzed using different number of modules increasing the slenderness ratio. In all configuration the structure is submitted to a punctual load of 500N in the mid of the end of the last column of nodes, as shown in Fig.7.

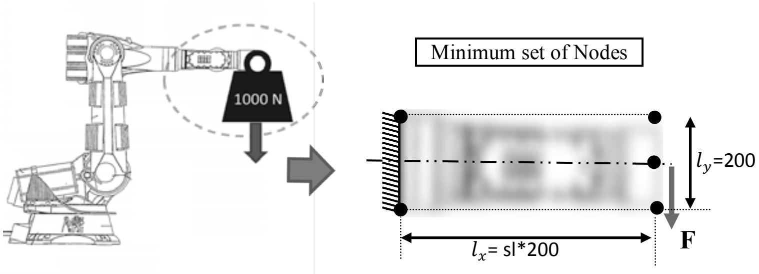
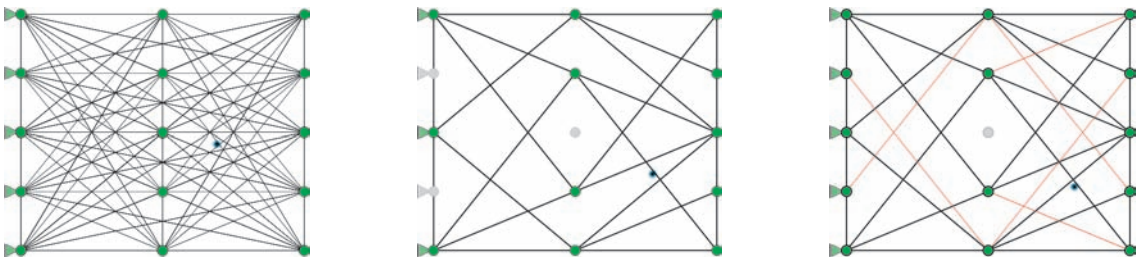


Fig. 7: Study case boundary conditions

The main input parameters for the algorithm are the cross section areas bars and cables which are $700\ mm^2$ and $40\ mm^2$ respectively, the strain on cables have been considered as 5.2e0-5, the population for simulations has been selected as 80 and the number of iterations as 5000, each simulation has been repeated 3 times and best results were selected for each structure.

The simulations compares the results in terms of stiffness-to-mass ratio for Cable-Truss and Truss structures under the same environment and boundary conditions, the target is to analyze which structure provides higher stiffness to mass ratio. Using a ground structure with fifteen nodes, as shown in Fig.8a., the method proposed searched for maximum stiffness-to-mass ratio. Same environment and boundary conditions are used for optimizing truss and cable-truss structures. Results of the topology optimization are shown in Fig.8.

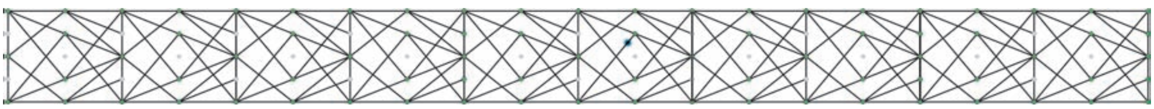


(a) (b) (c)

Fig.8: Optimal Structures obtained by discrete topology optimization

Structures obtained during topology optimization were used as initial modules to build more complex structures. It is important noting that by increasing the number of modules the slenderness ratio of the structure increases in the same proportion. Structures containing from 1 to 10 modules, where Fig. 9 depicts structures using 10 modules, were analyzed and results are shown in table 1.

(a)



(b)

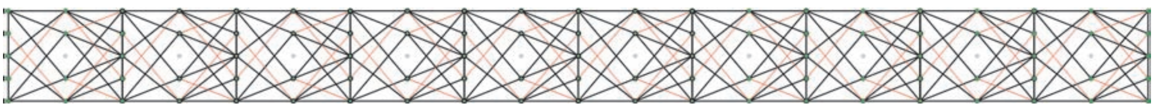


Fig. 9: a) Truss Structure using10 Modules b) Cable-truss Structure using10 Modules

Tab. 1: Stiffness to mass ratio of Optimized Structures.

	Truss (S-t-m)	Cable truss (S-t-m)
Sl=1	44996.79966	48149.45852
Sl=2	4835.850861	5193.808638
Sl=3	1019.162564	1103.527095
Sl=4	322.5478356	351.0791844
Sl=5	130.7810331	142.8265765
Sl=6	62.38712922	68.28822800
Sl=7	33.35000572	36.56401106
Sl=8	19.38779245	21.28215362
Sl=9	12.01891870	13.20573900
Sl=10	7.838589176	8.619098703

As it can be seen on the simulation results table, Cable-Truss structures presented higher stiffness-to-mass ratio in all cases, presenting average improvement 8.9% when compared to optimized Truss structures, as shown in Fig.10.

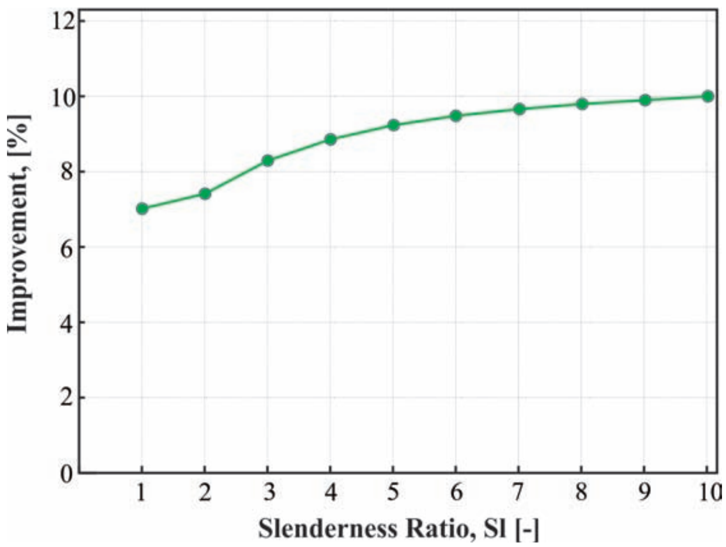


Fig. 10: Improvement in S-t-m of Cable-Trusses in comparison with Trusses

Furthermore, the length of the vector which encodes individuals increases quickly with the growth problems scale, leading the GAs to have troubles in convergence. Such problem is potentialized when analyzing cable-trusses since: a) the number of possible elements to interconnect to nodes increases the number of feasible solutions, b) cables cannot resist compression leading thus to a sharp increase in the number of kinematically instable structures during the stochastic search, and c) the evaluation of Eq.4 requires the use of iterative solution, which is computationally costly since inversion of the stiffness matrix must be performed several times.

6. Conclusions

In this article, a methodology for topology optimization of cable-trusses was presented. By combining Genetic Algorithm and Nonlinear Finite Element Method optimized cable-trusses were found. Such structures where obtained for an initial module, which was then patterned in order to form more complex structures.

Comparisons between optimized symmetric truss and cable-truss structures were performed for different number of modules. Results indicated that, in all cases, the stiffness-to-mass ratio of cable-trusses was higher than those obtained for trusses. Moreover, optimized symmetric cable-trusses have shown an average improvement of 8.9% when compared to optimized symmetric truss structures.

For further maximization of the stiffness-to-mass ratio complementary analysis are recommended. These include, for instance, optimization of cable-trusses considering the use of different modules as the slenderness ratio increases. As result, such study shall bring higher stiffness-to-mass ratio since the topology of the initial module may change as more modules are being used.

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