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# THE INVERSE KINEMATICS OF N-SERIAL ROBOTIC CHAIN IN DIFFERENT QUADRANTS OF WORKING SPACE

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**Abstract:** *Inverse kinematics of mechanic system is area of solution at many universities and scientific research institutions. This paper dealt with of solution problem of inverse kinematic by cyclic coordinate descent method (CCD). This method allows use n-link chain mechanism without complicated changing of control algorithm. The article is focuses on the issue of setting the number of elements of mechanism. Also presents a modified method the incremental CCD, which is compared with this method. From the perspective of number of cycles required to achieve the desired position with increasing number of links and from the perspective of changing quadrant of working space.*

**Keywords:** ***Inverse kinematic, n-serial, robotic chain.***

## 1. Inverse kinematic problem

Inverse kinematic problem can be described as „finding" the right parameters for each kinematic joint of mechanism to achieve the desired result in a pre-defined position of member. The available literature provides several definitions for this problem. In defined shape of mechanism, fig. 1, we know where its base is. With the control system and sensors know the current mechanism shape and position of the end point of the mechanism. If we define a new position in which to get the mechanism end point, for the computation algorithm, the actual end point position is starting position and its task is to calculate values for individual joints q1 to $q_n$, so the end point has the desired position with reasoning error limit (Wang, 1991).

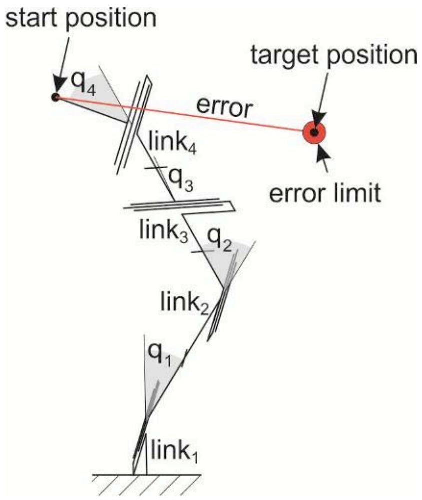


*Fig. 1: Problem of inverse kinematic solution*

There are several computation methods that are described in available literature. Whatever the method using Jacobian and their subsequent modifications (Wang, 1991; Mostýn, & Skařupa, 2000; Skařupa & Mostýn, 2002) or other methods (Mukundan, 2008), the biggest problem with these methods arises with the redundant manipulators, when the desired position is possible to achieve by the infinity combinations of $q_i$ values.

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## 2. Cyclic calculation

This method allows computing the inverse kinematics of redundant mechanisms with principle of gradual calculation parameters for each kinematics joint separately.

### 2.1 Cyclic coordinate descent

In this method there is gradual calculation for each kinematic joint separately from the last joint in a series of mechanism to the first. This cycle is repeated until it reaches desired position of the end point of mechanism respectively until value of error isn't smaller then error limit. The q value is a calculated difference between actual angle and desired angle. This q value can be limited by the maximum value of step size. The algorithm of calculation in basic form for one "finding" point with defined max step size and without optimization algorithm is shown (Bobovský, et al., 2011; Trebuňa & Bobovský, 2011).

### 2.2 Incremental cyclic coordinate descent

This method allows with last link "follow" desired point. Procedure of computation is shown (Bobovský, et al., 2011; Trebuňa & Bobovský, 2011). The algorithm is for one "finding" point with defined a max step size and without an optimized algorithms.

## 3. Number of cycles

Between a main criteria for choose the right inverse kinematics algorithm belongs a number of calculation cycles to achieve desired task and a time of computation. Computation times are changed depending on the type of computer on which the calculation is performed. Therefore, we only present the number of cycles required to achieve the desired position. The step size of the computation method CCD is 1° because of the twisting mechanism at a higher step value (Bobovský, et al. (2011)). The step size of method iCCD is 90°, when mechanism moves with a defined path between two points or without a defined path and the maximum possible number of cycles is n*100 for one point. Path is divided into parts of unit length.

Starting position of end point is $x_p = n$ a $y_p = 0$ and desired position of the mechanism is defined by fig. 2. The total number of 44 points distributed throughout the mechanism working space.

Link n has unit length and error limit is set to 0,01. The computation was made in application programmed in MATLAB©.

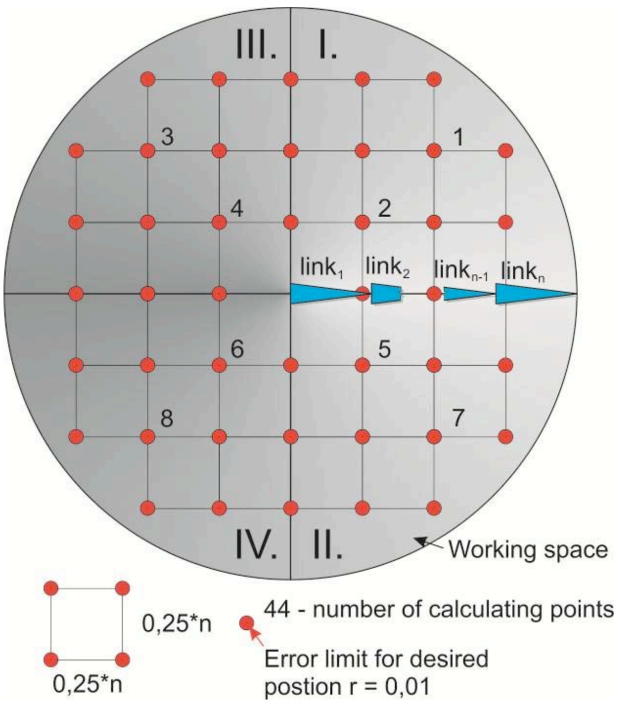



*Fig 2. Position of desired points*

### 3.1 Moving at defined path

When mechanism moves with defined path of end point for the total number of 44 required points, we obtained data on which we created the graphs, that shows the number of cycles required to achieve the desired point and the error from the desired point when reaching limits or error when reaching the maximum number of cycles for a given number of degrees of freedom (DOF). Fig. 3 shows the characteristics for point 1, shown in fig. 2. Based on the graph, for increasing number of DOF to achieve point 1, the CCD method leads to a sharp increase in required number of cycles. While the method iCCD lead to a linear increase in the required number of cycles. The positioning error in reaching the desired position leads to an increasing in the number of cycles of links in mechanism that don't reaches the desired position for n *100 cycles for one point of path.

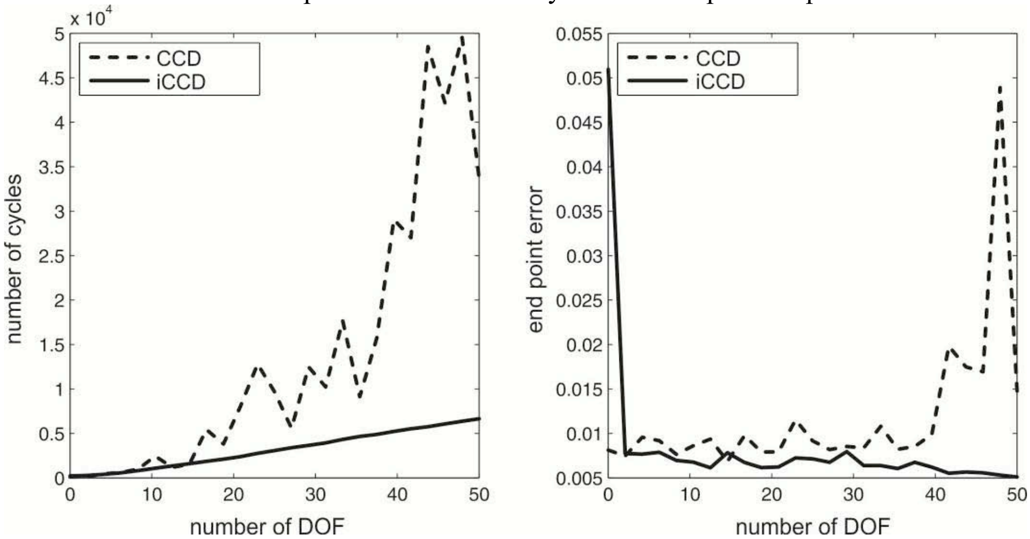



*Fig. 3 Comparison of CCD and iCCD method in achieving first point with defined path (a – number of cycles, b – error at end of calculation)*

Fig. 4 shows the characteristics for point 2. shown in fig. 2. Based on the graph, for increasing number of DOF, CCD method leads to faster achieving the desired point 2. While at the iCCD method there is a double number of cycles. The positioning error in reaching the desired position leads to an increasing in the number of cycles of links in mechanism that don't reaches the desired position for n *100 cycles for one point of path.

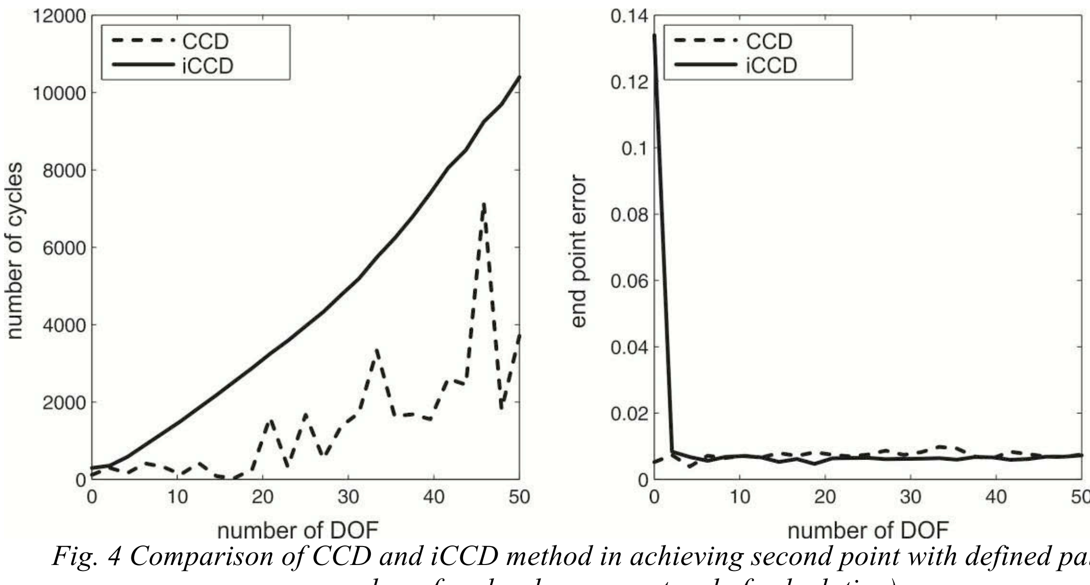



*Fig. 4 Comparison of CCD and iCCD method in achieving second point with defined path (a – number of cycles, b – error at end of calculation)*

### 3.2 Moving without defined path

When mechanism moves without defined path of end point for the total number of 44 required points, we obtained data on which we created the graphs, that shows the number of cycles required to achieve the desired point and the error from the desired point when reaching limits or error when reaching the maximum number of cycles for a given number of degrees of freedom (DOF). Fig. 5 shows the characteristics for point 1, shown in fig. 2. Based on the graph, for increasing number of DOF to achieve point 1, the CCD method leads to a faster achieving the desired point. But with higher DOF the mechanism don’t achieve the desired point. While the method iCCD lead to a linear increase in the required number of cycles. The positioning error in reaching the desired position leads to an increasing in the number of cycles of links in mechanism that don’t reaches the desired position for n *100 cycles for one point of path.

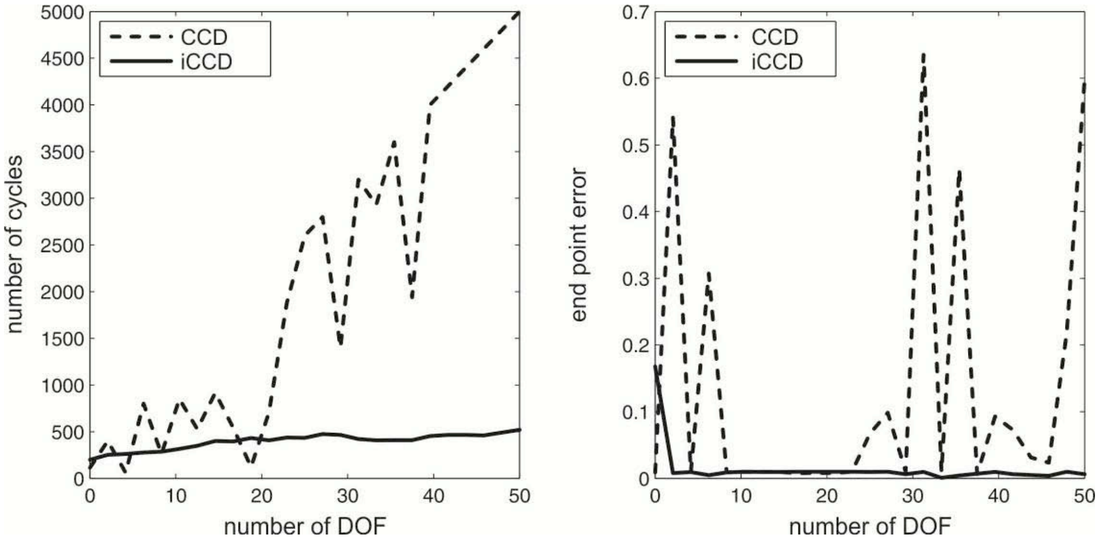



*Fig. 5 Comparison of CCD and iCCD method in achieving first point without defined path (a – number of cycles, b – error at end of calculation)*

Fig. 4 shows the characteristics for point 2. shown in fig. 2. Based on the graph, for increasing number of DOF, CCD method leads to faster achieving the desired point 2. While at the iCCD method there is an increasing number of cycles to 20%. The positioning error in reaching the desired position leads to an increasing in the number of cycles of links in mechanism that don’t reaches the desired position for n *100 cycles for one point of path.

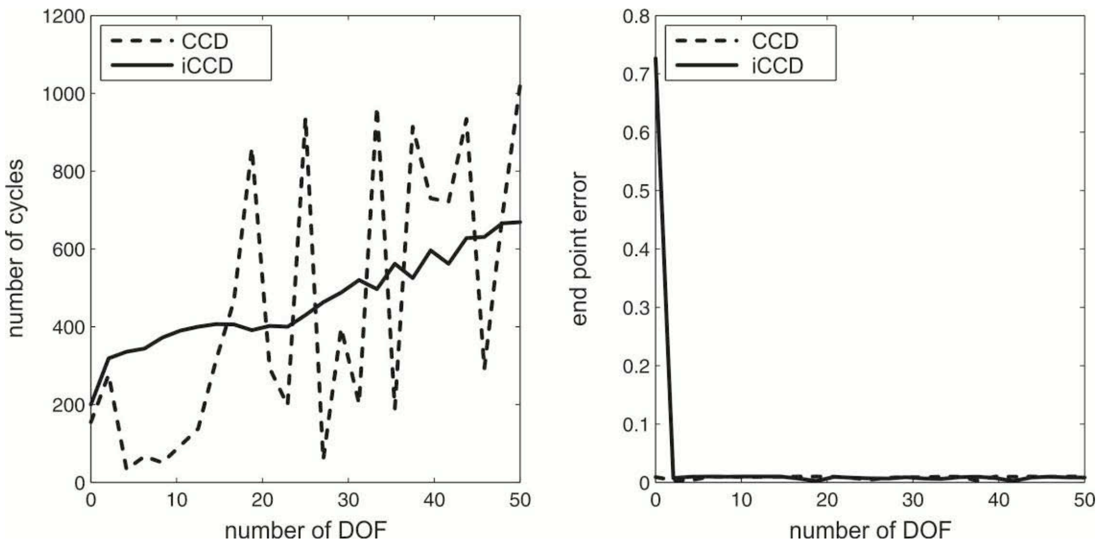


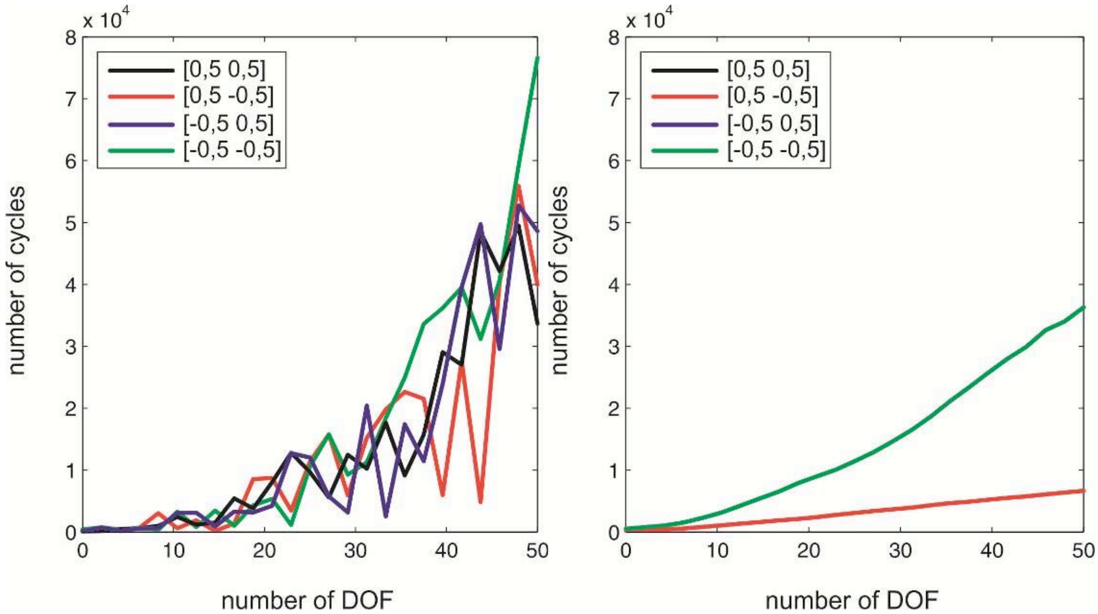

*Fig. 6 Comparison of CCD and iCCD method in achieving second point without defined path (a – number of cycles, b – error at end of calculation)*

## 4. Comparison

The following section compares the results for the desired points located in different quadrants. The goal is to show how changing the number of cycles required to achieve those points, depending on quadrant. In first case are compared points 1, 3, 5 and 7 shown in fig. 2 and in second case 2, 4, 5 and 6 also shown in fig. 2.

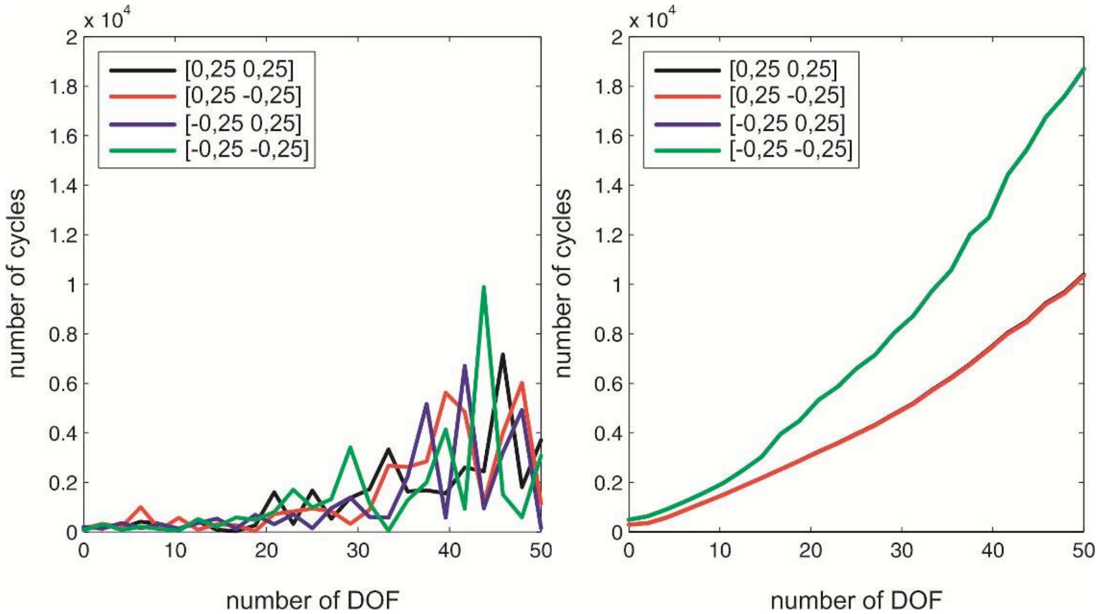
### 4.1 Moving at defined path

When moves on a defined path, which is divides into unit length. The method CCD occurs in different quadrants for different characteristic of graphs, fig. 7a) and fig. 8a). While characteristics at method iCCD have same behavior for quadrant I. and II., and the same for III. and IV. quadrant, fig. 7b) and fig. 8b). The same thing happened in both compared cases.




a) b)

*Fig. 7 Number of cycles required to achieve desired position with defined path for points 1, 3, 7 and 8 (a – CCD, b – iCCD)*

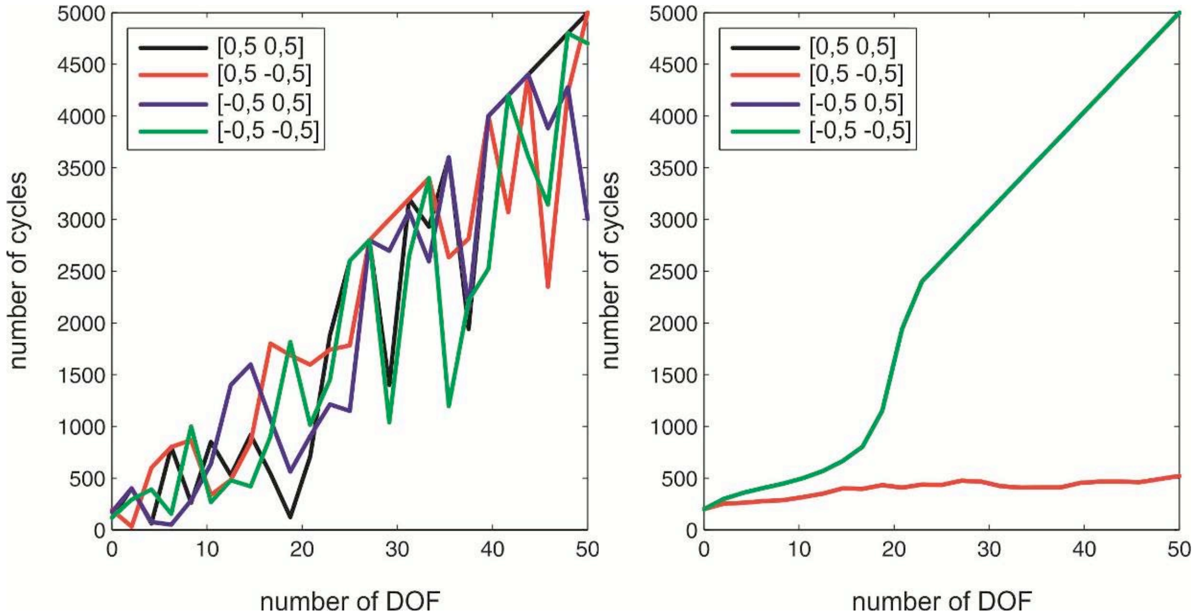



a) b)

*Fig. 8 Number of cycles required to achieve desired position with defined path for points 2, 4, 5 and 6 (a – CCD, b – iCCD)*

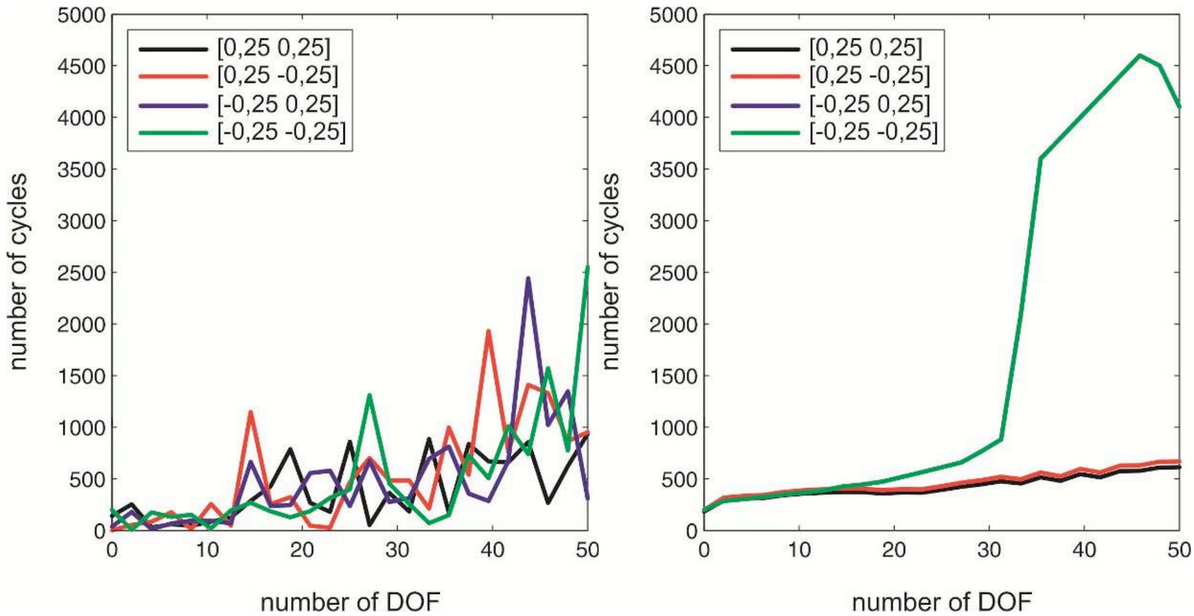
### 4.2 Moving without a defined path

When moves without a defined path. The method CCD occurs in different quadrants for different characteristic of graphs, fig. 9a) and fig. 10a). Limiting factor is the maximum number of cycles. While the method iCCD in the first case, fig. 9b) have same behavior for quadrant I. and II., and the same for III. and IV. quadrant. In second case, fig. 10) occurs for I. and II. quadrant of the subtle difference between characteristics. For III. And IV. quadrants are the characteristics same.




*a)* *b)*

*Fig. 9 Number of cycles required to achieve desired point without defined path for points 1, 3, 7 and 8 (a – CCD, b – iCCD)*




*a)* *b)*

*Fig. 10 Number of cycles required to achieve desired point without defined path for points 2, 4, 5 and 6 (a – CCD, b – iCCD)*

## 5. Conclusions

The inverse kinematics of mechanical systems is a complex issue. To date, there are several methods for its solution. Not all are applicable to mechanisms with more degrees of a freedom because of the complex and the subsequent compilation of Jacobian and his numerical complexity. This article presents a comparison of the CCD method and its modified version incremental CCD in terms of a cycle number for different quadrants of mechanism working space.

When comparing these methods the results obtained we conclude that the calculation using method of iCCD is convenient at "finding" a single position of moving at the defined path if the desired point is close to actual position of the end point. Method of CCD is useful for distance points. To achieving the final position is to perform additional calculations at higher density of points within the mechanism working space.

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